

Planning and Optimization

C2. Delete Relaxation: Properties of Relaxed Planning Tasks

Malte Helmert and Gabriele Röger

Universität Basel

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— C2. Delete Relaxation: Properties of Relaxed Planning Tasks

C2.1 The Domination Lemma

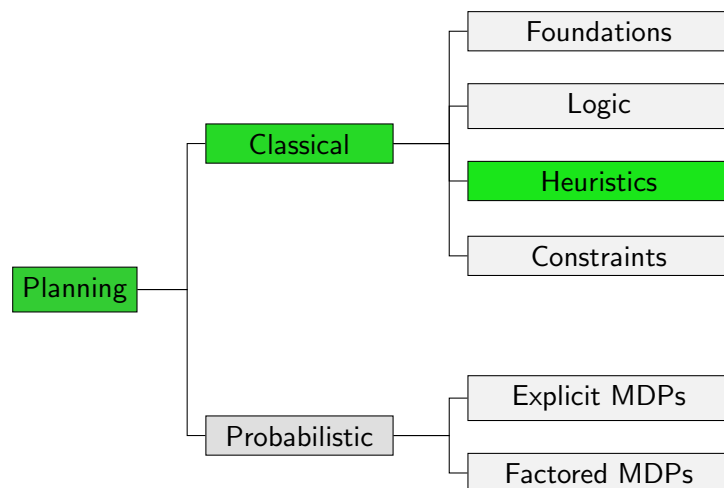
C2.2 The Relaxation Lemma

C2.3 Further Properties

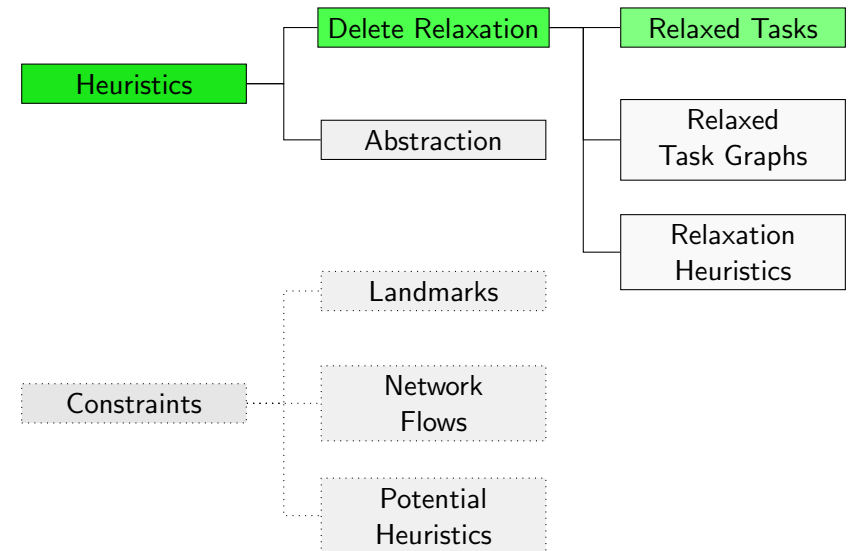
C2.4 Greedy Algorithm

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Content of this Course



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C2.1 The Domination Lemma

On-Set and Dominating States

Definition (On-Set)

The **on-set** of a valuation s is the set of propositional variables that are true in s , i.e., $on(s) = s^{-1}(\{\mathbf{T}\})$.

↔ for **states** of propositional planning tasks:
states can be viewed as **sets** of (true) state variables

Definition (Dominate)

A valuation s' **dominates** a valuation s if $on(s) \subseteq on(s')$.

↔ all state variables true in s are also true in s'

Domination Lemma (1)

Lemma (Domination)

Let s and s' be valuations of a set of propositional variables V , and let χ be a propositional formula over V which does not contain negation symbols.

If $s \models \chi$ and s' dominates s , then $s' \models \chi$.

Proof.

Proof by induction over the structure of χ .

- ▶ Base case $\chi = \top$: then $s' \models \top$.
- ▶ Base case $\chi = \perp$: then $s \not\models \perp$.

...

Domination Lemma (2)

Proof (continued).

