# Planning and Optimization

## E7. Post-hoc Optimization

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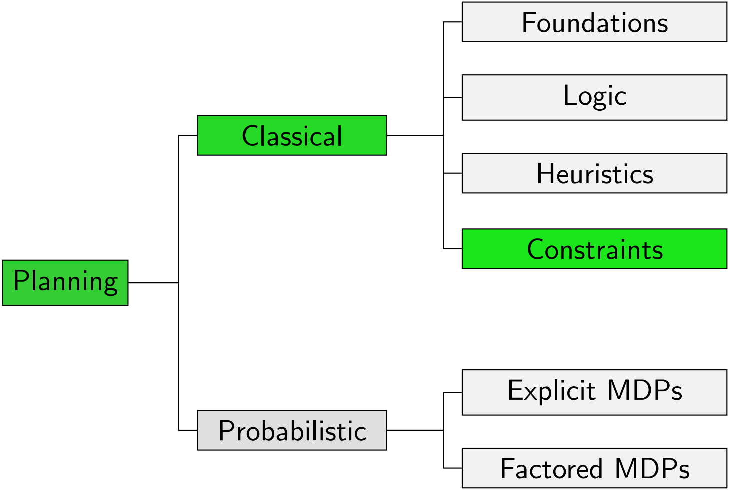
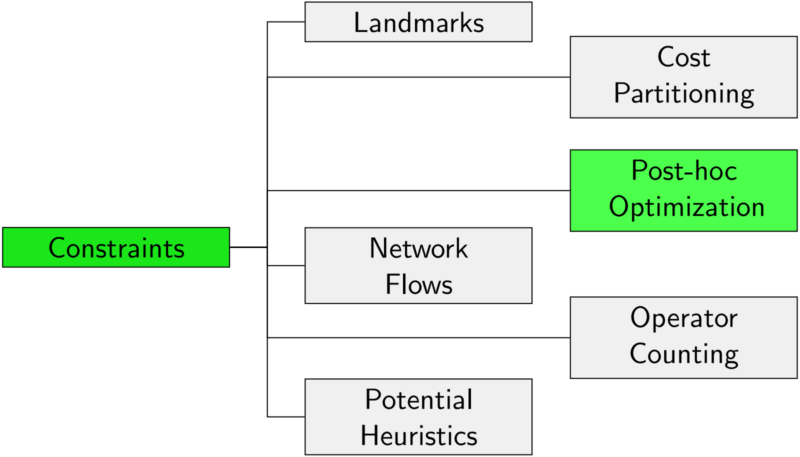
# Content of this Course



# Content of this Course: Constraints



# Introduction

# Puzzle

$\rightsquigarrow$ Blackboard

# Connection to Planning

Let us now rephrase the puzzle slightly

## Example

- the 6 kinds of sweets are 6 operators
- and the 3 hints are about admissible heuristics $h_1$, $h_2$ and $h_3$, stating that:
  - only operators $o_1, o_2, o_3$ and $o_4$ are relevant for $h_1$ and $h_1(s_0) = 11$
  - only operators $o_3, o_4, o_5$ and $o_6$ are relevant for $h_2$ and $h_2(s_0) = 11$
  - only operators $o_1, o_2$ and $o_6$ are relevant for $h_3$ and $h_3(s_0) = 8$

What is the highest possible admissible heuristic estimate for $s_0$ with this information?

# LP Formalization of the Example

We can express the puzzle (rephrased or not) as an LP

**Variables**

Non-negative variable $X_1, \ldots, X_6$ for operator $o_1, \ldots, o_6$

Minimize $X_1 + X_2 + X_3 + X_4 + X_5 + X_6$ subject to

$$
\begin{aligned}
X_1 + X_2 + X_3 + X_4 &\geq 11 \\
X_3 + X_4 + X_5 + X_6 &\geq 11 \\
X_1 + X_2 + X_6 &\geq 8 \\
X_i &\geq 0 \quad \text{for } i \in \{1, \ldots, 6\}
\end{aligned}
$$

# Post-hoc Optimization

# Post-hoc Optimization

The heuristic that generalizes this kind of reasoning
is the Post-hoc Optimization Heuristic (PhO)

- can be computed for any kind of heuristic ...
- ... as long as we are able to determine relevance of operators
- if in doubt, it's always safe to assume an operator is relevant for a heuristic
- but for PhO to work well, it's important that the set of relevant operators is as small as possible

# Operator Relevance in Abstractions

**Definition (Reminder: Affecting Transition Labels)**

Let $\mathcal{T}$ be a transition system, and let $\ell$ be one of its labels.

