Planning and Optimization

E7. Post-hoc Optimization

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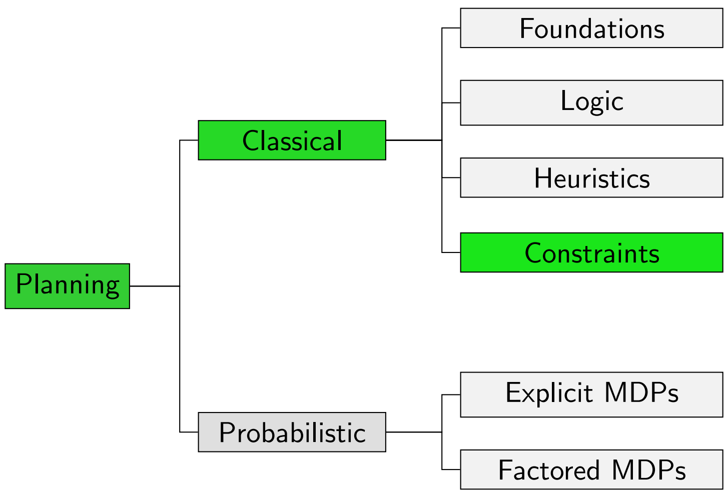
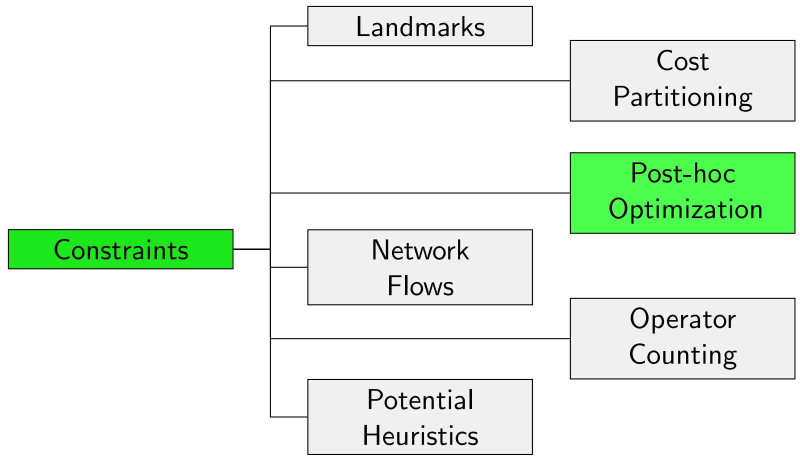
November 20, 2019

Content of this Course



Content of this Course: Constraints



E7.1 Introduction

Puzzle

$\rightsquigarrow$ Blackboard

Connection to Planning

Let us now rephrase the puzzle slightly

Example

- the 6 kinds of sweets are 6 operators
- and the 3 hints are about admissible heuristics h_1, h_2 and h_3, stating that:
  - only operators o_1, o_2, o_3 and o_4 are relevant for h_1 and $h_1(s_0) = 11$
  - only operators o_3, o_4, o_5 and o_6 are relevant for h_2 and $h_2(s_0) = 11$
  - only operators o_1, o_2 and o_6 are relevant for h_3 and $h_3(s_0) = 8$

What is the highest possible admissible heuristic estimate for s_0 with this information?

LP Formalization of the Example

We can express the puzzle (rephrased or not) as an LP

Variables

Non-negative variable $X_1, \dots, X_6$ for operator $o_1, \dots, o_6$

Minimize $X_1 + X_2 + X_3 + X_4 + X_5 + X_6$ subject to

$$
\begin{aligned}
X_1 + X_2 + X_3 + X_4 &\geq 11 \\
X_3 + X_4 + X_5 + X_6 &\geq 11 \\
X_1 + X_2 + X_6 &\geq 8 \\
X_i &\geq 0 \quad \text{for } i \in \{1, \dots, 6\}
\end{aligned}
$$

E7.2 Post-hoc Optimization

Post-hoc Optimization

The heuristic that generalizes this kind of reasoning is the Post-hoc Optimization Heuristic (PhO)

- can be computed for any kind of heuristic ...
- ... as long as we are able to determine relevance of operators
- if in doubt, it's always safe to assume an operator is relevant for a heuristic
- but for PhO to work well, it's important that the set of relevant operators is as small as possible

Operator Relevance in Abstractions

Definition (Reminder: Affecting Transition Labels)
Let $\mathcal{T}$ be a transition system, and let ℓ be one of its labels.
We say that ℓ affects $\mathcal{T}$ if $\mathcal{T}$ has a transition $s \xrightarrow{\ell} t$ with $s \neq t$.

Definition (Operator Relevance in Abstractions)
An operator o is relevant for an abstraction α if o affects $\mathcal{T}^\alpha$.

We can efficiently determine operator relevance for abstractions.

