

# Discrete Mathematics in Computer Science

## Tuples and the Cartesian Product

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# Tuples

- **$k$ -tuple**: ordered sequence of  $k$  objects ( $k \in \mathbb{N}_0$ )
- written  $(o_1, \dots, o_k)$  or  $\langle o_1, \dots, o_k \rangle$
- unlike sets, **order matters** ( $\langle 1, 2 \rangle \neq \langle 2, 1 \rangle$ )
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- objects may occur multiple times in a tuple
- objects contained in tuples are called **components**
- terminology:
  - $k = 2$ : (ordered) pair
  - $k = 3$ : triple
  - more rarely: quadruple, quintuple, sextuple, septuple, ...
- if  $k$  is clear from context (or does not matter), often just called **tuple**

# Equality of Tuples

## Definition (Equality of Tuples)

Two  $n$ -tuples  $t = \langle o_1, \dots, o_n \rangle$  and  $t' = \langle o'_1, \dots, o'_n \rangle$  are **equal** ( $t = t'$ ) if for  $i \in \{1, \dots, n\}$  it holds that  $o_i = o'_i$ .

# Cartesian Product

## Definition (Cartesian Product and Cartesian Power)

Let  $S_1, \dots, S_n$  be sets. The **Cartesian product**  $S_1 \times \dots \times S_n$  is the following set of  $n$ -tuples:

$$S_1 \times \dots \times S_n = \{ \langle x_1, \dots, x_n \rangle \mid x_1 \in S_1, x_2 \in S_2, \dots, x_n \in S_n \}.$$

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The  $k$ -ary **Cartesian power** of a set  $S$  (with  $k \in \mathbb{N}_1$ ) is the set  $S^k = \{ \langle o_1, \dots, o_k \rangle \mid o_i \in S \text{ for all } i \in \{1, \dots, k\} \} = \underbrace{S \times \dots \times S}_{k \text{ times}}.$

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**Example:**  $A = \{a, b\}$ ,  $B = \{1, 2, 3\}$

$A^2 =$

## (Non-)properties of the Cartesian Product

The Cartesian product is

- **not commutative**, in most cases  $A \times B \neq B \times A$ .
- **not associative**, in most cases  $(A \times B) \times C \neq A \times (B \times C)$

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Why? Exceptions?