We say that $\ell$ affects $\mathcal{T}$ if $\mathcal{T}$ has a transition $s \xrightarrow{\ell} t$ with $s \neq t$.

**Definition (Operator Relevance in Abstractions)**

An operator $o$ is relevant for an abstraction $\alpha$ if $o$ affects $\mathcal{T}^\alpha$.

We can efficiently determine operator relevance for abstractions.

# Linear Program (1)

Construct linear program for set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

- variable $X_o$ for each operator $o \in O$
- intuitively, $X_o$ is cost incurred by operator $o$
- abstraction heuristics are admissible

$$\sum_{o \in O} X_o \geq h^\alpha(s) \quad \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

- can tighten these constraints to

$$\sum_{o \in O : o \text{ affects } \mathcal{T}^\alpha} X_o \geq h^\alpha(s) \quad \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

# Linear Program (2)

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

**Variables**

$X_o$ for each operator $o \in O$

**Objective**

Minimize $\sum_{o \in O} X_o$

**Subject to**

$$\begin{array}{ll} \sum_{o \in O : o \text{ affects } \mathcal{T}^\alpha} X_o \geq h^\alpha(s) & \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\} \\ X_o \geq 0 & \text{for all } o \in O \end{array}$$

# Simplifying the LP

- Reduce size of LP by aggregating variables which always occur together in constraints.
- Happens if several operators are relevant for exactly the same heuristics.
- Partitioning $O/\!\sim$ induced by this equivalence relation
- One variable $X_{[o]}$ for each $[o] \in O/\!\sim$

# Example

## Example

- only operators $o_1, o_2, o_3$ and $o_4$ are relevant for $h_1$ and $h_1(s_0) = 11$
- only operators $o_3, o_4, o_5$ and $o_6$ are relevant for $h_2$ and $h_2(s_0) = 11$
- only operators $o_1, o_2$ and $o_6$ are relevant for $h_3$ and $h_3(s_0) = 8$

Which operators affect the same heuristics?

# Example

## Example

- only operators $o_1, o_2, o_3$ and $o_4$ are relevant for $h_1$ and $h_1(s_0) = 11$
- only operators $o_3, o_4, o_5$ and $o_6$ are relevant for $h_2$ and $h_2(s_0) = 11$
- only operators $o_1, o_2$ and $o_6$ are relevant for $h_3$ and $h_3(s_0) = 8$

Which operators affect the same heuristics?

Answer: $o_1 \sim o_2$ and $o_3 \sim o_4$

$\Rightarrow O/\!\sim = \{[o_1], [o_3], [o_5], [o_6]\}$

# Simplifying the LP: Example

LP before aggregation

**Variables**

Non-negative variable $X_1, \dots, X_6$
for operator $o_1, \dots, o_6$

$$
\begin{aligned}
\text{Minimize} \quad & X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \quad \text{subject to} \\
& X_1 + X_2 + X_3 + X_4 \geq 11 \\
& X_3 + X_4 + X_5 + X_6 \geq 11 \\
& X_1 + X_2 + X_6 \geq 8 \\
& X_i \geq 0 \quad \text{for } i \in \{1, \dots, 6\}
\end{aligned}
$$

# Simplifying the LP: Example

LP after aggregation

**Variables**

Non-negative variable $X_{[1]}, X_{[3]}, X_{[5]}, X_{[6]}$
for equivalence classes $[o_1], [o_3], [o_5], [o_6]$

$$\text{Minimize} \quad X_{[1]} + X_{[3]} + X_{[5]} + X_{[6]} \quad \text{subject to}$$

$$\begin{aligned}
X_{[1]} + X_{[3]} \qquad\qquad\qquad &\geq 11 \\
X_{[3]} + X_{[5]} + X_{[6]} &\geq 11 \\
X_{[1]} + \qquad\qquad + X_{[6]} &\geq 8 \\
X_i &\geq 0 \quad \text{for } i \in \{[1], [3], [5], [6]\}
\end{aligned}$$