Linear Program (1)

Construct linear program for set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

- variable X_o for each operator $o \in O$
- intuitively, X_o is cost incurred by operator o
- abstraction heuristics are admissible

$$\sum_{o \in O} X_o \geq h^\alpha(s) \quad \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

- can tighten these constraints to

$$\sum_{o \in O : o \text{ affects } \mathcal{T}^\alpha} X_o \geq h^\alpha(s) \quad \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

Linear Program (2)

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

Variables
X_o for each operator $o \in O$

Objective
Minimize $\sum_{o \in O} X_o$

Subject to

$$\begin{aligned} \sum_{o \in O : o \text{ affects } \mathcal{T}^\alpha} X_o &\geq h^\alpha(s) && \text{for } \alpha \in \{\alpha_1, \dots, \alpha_n\} \\ X_o &\geq 0 && \text{for all } o \in O \end{aligned}$$

Simplifying the LP

- Reduce size of LP by aggregating variables which always occur together in constraints.
- Happens if several operators are relevant for exactly the same heuristics.
- Partitioning $O/\sim$ induced by this equivalence relation
- One variable $X_{[o]}$ for each $[o] \in O/\sim$

Example

Example
- only operators o_1, o_2, o_3 and o_4 are relevant for h_1 and $h_1(s_0) = 11$
- only operators o_3, o_4, o_5 and o_6 are relevant for h_2 and $h_2(s_0) = 11$
- only operators o_1, o_2 and o_6 are relevant for h_3 and $h_3(s_0) = 8$

Which operators affect the same heuristics?

Answer: $o_1 \sim o_2$ and $o_3 \sim o_4$
$\Rightarrow O/\sim = \{[o_1], [o_3], [o_5], [o_6]\}$

Simplifying the LP: Example

LP before aggregation

Variables
Non-negative variable $X_1, \dots, X_6$ for operator $o_1, \dots, o_6$

$$\text{Minimize} \quad X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \quad \text{subject to}$$

$$\begin{aligned} X_1 + X_2 + X_3 + X_4 \qquad\qquad &\geq 11 \\ X_3 + X_4 + X_5 + X_6 &\geq 11 \\ X_1 + X_2 \qquad\qquad + X_6 &\geq 8 \\ X_i &\geq 0 \quad \text{for } i \in \{1, \dots, 6\} \end{aligned}$$

Simplifying the LP: Example

LP after aggregation

Variables
Non-negative variable $X_{[1]}, X_{[3]}, X_{[5]}, X_{[6]}$
for equivalence classes $[o_1], [o_3], [o_5], [o_6]$

Minimize $X_{[1]} + X_{[3]} + X_{[5]} + X_{[6]}$ subject to

$$
\begin{aligned}
X_{[1]} + X_{[3]} \quad\quad\quad\quad & \geq 11 \\
X_{[3]} + X_{[5]} + X_{[6]} & \geq 11 \\
X_{[1]} + \quad\quad\quad\quad + X_{[6]} & \geq 8 \\
X_i & \geq 0 \quad \text{for } i \in \{[1],[3],[5],[6]\}
\end{aligned}
$$

PhO Heuristic

Definition (Post-hoc Optimization Heuristic)
The post-hoc optimization heuristic $h^{\text{PhO}}_{\{\alpha_1,\dots,\alpha_n\}}$ for abstractions $\alpha_1, \dots, \alpha_n$ is the objective value of the following linear program:

$$\text{Minimize} \sum_{[o] \in O/\sim} X_{[o]} \text{ subject to}$$

$$
\begin{aligned}
\sum_{[o] \in O/\sim : o \text{ affects } \alpha} X_{[o]} & \geq h^{\alpha}(s) && \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\} \\
X_{[o]} & \geq 0 && \text{for all } [o] \in O/\sim,
\end{aligned}
$$

where $o \sim o'$ iff o and o' affect the same transition systems in $\mathcal{T}^{\alpha_1}, \dots, \mathcal{T}^{\alpha_n}$.

PhO Heuristic

h^{PhO}: Pommerening, Röger & Helmert (2013)

1. Precompute all abstraction heuristics $h^{\alpha_1}, \dots, h^{\alpha_n}$.
2. Create LP for initial state s_0.
3. For each new state s:
   - Look up $h^{\alpha}(s)$ for all $\alpha \in \{\alpha_1, \dots, \alpha_n\}$.
   - Adjust LP by replacing bounds with the $h^{\alpha}(s)$.

Post-hoc Optimization Heuristic: Admissibility

Theorem (Admissibility)
The post-hoc optimization heuristic is admissible.

Proof.
Let Π be a planning task and $\{\alpha_1, \dots, \alpha_n\}$ be a set of abstractions.
We show that there is a feasible variable assignment with objective value equal to the cost of an optimal plan.
Let π be an optimal plan for state s and let $cost_{\pi}(O')$ be the cost incurred by operators from $O' \subseteq O$ in π.
Setting each $X_{[o]}$ to $cost_{\pi}([o])$ is a feasible variable assignment:
Constraints $X_{[o]} \geq 0$ are satisfied. ...

Post-hoc Optimization Heuristic: Admissibility

Theorem (Admissibility)
The post-hoc optimization heuristic is admissible.