- ▶ **Base case** $\chi = v \in V$: if $s \models v$, then $v \in on(s)$.
With $on(s) \subseteq on(s')$, we get $v \in on(s')$ and hence $s' \models v$.
- ▶ **Inductive case** $\chi = \chi_1 \wedge \chi_2$: by induction hypothesis, our claim holds for the proper subformulas χ_1 and χ_2 of χ .

$$\begin{aligned}
 s \models \chi &\implies s \models \chi_1 \wedge \chi_2 \\
 &\implies s \models \chi_1 \text{ and } s \models \chi_2 \\
 &\stackrel{\text{I.H. (twice)}}{\implies} s' \models \chi_1 \text{ and } s' \models \chi_2 \\
 &\implies s' \models \chi_1 \wedge \chi_2 \\
 &\implies s' \models \chi.
 \end{aligned}$$

- ▶ **Inductive case** $\chi = \chi_1 \vee \chi_2$: analogous

□

C2.2 The Relaxation Lemma

Add Sets and Delete Sets

Definition (Add Set and Delete Set for an Effect)

Consider a propositional planning task with state variables V . Let e be an effect over V , and let s be a state over V . The **add set** of e in s , written $addset(e, s)$, and the **delete set** of e in s , written $delset(e, s)$, are defined as the following sets of state variables:

$$addset(e, s) = \{v \in V \mid s \models effcond(v, e)\}$$

$$delset(e, s) = \{v \in V \mid s \models effcond(\neg v, e)\}$$

Note: For all states s and operators o applicable in s , we have $on(s[o]) = (on(s) \setminus delset(eff(o), s)) \cup addset(eff(o), s)$.

Relaxation Lemma

For this and the following chapters on delete relaxation, we assume implicitly that we are working with **propositional planning tasks in positive normal form**.

Lemma (Relaxation)

Let s be a state, and let s' be a state that dominates s .

- 1 If o is an operator applicable in s , then o^+ is applicable in s' and $s'[o^+]$ dominates $s[o]$.
- 2 If π is an operator sequence applicable in s , then π^+ is applicable in s' and $s'[\pi^+]$ dominates $s[\pi]$.
- 3 If additionally π leads to a goal state from state s , then π^+ leads to a goal state from state s' .

Proof of Relaxation Lemma (1)

Proof.

Let V be the set of state variables.

Part 1: Because o is applicable in s , we have $s \models pre(o)$.

Because $pre(o)$ is negation-free and s' dominates s , we get $s' \models pre(o)$ from the domination lemma.

Because $pre(o^+) = pre(o)$, this shows that o^+ is applicable in s' .

...

Proof of Relaxation Lemma (2)

Proof (continued).

To prove that $s' \llbracket o^+ \rrbracket$ dominates $s \llbracket o \rrbracket$,
we first compare the relevant add sets:

$$\begin{aligned} \text{addset}(\text{eff}(o), s) &= \{v \in V \mid s \models \text{effcond}(v, \text{eff}(o))\} \\ &= \{v \in V \mid s \models \text{effcond}(v, \text{eff}(o^+))\} \quad (1) \end{aligned}$$

$$\begin{aligned} &\subseteq \{v \in V \mid s' \models \text{effcond}(v, \text{eff}(o^+))\} \quad (2) \\ &= \text{addset}(\text{eff}(o^+), s'), \end{aligned}$$

where (1) uses $\text{effcond}(v, \text{eff}(o)) \equiv \text{effcond}(v, \text{eff}(o^+))$
and (2) uses the dominance lemma (note that effect conditions
are negation-free for operators in positive normal form). ...

Proof of Relaxation Lemma (3)

Proof (continued).

We then get:

$$\begin{aligned} \text{on}(s \llbracket o \rrbracket) &= (\text{on}(s) \setminus \text{delset}(\text{eff}(o), s)) \cup \text{addset}(\text{eff}(o), s) \\ &\subseteq \text{on}(s) \cup \text{addset}(\text{eff}(o), s) \\ &\subseteq \text{on}(s') \cup \text{addset}(\text{eff}(o^+), s') \\ &= \text{on}(s' \llbracket o^+ \rrbracket), \end{aligned}$$

and thus $s' \llbracket o^+ \rrbracket$ dominates $s \llbracket o \rrbracket$.

This concludes the proof of Part 1. ...

Proof of Relaxation Lemma (4)

Proof (continued).

Part 2: by induction over $n = |\pi|$

Base case: $\pi = \langle \rangle$

The empty plan is trivially applicable in s' , and
 $s' \llbracket \langle \rangle^+ \rrbracket = s'$ dominates $s \llbracket \langle \rangle \rrbracket = s$ by prerequisite.

Inductive case: $\pi = \langle o_1, \dots, o_{n+1} \rangle$

By the induction hypothesis, $\langle o_1^+, \dots, o_n^+ \rangle$ is applicable in s' ,
and $t' = s' \llbracket \langle o_1^+, \dots, o_n^+ \rangle \rrbracket$ dominates $t = s \llbracket \langle o_1, \dots, o_n \rangle \rrbracket$.

Also, o_{n+1} is applicable in t .

Using Part 1, o_{n+1}^+ is applicable in t' and $s' \llbracket \pi^+ \rrbracket = t' \llbracket o_{n+1}^+ \rrbracket$
dominates $s \llbracket \pi \rrbracket = t \llbracket o_{n+1} \rrbracket$.

This concludes the proof of Part 2. ...

Proof of Relaxation Lemma (5)

Proof (continued).

