

Planning and Optimization

A6. Positive Normal Form, STRIPS and SAS⁺

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September 28, 2020

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A6.1 Motivation

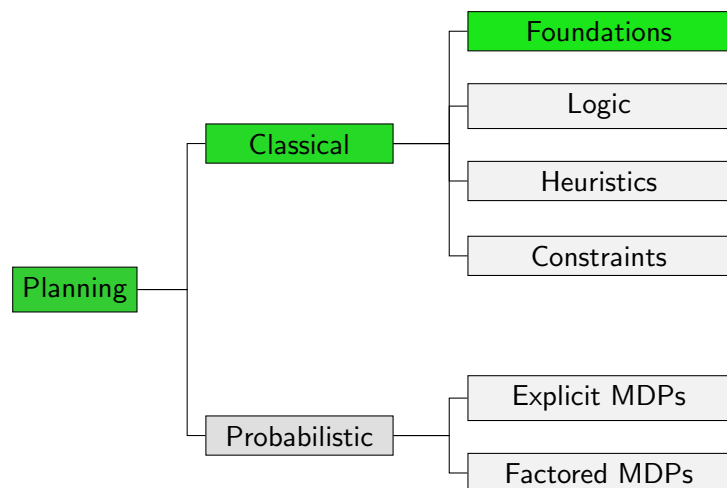
A6.2 Positive Normal Form

A6.3 STRIPS

A6.4 SAS⁺

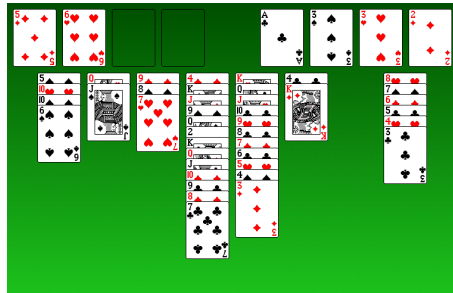
A6.5 Summary

Content of this Course



A6.1 Motivation

Example: Freecell



Example (Good and Bad Effects)

If we move $K♦$ to a free tableau position,

the **good effect** is that $4♣$ is now accessible.

The **bad effect** is that we lose one free tableau position.

What is a Good or Bad Effect?

Question: Which operator effects are good, and which are bad?

Difficult to answer in general, because it depends on context:

- ▶ Locking our door is **good** if we want to keep burglars out.
- ▶ Locking our door is **bad** if we want to enter.

We now consider a reformulation of propositional planning tasks that makes the distinction between good and bad effects obvious.

A6.2 Positive Normal Form

Positive Formulas, Operators and Tasks

Definition (Positive Formula)

A logical formula φ is **positive** if no negation symbols appear in φ .

Note: This includes the negation symbols implied by \rightarrow and \leftrightarrow .

Definition (Positive Operator)

An operator o is **positive** if $pre(o)$ and all effect conditions in $eff(o)$ are positive.

Definition (Positive Propositional Planning Task)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is **positive** if all operators in O and the goal γ are positive.

Positive Normal Form

Definition (Positive Normal Form)

A propositional planning task is in **positive normal form** if it is positive and all operator effects are flat.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
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 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$

Identify state variable v occurring negatively in conditions.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$

Introduce new variable \hat{v} with complementary initial value.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$
Identify effects on variable v .

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \wedge \neg bike-unlocked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$
Introduce complementary effects for \hat{v} .

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \wedge \neg bike-unlocked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$
Identify negative conditions for v .

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge bike-unlocked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\
 &\quad \langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge bike-unlocked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$
Replace by positive condition \hat{v} .

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$$

$$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\ uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\}$$

$$O = \{\langle home \wedge bike \wedge bike-unlocked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\ \langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge bike-unlocked) \triangleright \neg bike) \rangle\}$$

$$\gamma = lecture \wedge bike$$

Positive Normal Form: Existence

Theorem (Positive Normal Form)

For every propositional planning task Π , there is an equivalent propositional planning task Π' in positive normal form. Moreover, Π' can be computed from Π in polynomial time.

Note: Equivalence here means that the transition systems induced by Π and Π' , **restricted to the reachable states**, are isomorphic.

We prove the theorem by describing a suitable algorithm. (However, we do not prove its correctness or complexity.)

Positive Normal Form: Algorithm

Transformation of $\langle V, I, O, \gamma \rangle$ to Positive Normal Form

Replace all operators with equivalent conflict-free operators.

Convert all conditions to negation normal form (NNF).

while any condition contains a negative literal $\neg v$:

Let v be a variable which occurs negatively in a condition.

$V := V \cup \{\hat{v}\}$ for some new propositional state variable \hat{v}

$$I(\hat{v}) := \begin{cases} \mathbf{F} & \text{if } I(v) = \mathbf{T} \\ \mathbf{T} & \text{if } I(v) = \mathbf{F} \end{cases}$$

Replace the effect v by $(v \wedge \neg \hat{v})$ in all operators $o \in O$.

Replace the effect $\neg v$ by $(\neg v \wedge \hat{v})$ in all operators $o \in O$.

Replace $\neg v$ by \hat{v} in all conditions.

Convert all operators $o \in O$ to flat operators.

Here, **all conditions** refers to all operator preconditions, operator effect conditions and the goal.

