# Planning and Optimization

## E2. Landmarks: RTG Landmarks & MHS Heuristic

Malte Helmert and Thomas Keller

Universität Basel

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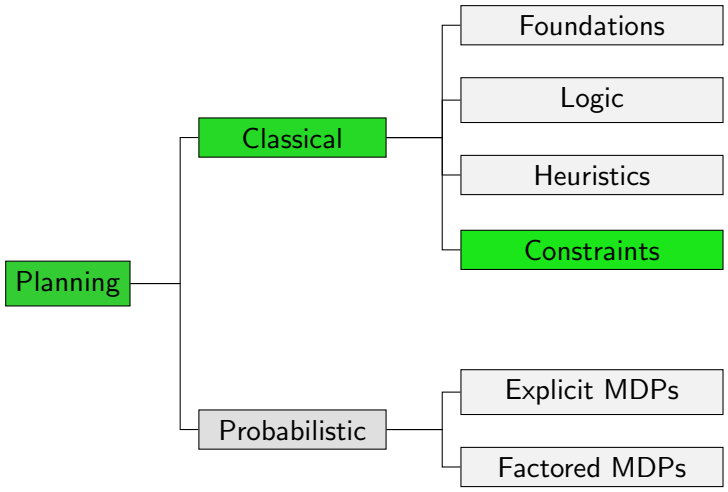
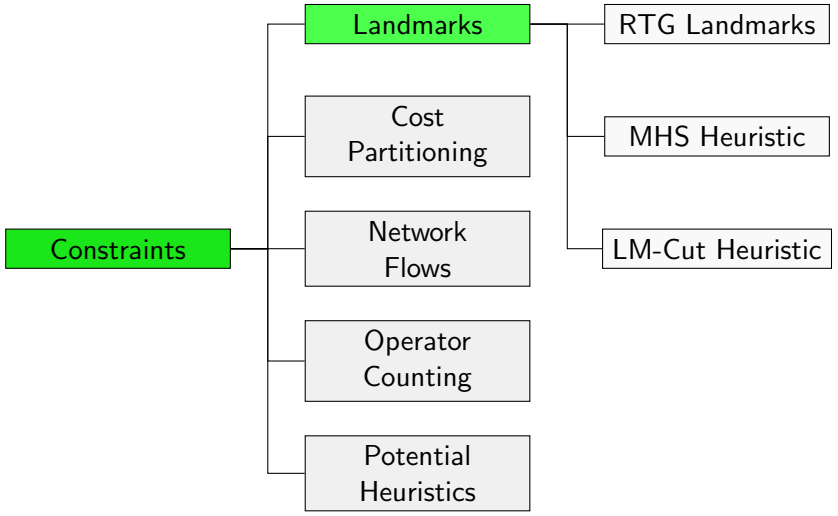
## Content of this Course






## Content of this Course: Constraints



# E2.1 Landmarks





## Landmarks

Basic Idea: Something that must happen in every solution

For example

- some operator must be applied (action landmark)
- some atomic proposition must hold (fact landmark)
- some formula must be true (formula landmark)

$\rightarrow$ Derive heuristic estimate from this kind of information.

We only consider fact and disjunctive action landmarks.





## Definition

**Definition (Disjunctive Action Landmark)**

Let $s$ be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.

A disjunctive action landmark for $s$ is a set of operators $L \subseteq O$ such that every label path from $s$ to a goal state contains an operator from $L$.
The cost of landmark $L$ is $cost(L) = \min_{o \in L} cost(o)$.

**Definition (Fact Landmark)**

Let $s$ be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.

An atomic proposition $v = d$ for $v \in V$ and $d \in \mathrm{dom}(v)$ is a fact landmark for $s$ if every state path from $s$ to a goal state contains a state $s'$ with $s'(v) = d$.

If we talk about landmarks for the initial state, we omit "for $I$".