# PhO Heuristic

**Definition (Post-hoc Optimization Heuristic)**

The post-hoc optimization heuristic $h^{\mathrm{PhO}}_{\{\alpha_1,\dots,\alpha_n\}}$ for abstractions $\alpha_1,\dots,\alpha_n$ is the objective value of the following linear program:

$$\text{Minimize} \sum_{[o]\in O/\sim} X_{[o]} \text{ subject to}$$

$$\begin{aligned} \sum\nolimits_{[o]\in O/\sim : o \text{ affects } \alpha} X_{[o]} &\geq h^{\alpha}(s) && \text{for all } \alpha \in \{\alpha_1,\dots,\alpha_n\} \\ X_{[o]} &\geq 0 && \text{for all } [o] \in O/\sim, \end{aligned}$$

where $o \sim o'$ iff $o$ and $o'$ affect the same transition systems in $\mathcal{T}^{\alpha_1},\dots,\mathcal{T}^{\alpha_n}$.

# PhO Heuristic

**$h^{\text{PhO}}$: Pommerening, Röger & Helmert (2013)**

1. Precompute all abstraction heuristics $h^{\alpha_1}, \ldots, h^{\alpha_n}$.
2. Create LP for initial state $s_0$.
3. For each new state $s$:
   - Look up $h^{\alpha}(s)$ for all $\alpha \in \{\alpha_1, \ldots, \alpha_n\}$.
   - Adjust LP by replacing bounds with the $h^{\alpha}(s)$.

# Post-hoc Optimization Heuristic: Admissibility

**Theorem (Admissibility)**

*The post-hoc optimization heuristic is admissible.*

**Proof.**

Let $\Pi$ be a planning task and $\{\alpha_1, \dots, \alpha_n\}$ be a set of abstractions. We show that there is a feasible variable assignment with objective value equal to the cost of an optimal plan.
Let $\pi$ be an optimal plan for state $s$ and let $cost_\pi(O')$ be the cost incurred by operators from $O' \subseteq O$ in $\pi$.

Setting each $X_{[o]}$ to $cost_\pi([o])$ is a feasible variable assignment:
Constraints $X_{[o]} \geq 0$ are satisfied. ...

# Post-hoc Optimization Heuristic: Admissibility

**Theorem (Admissibility)**

*The post-hoc optimization heuristic is admissible.*

**Proof (continued).**

For each $\alpha \in \{\alpha_1, \dots, \alpha_n\}$, $\pi$ is a solution in the abstract transition system and the sum in the corresponding constraint equals the cost of the state-changing abstract state transitions (i.e.. not accounting for self-loops). As $h^\alpha(s)$ corresponds to the cost of an optimal solution in the abstraction, the inequality holds.

For this assignment, the objective function has value $h^*(s)$ (cost of $\pi$), so the objective value of the LP is admissible. $\square$

# Comparison

# Combining Estimates from Abstraction Heuristics

We have seen two alternatives to combine abstraction heuristics admissibly:

- Canonical heuristic (for PDBs)
- Optimal cost partitioning

How does PhO compare to these?

# Reminder: The Canonical Heuristic Function

If for a set of patterns no operator affects more than one pattern, the sum of the heuristic estimates is admissible.

## Definition (Canonical Heuristic Function)

Let $\mathcal{C}$ be a pattern collection for an FDR planning task.

The canonical heuristic $h^{\mathcal{C}}$ for pattern collection $\mathcal{C}$ is defined as

$$h^{\mathcal{C}}(s) = \max_{\mathcal{D} \in cliques(\mathcal{C})} \sum_{P \in \mathcal{D}} h^P(s),$$

where $cliques(\mathcal{C})$ is the set of all maximal cliques in the compatibility graph for $\mathcal{C}$.

For a given pattern collection, the canonical heuristic is the best possible admissible heuristic not using cost partitioning.

# Reminder: Optimal Cost Partitioning for Abstractions

Optimal cost partitioning for abstractions...

- ...uses a state-specific LP to find the best possible cost partitioning, and sums up the heuristic estimates.
- ...dominates the canonical heuristic, i.e.. for the same pattern collection, it never gives lower estimates than $h^{\mathcal{C}}$.
- ...is very expensive to compute (recomputing the abstractions in every state).

