

Planning and Optimization

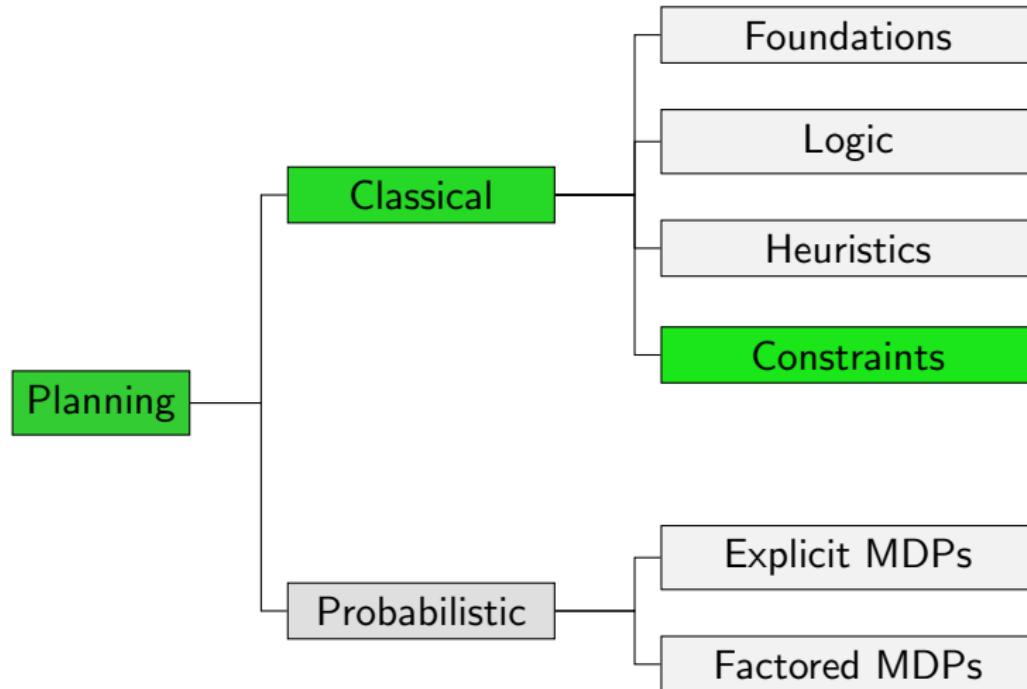
E1. Constraints: Introduction

Malte Helmert and Thomas Keller

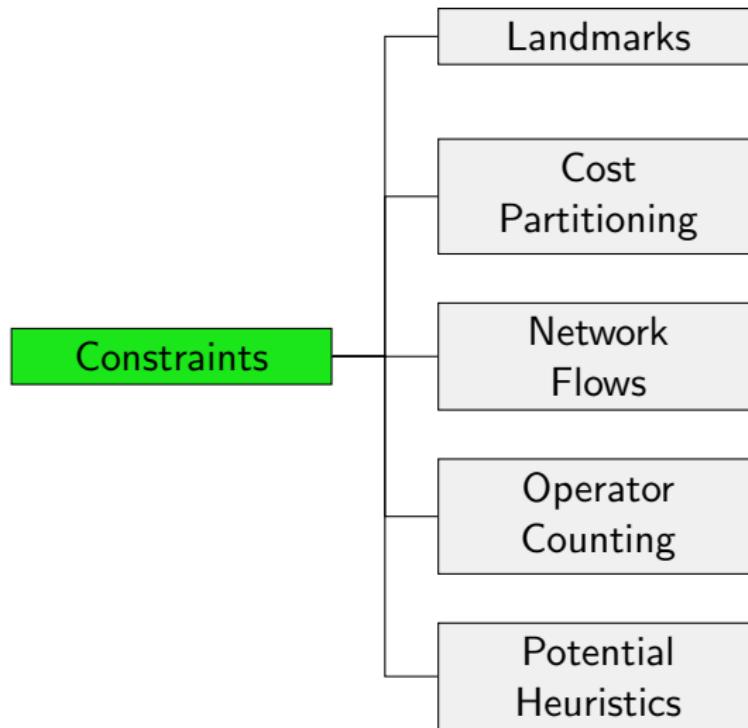
Universität Basel

November 11, 2019

Content of this Course



Content of this Course: Constraints



Constraint-based Heuristics

Coming Up with Heuristics in a Principled Way

General Procedure for Obtaining a Heuristic

Solve a simplified version of the problem.