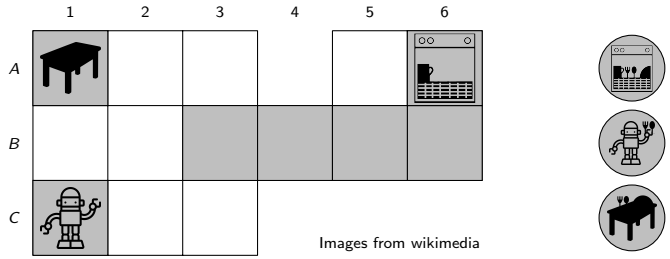
## Landmarks: Example

**Example**

Consider a FDR planning task $\langle V, I, O, \gamma \rangle$ with

- $V = \{\textit{robot-at}, \textit{dishes-at}\}$ with
  - $\mathrm{dom}(\textit{robot-at}) = \{\mathrm{A1}, \ldots, \mathrm{C3}, \mathrm{B4}, \mathrm{A5}, \ldots, \mathrm{B6}\}$
  - $\mathrm{dom}(\textit{dishes-at}) = \{\mathrm{Table}, \mathrm{Robot}, \mathrm{Dishwasher}\}$
- $I = \{\textit{robot-at} \mapsto \mathrm{C1}, \textit{dishes-at} \mapsto \mathrm{Table}\}$
- operators
  - move-$x$-$y$ to move from cell $x$ to adjacent cell $y$
  - pickup dishes, and
  - load dishes into the dishwasher.
- $\gamma = (\textit{robot-at} = B6) \wedge (\textit{dishes-at} = \mathrm{Dishwasher})$

## Fact Landmarks: Example



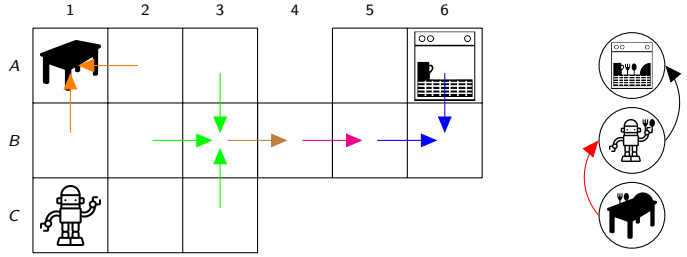

Each fact in gray is a fact landmark:

- `robot-at` $= x$ for $x \in \{\text{A1}, \text{A6}, \text{B3}, \text{B4}, \text{B5}, \text{B6}, \text{C1}\}$
- `dishes-at` $= x$ for $x \in \{\text{Dishwasher}, \text{Robot}, \text{Table}\}$





## Disjunctive Action Landmarks: Example




Actions of same color form disjunctive action landmark:

- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}
- {move-A3-B3, move-B2-B3, move-C3-B3}
- {move-B1-A1, move-A2-A1}
- ...





## Remarks

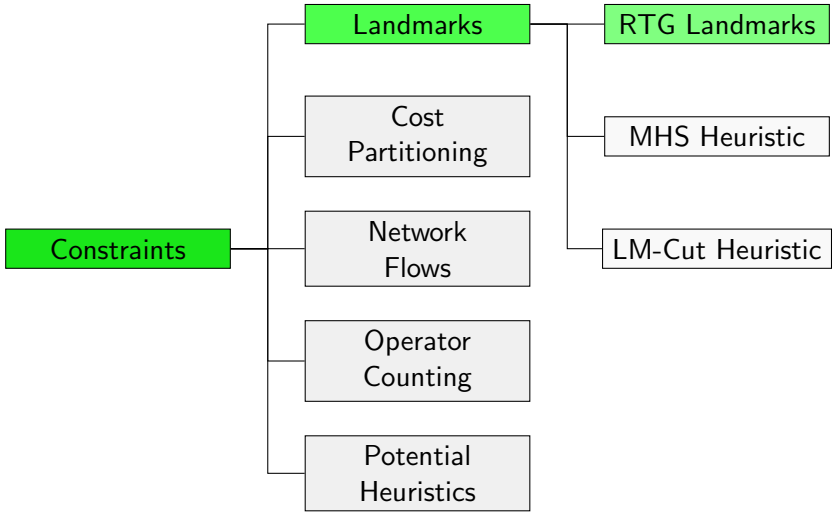
- Not every landmark is informative. Some examples:
  - The set of all operators is a disjunctive action landmark unless the initial state is already a goal state.
  - Every variable that is initially true is a fact landmark.
- Deciding whether a given variable is a fact landmark is as hard as the plan existence problem.
- Deciding whether a given operator set is a disjunctive action landmark is as hard as the plan existence problem.
- Every fact landmark $v$ that is initially false induces a disjunctive action landmark consisting of all operators that possibly make $v$ true.