Proof (continued).
For each $\alpha \in \{\alpha_1, \dots, \alpha_n\}$, π is a solution in the abstract transition system and the sum in the corresponding constraint equals the cost of the state-changing abstract state transitions (i.e.. not accounting for self-loops). As $h^\alpha(s)$ corresponds to the cost of an optimal solution in the abstraction, the inequality holds.

For this assignment, the objective function has value $h^*(s)$ (cost of π), so the objective value of the LP is admissible. □

E7.3 Comparison

Combining Estimates from Abstraction Heuristics

We have seen two alternatives to combine abstraction heuristics admissibly:

- Canonical heuristic (for PDBs)
- Optimal cost partitioning

How does PhO compare to these?

Reminder: The Canonical Heuristic Function

If for a set of patterns no operator affects more than one pattern, the sum of the heuristic estimates is admissible.

Definition (Canonical Heuristic Function)
Let $\mathcal{C}$ be a pattern collection for an FDR planning task.

The canonical heuristic $h^{\mathcal{C}}$ for pattern collection $\mathcal{C}$ is defined as

$$h^{\mathcal{C}}(s) = \max_{\mathcal{D} \in cliques(\mathcal{C})} \sum_{P \in \mathcal{D}} h^P(s),$$

where $cliques(\mathcal{C})$ is the set of all maximal cliques in the compatibility graph for $\mathcal{C}$.

For a given pattern collection, the canonical heuristic is the best possible admissible heuristic not using cost partitioning.

Reminder: Optimal Cost Partitioning for Abstractions

Optimal cost partitioning for abstractions...

- ... uses a state-specific LP to find the best possible cost partitioning, and sums up the heuristic estimates.
- ... dominates the canonical heuristic, i.e.. for the same pattern collection, it never gives lower estimates than $h^{\mathcal{C}}$.
- ... is very expensive to compute (recomputing the abstractions in every state).

PhO: Linear Program

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

Variables
$X_{[o]}$ for all equivalence classes $[o] \in O/\!\sim$

Objective
Minimize $\sum_{[o] \in O/\sim} X_{[o]}$

Subject to

$$[Y_\alpha] \quad \sum_{[o] \in O/\sim : o \text{ affects } \mathcal{T}^\alpha} X_{[o]} \geq h^\alpha(s) \quad \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

$$X_{[o]} \geq 0 \quad \text{for all } [o] \in O/\!\sim$$

PhO: Dual Linear Program

For set of abstractions $\{\alpha_1, \dots, \alpha_n\}$:

Variables
Y_α for each abstraction $\alpha \in \{\alpha_1, \dots, \alpha_n\}$

Objective
Maximize $\sum_{\alpha \in \{\alpha_1, \dots, \alpha_n\}} h^\alpha(s) Y_\alpha$

Subject to

$$[X_{[o]}] \quad \sum_{\alpha \in \{\alpha_1, \dots, \alpha_n\} : o \text{ affects } \mathcal{T}^\alpha} Y_\alpha \leq 1 \quad \text{for all } [o] \in O/\!\sim$$

$$Y_\alpha \geq 0 \quad \text{for all } \alpha \in \{\alpha_1, \dots, \alpha_n\}$$

We compute a state-specific cost partitioning that can only scale the operator costs within each heuristic by a factor $0 \leq Y_\alpha \leq 1$.

Relation to Optimal Cost Partitioning

Theorem
Optimal cost partitioning dominates post-hoc optimization.

Proof Sketch.
Consider the assignment $\langle Y_{\alpha_1}, \dots, Y_{\alpha_n} \rangle$ of the dual of the LP solved by the post-hoc optimization heuristic in state s. Its solution value is equivalent to the solution value of the cost partitioned heuristic induced by the cost partitioning $\langle Y_{\alpha_1} cost, \dots, Y_{\alpha_n} cost \rangle$.

Relation to Canonical Heuristic

Theorem

Consider the dual D of the LP solved by the post-hoc optimization heuristic in state s for a given set of abstractions. If we restrict the variables in D to integers, the objective value is the canonical heuristic value $h^{\mathcal{C}}(s)$.

Corollary

The post-hoc optimization heuristic dominates the canonical heuristic for the same set of abstractions.

h^{PhO} vs $h^{\mathcal{C}}$

- For the canonical heuristic, we need to find all maximal cliques, which is an NP-hard problem.
- The post-hoc optimization heuristic dominates the canonical heuristic and can be computed in polynomial time.
- The post-hoc optimization heuristic solves an LP in each state but does not require a preprocessing step
- With post-hoc optimization, a large number of small patterns works well.

E7.4 Summary

Summary

- Post-hoc optimization heuristic constraints express admissibility of heuristics
- exploits (ir-)relevance of operators for heuristics
- explores the middle ground between canonical heuristic and optimal cost partitioning.
- For the same set of abstractions, the post-hoc optimization heuristic dominates the canonical heuristic.
- The computation can be done in polynomial time.