# PhO: Linear Program

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

## Variables

$X_{[o]}$ for all equivalence classes $[o] \in O/\sim$

## Objective

Minimize $\sum_{[o] \in O/\sim} X_{[o]}$

## Subject to

$$
\begin{array}{lll}
[Y_\alpha] & \sum_{[o] \in O/\sim : o \text{ affects } \mathcal{T}^\alpha} X_{[o]} \geq h^\alpha(s) & \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\} \\
 & X_{[o]} \geq 0 & \text{for all } [o] \in O/\sim
\end{array}
$$

# PhO: Dual Linear Program

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

**Variables**

$Y_\alpha$ for each abstraction $\alpha \in \{\alpha_1, \dots, \alpha_n\}$

**Objective**

Maximize $\sum_{\alpha \in \{\alpha_1, \dots, \alpha_n\}} h^\alpha(s) Y_\alpha$

**Subject to**

$$[X_{[o]}] \quad \sum_{\alpha \in \{\alpha_1, \dots, \alpha_n\} : o \text{ affects } \mathcal{T}^\alpha} Y_\alpha \leq 1 \quad \text{for all } [o] \in O/\sim$$

$$Y_\alpha \geq 0 \quad \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

# PhO: Dual Linear Program

For set of abstractions $\{\alpha_1, \ldots, \alpha_n\}$:

**Variables**

$Y_\alpha$ for each abstraction $\alpha \in \{\alpha_1, \ldots, \alpha_n\}$

**Objective**

Maximize $\sum_{\alpha \in \{\alpha_1, \ldots, \alpha_n\}} h^\alpha(s) Y_\alpha$

**Subject to**

$$[X_{[o]}] \quad \sum_{\alpha \in \{\alpha_1, \ldots, \alpha_n\} : o \text{ affects } \mathcal{T}^\alpha} Y_\alpha \leq 1 \quad \text{for all } [o] \in O/\!\sim$$

$$Y_\alpha \geq 0 \quad \text{for all } \alpha \in \{\alpha_1, \ldots, \alpha_n\}$$

We compute a state-specific cost partitioning that can only scale the operator costs within each heuristic by a factor $0 \leq Y_\alpha \leq 1$.

# Relation to Optimal Cost Partitioning

## Theorem

*Optimal cost partitioning dominates post-hoc optimization.*

## Proof Sketch.

Consider the assignment $\langle Y_{\alpha_1}, \ldots, Y_{\alpha_n} \rangle$ of the dual of the LP solved by the post-hoc optimization heuristic in state $s$. Its solution value is equivalent to the solution value of the cost partitioned heuristic induced by the cost partitioning $\langle Y_{\alpha_1} cost, \ldots, Y_{\alpha_n} cost \rangle$.

# Relation to Canonical Heuristic

**Theorem**

*Consider the dual D of the LP solved by the post-hoc optimization heuristic in state s for a given set of abstractions. If we restrict the variables in D to integers, the objective value is the canonical heuristic value $h^{\mathcal{C}}(s)$.*

# Relation to Canonical Heuristic

## Theorem

*Consider the dual $D$ of the LP solved by the post-hoc optimization heuristic in state $s$ for a given set of abstractions. If we restrict the variables in $D$ to integers, the objective value is the canonical heuristic value $h^{\mathcal{C}}(s)$.*

## Corollary

*The post-hoc optimization heuristic dominates the canonical heuristic for the same set of abstractions.*

# $h^{\mathrm{PhO}}$ vs $h^{\mathcal{C}}$

- For the canonical heuristic, we need to find all maximal cliques, which is an NP-hard problem.
- The post-hoc optimization heuristic dominates the canonical heuristic and can be computed in polynomial time.
- The post-hoc optimization heuristic solves an LP in each state but does not require a preprocessing step
- With post-hoc optimization, a large number of small patterns works well.

# Summary

# Summary

- Post-hoc optimization heuristic constraints express admissibility of heuristics
- exploits (ir-)relevance of operators for heuristics
- explores the middle ground between canonical heuristic and optimal cost partitioning.
- For the same set of abstractions, the post-hoc optimization heuristic dominates the canonical heuristic.
- The computation can be done in polynomial time.