# E2.2 Landmarks from RTGs

## Content of this Course: Constraints








## Computing Landmarks

### How can we come up with landmarks?

Most landmarks are derived from the relaxed task graph:

- RHW landmarks: Richter, Helmert & Matthias Westphal. Landmarks Revisited. (AAAI 2008)
- LM-Cut: Helmert & Domshlak. Landmarks, Critical Paths and Abstractions: What's the Difference Anyway? (ICAPS 2009)
- $h^m$ landmarks: Keyder, Richter & Helmert: Sound and Complete Landmarks for And/Or Graphs (ECAI 2010)

We discuss $h^m$ landmarks restricted to $m = 1$
and to STRIPS planning tasks.





## Incidental Landmarks: Example

**Example (Incidental Landmarks)**

Consider a STRIPS planning task $\langle V, I, \{o_1, o_2\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f\},$$
$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$
$$o_1 = \langle \{a\}, \{c, d, e\}, \{b\} \rangle,$$
$$o_2 = \langle \{d, e\}, \{f\}, \{a\} \rangle, \text{ and}$$
$$\gamma = \{e, f\}.$$

Single solution: $\langle o_1, o_2 \rangle$

- All variables are fact landmarks.
- Variable $b$ is initially true but irrelevant for the plan.
- Variable $c$ gets true as "side effect" of $o_1$ but it is not necessary for the goal or to make an operator applicable.





## Causal Landmarks

**Definition (Causal Fact Landmark)**

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a STRIPS planning task.

An atomic proposition $v = \mathbf{T}$ for $v \in V$ is a causal fact landmark

- if $v \in \gamma$
- or if for all goal paths $\pi = \langle o_1, \ldots, o_n \rangle$ there is an $o_i$ with $v \in pre(o_i)$.

## Causal Landmarks: Example

**Example (Causal Landmarks)**

Consider a STRIPS planning task $\langle V, I, \{o_1, o_2\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f\},$$
$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$
$$o_1 = \langle \{a\}, \{c, d, e\}, \{b\} \rangle,$$
$$o_2 = \langle \{d, e\}, \{f\}, \{a\} \rangle, \text{ and}$$
$$\gamma = \{e, f\}.$$

Single solution: $\langle o_1, o_2 \rangle$

- All variables are fact landmarks for the initial state.
- Only $a, d, e$ and $f$ are causal landmarks.





## What We Are Doing Next

- Causal landmarks are the desirable landmarks.
- We can use a simplified version of RTGs to compute causal landmarks for STRIPS planning tasks.
- We will define landmarks of AND/OR graphs, ...
- and show how they can be computed.
- Afterwards we establish that these are landmarks of the planning task.





## Simplified Relaxed Task Graph

**Definition**

For a STRIPS planning task $\langle V, I, O, \gamma \rangle$, the simplified relaxed task graph $sRTG(\Pi^+)$ is the AND/OR graph $\langle N_{\text{and}} \cup N_{\text{or}}, A, type \rangle$ with

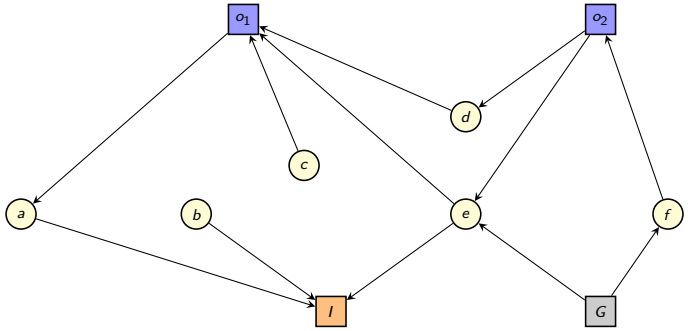
- $N_{\text{and}} = \{n_o \mid o \in O\} \cup \{v_I, v_G\}$
  with $type(n) = \wedge$ for all $n \in N_{\text{and}}$,
- $N_{\text{or}} = \{n_v \mid v \in V\}$
  with $type(n) = \vee$ for all $n \in N_{\text{or}}$, and
- $A = \{\langle n_a, n_o \rangle \mid o \in O, a \in add(o)\} \cup$
  $\{\langle n_o, n_p \rangle \mid o \in O, p \in pre(o)\} \cup$
  $\{\langle n_v, n_I \rangle \mid v \in I\} \cup$
  $\{\langle n_G, n_v \rangle \mid v \in G\}$