Part 3: Let γ be the goal formula.

From Part 2, we obtain that $t' = s' \llbracket \pi^+ \rrbracket$ dominates $t = s \llbracket \pi \rrbracket$.
By prerequisite, t is a goal state and hence $t \models \gamma$.

Because the task is in positive normal form, γ is negation-free,
and hence $t' \models \gamma$ because of the domination lemma.

Therefore, t' is a goal state. \square

C2.3 Further Properties

Further Properties of Delete Relaxation

- ▶ The relaxation lemma is the main technical result that we will use to study delete relaxation.
- ▶ Next, we derive some further properties of delete relaxation that will be useful for us.
- ▶ Two of these are direct consequences of the relaxation lemma.

Consequences of the Relaxation Lemma (1)

Corollary (Relaxation Preserves Plans and Leads to Dominance)

Let π be an operator sequence that is applicable in state s .

Then π^+ is applicable in s and $s[\pi^+]$ dominates $s[\pi]$.

If π is a plan for Π , then π^+ is a plan for Π^+ .

Proof.

Apply relaxation lemma with $s' = s$. □

- ↪ Relaxations of plans are relaxed plans.
- ↪ Delete relaxation is no harder to solve than original task.
- ↪ Optimal relaxed plans are never more expensive than optimal plans for original tasks.

Consequences of the Relaxation Lemma (2)

Corollary (Relaxation Preserves Dominance)

Let s be a state, let s' be a state that dominates s ,

and let π^+ be a relaxed operator sequence applicable in s .

Then π^+ is applicable in s' and $s'[\pi^+]$ dominates $s[\pi^+]$.

Proof.

Apply relaxation lemma with π^+ for π , noting that $(\pi^+)^+ = \pi^+$. □

- ↪ If there is a relaxed plan starting from state s , the same plan can be used starting from a dominating state s' .
- ↪ Dominating states are always “better” in relaxed tasks.

Monotonicity of Relaxed Planning Tasks

Lemma (Monotonicity)

Let s be a state in which relaxed operator o^+ is applicable.
Then $s[[o^+]]$ dominates s .

Proof.

Since relaxed operators only have positive effects, we have $on(s) \subseteq on(s) \cup addset(eff(o^+), s) = on(s[[o^+]])$. \square

\rightsquigarrow Together with our previous results, this means that making a transition in a relaxed planning task **never** hurts.

Finding Relaxed Plans

Using the theory we developed, we are now ready to study the problem of **finding plans** for **relaxed planning tasks**.

C2.4 Greedy Algorithm

Greedy Algorithm for Relaxed Planning Tasks

The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

Greedy Planning Algorithm for $\langle V, I, O^+, \gamma \rangle$

$s := I$

$\pi^+ := \langle \rangle$

loop forever:

if $s \models \gamma$:

return π^+

else if there is an operator $o^+ \in O^+$ applicable in s

 with $s[[o^+]] \neq s$:

 Append such an operator o^+ to π^+ .

$s := s[[o^+]]$

else:

return unsolvable

Correctness of the Greedy Algorithm

The algorithm is **sound**:

- ▶ If it returns a plan, this is indeed a correct solution.
- ▶ If it returns “unsolvable”, the task is indeed unsolvable
 - ▶ Upon termination, there clearly is no relaxed plan from s .
 - ▶ By iterated application of the monotonicity lemma, s dominates l .
 - ▶ By the relaxation lemma, there is no solution from l .

What about **completeness** (termination) and **runtime**?

- ▶ Each iteration of the loop adds at least one atom to $on(s)$.
- ▶ This guarantees termination after at most $|V|$ iterations.
- ▶ Thus, the algorithm can clearly be implemented to run in polynomial time.
 - ▶ A good implementation runs in $O(|\Pi|)$.

Using the Greedy Algorithm as a Heuristic

We can apply the greedy algorithm within heuristic search:

- ▶ When evaluating a state s in progression search, solve relaxation of planning task with initial state s .
- ▶ When evaluating a subgoal φ in regression search, solve relaxation of planning task with goal φ .
- ▶ Set $h(s)$ to the cost of the generated relaxed plan.

Is this an **admissible** heuristic?

- ▶ Yes if the relaxed plans are **optimal** (due to the plan preservation corollary).
- ▶ However, usually they are not, because our greedy relaxed planning algorithm is very poor.

(What about safety? Goal-awareness? Consistency?)

C2.5 Summary

Summary

- ▶ Delete relaxation is a **simplification** in the sense that it is never harder to solve a relaxed task than the original one.
- ▶ Delete-relaxed tasks have a **domination** property: it is always beneficial to make more state variables true.
- ▶ Because of their **monotonicity** property, delete-relaxed tasks can be solved in polynomial time by a greedy algorithm.
- ▶ However, the solution quality of this algorithm is poor.