Why Positive Normal Form is Interesting

In the **absence of nontrivial conditional effects**, positive normal form allows us to distinguish good and bad effects easily:

- ▶ Effects that make state variables true (**add effects**) are good.
- ▶ Effects that make state variables false (**delete effects**) are bad.

This is particularly useful for planning algorithms based on **delete relaxation**, which we will study later in this course.

(Why restriction “in the absence of nontrivial conditional effects”?)

A6.3 STRIPS

STRIPS Operators and Planning Tasks

Definition (STRIPS Operator)

An operator o of a prop. planning task is a **STRIPS operator** if

- ▶ $pre(o)$ is a conjunction of state variables, and
- ▶ $eff(o)$ is a conflict-free conjunction of atomic effects.

Definition (STRIPS Planning Task)

A propositional planning task $\langle V, O, I, \gamma \rangle$ is a **STRIPS planning task** if all operators $o \in O$ are STRIPS operators and γ is a conjunction of state variables.

Note: STRIPS operators are conflict-free and flat.
(For “flat”, we think of atomic effects ℓ as $\top \triangleright \ell$ here.)
STRIPS is a special case of positive normal form.

STRIPS Operators: Remarks

- ▶ Every STRIPS operator is of the form

$$\langle v_1 \wedge \dots \wedge v_n, \ell_1 \wedge \dots \wedge \ell_m \rangle$$

where v_i are state variables and ℓ_j are atomic effects.

- ▶ Often, STRIPS operators o are described via three **sets of state variables**:
 - ▶ the **preconditions** (state variables occurring in $pre(o)$)
 - ▶ the **add effects** (state variables occurring positively in $eff(o)$)
 - ▶ the **delete effects** (state variables occurring negatively in $eff(o)$)
- ▶ Definitions of STRIPS in the literature often do **not** require conflict-freeness. But it is easy to achieve and makes many things simpler.
- ▶ There exists a variant called **STRIPS with negation** where negative literals are also allowed in conditions.

Why STRIPS is Interesting

- ▶ STRIPS is **particularly simple**, yet expressive enough to capture general planning tasks.
- ▶ In particular, STRIPS planning is **no easier** than planning in general (as we will see in Chapter A8).
- ▶ Many algorithms in the planning literature are **only presented for STRIPS planning tasks** (generalization is often, but not always, obvious).

STRIPS

Stanford Research Institute Problem Solver
(Fikes & Nilsson, 1971)

Transformation to STRIPS

- ▶ Not every operator is equivalent to a STRIPS operator.
- ▶ However, each operator can be transformed into a **set** of STRIPS operators whose “combination” is equivalent to the original operator. (How?)
- ▶ However, this transformation may exponentially increase the number of operators. There are planning tasks for which such a blow-up is unavoidable.
- ▶ There are polynomial transformations of propositional planning tasks to STRIPS, but these do not lead to isomorphic transition systems (auxiliary states are needed). (They are, however, equivalent in a weaker sense.)

A6.4 SAS⁺

SAS⁺ Operators and Planning Tasks

Definition (SAS⁺ Operator)

An operator o of an FDR planning task is a **SAS⁺ operator** if

- ▶ $pre(o)$ is a satisfiable conjunction of atoms, and
- ▶ $eff(o)$ is a conflict-free conjunction of atomic effects.

Definition (SAS⁺ Planning Task)

An FDR planning task $\langle V, O, I, \gamma \rangle$ is a **SAS⁺ planning task** if all operators $o \in O$ are SAS⁺ operators and γ is a satisfiable conjunction of atoms.

Note: SAS⁺ operators are conflict-free and flat.
(Same comments as for STRIPS operators apply.)

SAS⁺ Operators: Remarks

- ▶ Every SAS⁺ operator is of the form

$$\langle v_1 = d_1 \wedge \dots \wedge v_n = d_n, \quad v'_1 := d'_1 \wedge \dots \wedge v'_m := d'_m \rangle$$

where all v_i are distinct and all v'_j are distinct.

- ▶ Often, SAS⁺ operators o are described via two **sets of partial assignments**:
 - ▶ the **preconditions** $\{v_1 \mapsto d_1, \dots, v_n \mapsto d_n\}$
 - ▶ the **effects** $\{v'_1 \mapsto d'_1, \dots, v'_m \mapsto d'_m\}$

SAS⁺ vs. STRIPS

- ▶ SAS⁺ is an analogue of STRIPS planning tasks for FDR, but there is no special role of “positive” conditions.
- ▶ Apart from this difference, all comments for STRIPS apply analogously.
- ▶ If all variable domains are binary, SAS⁺ is essentially STRIPS with negation.

SAS⁺

Derives from SAS = Simplified Action Structures
(Bäckström & Klein, 1991)

A6.5 Summary

Summary

- ▶ A **positive** task helps distinguish good and bad effects. The notion of positive tasks only exists for **propositional** tasks.
- ▶ A positive task with flat operators is in **positive normal form**.
- ▶ **STRIPS** is even more restrictive than positive normal form, forbidding complex preconditions and conditional effects.
- ▶ Both forms are expressive enough to capture general propositional planning tasks.
- ▶ Transformation to positive normal form is possible with polynomial size increase.
- ▶ Isomorphic transformations of propositional planning tasks to STRIPS can increase the number of operators exponentially; non-isomorphic polynomial transformations exist.
- ▶ **SAS⁺** is the analogue of STRIPS for FDR planning tasks.