## Simplified RTG: Example

The simplified RTG for our example task is:



## Characterizing Equation System

**Theorem**

*Let $G = \langle N, A, type \rangle$ be an AND/OR graph. Consider the following system of equations:*

$$LM(n) = \{n\} \cup \bigcap_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \vee$$

$$LM(n) = \{n\} \cup \bigcup_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \wedge$$

*The equation system has a unique maximal solution (maximal with regard to set inclusion), and for this solution it holds that*

*$n' \in LM(n)$ iff $n'$ is a landmark for reaching $n$ in $G$.*





## Computation of Maximal Solution

**Theorem**

*Let $G = \langle N, A, type \rangle$ be an AND/OR graph. Consider the following system of equations:*

$$LM(n) = \{n\} \cup \bigcap_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \vee$$

$$LM(n) = \{n\} \cup \bigcup_{\langle n,n' \rangle \in A} LM(n') \quad type(n) = \wedge$$

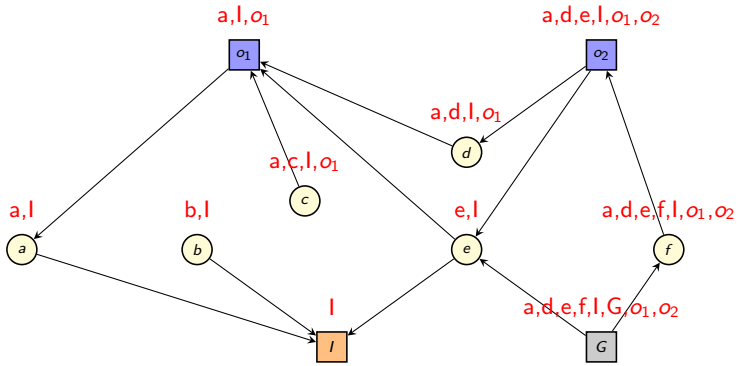
*The equation system has a unique maximal solution (maximal with regard to set inclusion).*

Computation: Initialize landmark sets as $LM(n) = N_{\text{and}} \cup N_{\text{or}}$ and apply equations as update rules until fixpoint.





## Computation: Example




(cf. screen version of slides for step-wise computation)





## Relation to Planning Task Landmarks

**Theorem**

*Let $\Pi = \langle V, I, O, \gamma \rangle$ be a STRIPS planning task and let $\mathcal{L}$ be the set of landmarks for reaching $n_G$ in $sRTG(\Pi^+)$.*

*The set $\{v = \mathbf{T} \mid v \in V \text{ and } n_v \in \mathcal{L}\}$ is exactly the set of causal fact landmarks in $\Pi^+$.*

*For operators $o \in O$, if $n_o \in \mathcal{L}$ then $\{o\}$ is a disjunctive action landmark in $\Pi^+$.*
*There are no other disjunctive action landmarks of size 1.*

(Proofs omitted.)

## Computed RTG Landmarks: Example

**Example (Computed RTG Landmarks)**

Consider a STRIPS planning task $\langle V, I, \{o_1, o_2\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f\},$$
$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{T}, f \mapsto \mathbf{F}\},$$
$$o_1 = \langle \{a\}, \{c, d, e\}, \{b\} \rangle,$$
$$o_2 = \langle \{d, e\}, \{f\}, \{a\} \rangle, \text{ and}$$
$$\gamma = \{e, f\}.$$

- $LM(n_G) = \{a, d, e, f, I, G, o_1, o_2\}$
- $a, d, e$, and $f$ are causal fact landmarks of $\Pi^+$.
- $\{o_1\}$ and $\{o_2\}$ are disjunctive action landmarks of $\Pi^+$.





## Landmarks of $\Pi^+$ Are Landmarks of $\Pi$

**Theorem**

*Let $\Pi$ be a STRIPS planning task.*

*All fact landmarks of $\Pi^+$ are fact landmarks of $\Pi$ and all disjunctive action landmarks of $\Pi^+$ are disjunctive action landmarks of $\Pi$.*

**Proof.**

Let $L$ be a disjunctive action landmark of $\Pi^+$ and $\pi$ be a plan for $\Pi$. Then $\pi$ is also a plan for $\Pi^+$ and, thus, $\pi$ contains an operator from $L$.

Let $f$ be a fact landmark of $\Pi^+$. If $f$ is already true in the initial state, then it is also a landmark of $\Pi$. Otherwise, every plan for $\Pi^+$ contains an operator that adds $f$ and the set of all these operators is a disjunctive action landmark of $\Pi^+$. Therefore, also each plan of $\Pi$ contains such an operator, making $f$ a fact landmark of $\Pi$. □



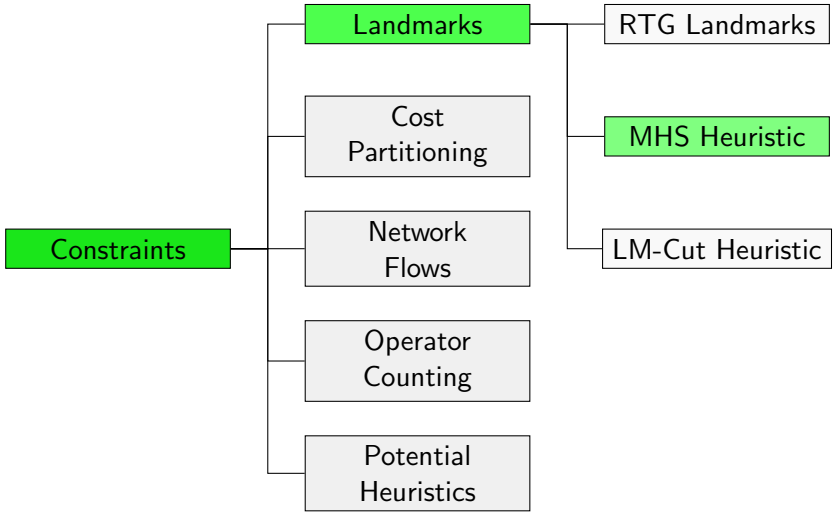


# E2.3 Minimum Hitting Set Heuristic





## Content of this Course: Constraints



## Exploiting Disjunctive Action Landmarks

- The cost $cost(L)$ of a disjunctive action landmark $L$ is an admissible heuristic, but it is usually not very informative.
- Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set $\mathcal{L}$ of disjunctive action landmarks?

- Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ not admissible!
- Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ usually very weak heuristic
- better: Hitting sets





## Hitting Sets

**Definition (Hitting Set)**

Let $X$ be a set, $\mathcal{F} = \{F_1, \ldots, F_n\} \subseteq 2^X$ be a family of subsets of $X$ and $c : X \to \mathbb{R}_0^+$ be a cost function for $X$.

A hitting set is a subset $H \subseteq X$ that "hits" all subsets in $\mathcal{F}$, i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The cost of $H$ is $\sum_{x \in H} c(x)$.

A minimum hitting set (MHS) is a hitting set with minimal cost.

MHS is a "classical" NP-complete problem (Karp, 1972)





## Example: Hitting Sets

**Example**

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$

What is minimum hitting set?

Solution: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$





## Hitting Sets for Disjunctive Action Landmarks

Idea: disjunctive action landmarks are interpreted as instance of minimum hitting set

**Definition (Hitting Set Heuristic)**

Let $\mathcal{L}$ be a set of disjunctive action landmarks. The hitting set heuristic $h^{\text{MHS}}(\mathcal{L})$ is defined as the cost of a minimum hitting set for $\mathcal{L}$ with $c(o) = cost(o)$.

**Proposition (Hitting Set Heuristic is Admissible)**

*Let $\mathcal{L}$ be a set of disjunctive action landmarks for state $s$. Then $h^{MHS}(\mathcal{L})$ is an admissible estimate for $s$.*

## Hitting Set Heuristic: Discussion

- The hitting set heuristic is the best possible heuristic that only uses the given information...
- ...but is NP-hard to compute.
- $\rightsquigarrow$ Use approximations that can be efficiently computed.
  $\Rightarrow$ LP-relaxation, cost partitioning (both discussed later)





# E2.4 Summary





## Summary

- Fcat landmark: atomic proposition that is true in each state path to a goal
- Disjunctive action landmark: set $L$ of operators such that every plan uses some operator from $L$
- Relaxed task graphs allows efficient computation of landmarks
- Hitting sets yield the most accurate heuristic for a given set of disjunctive action landmarks
- Computation of minimal hitting set is NP-hard