Major ideas for heuristics in the planning literature:

- delete relaxation
- abstraction
- landmarks
- critical paths
- network flows
- potential heuristic

Landmarks, network flows and potential heuristics are based on **constraints** that can be specified for a planning task.

Constraints: Example

Example

Consider a FDR planning task $\langle V, I, O, \gamma \rangle$ with

- $V = \{robot\text{-}at, dishes\text{-}at\}$ with
 - $\text{dom}(robot\text{-}at) = \{\text{A1}, \dots, \text{C3}, \text{B4}, \text{A5}, \dots, \text{B6}\}$
 - $\text{dom}(dishes\text{-}at) = \{\text{Table}, \text{Robot}, \text{Dishwasher}\}$
- $I = \{robot\text{-}at \mapsto \text{C1}, dishes\text{-}at \mapsto \text{Table}\}$
- operators
 - move- x - y to move from cell x to adjacent cell y
 - pickup dishes, and
 - load dishes into the dishwasher.
- $\gamma = (robot\text{-}at = \text{B6}) \wedge (dishes\text{-}at = \text{Dishwasher})$

Constraints: Example

	1	2	3	4	5	6
A						
B						
C						

Images from wikipedia

Constraints

Some heuristics exploit **constraints** that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some value in at least one visited state.
(a **fact landmark** constraint)

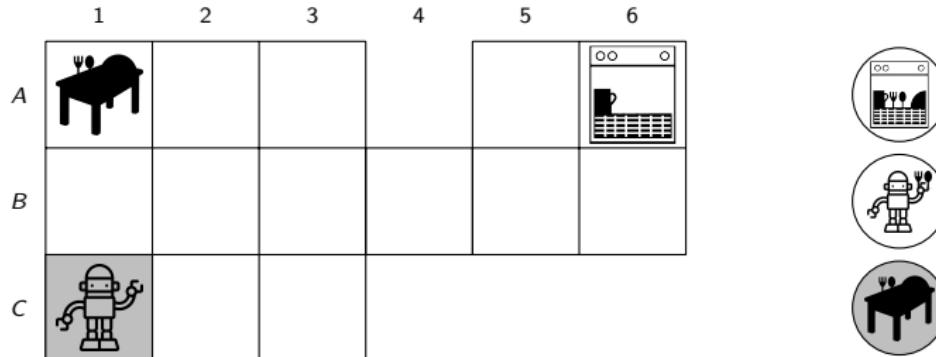
Fact Landmarks: Example

Which values do *robot-at* and *dishes-at* take in every solution?

	1	2	3	4	5	6	
A							
B							
C							

Fact Landmarks: Example

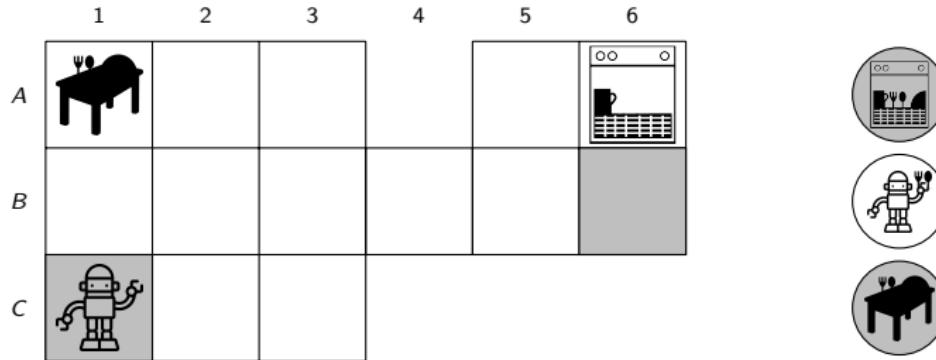
Which values do *robot-at* and *dishes-at* take in every solution?



- $\text{robot-at} = \text{C1}$, $\text{dishes-at} = \text{Table}$ (initial state)

Fact Landmarks: Example

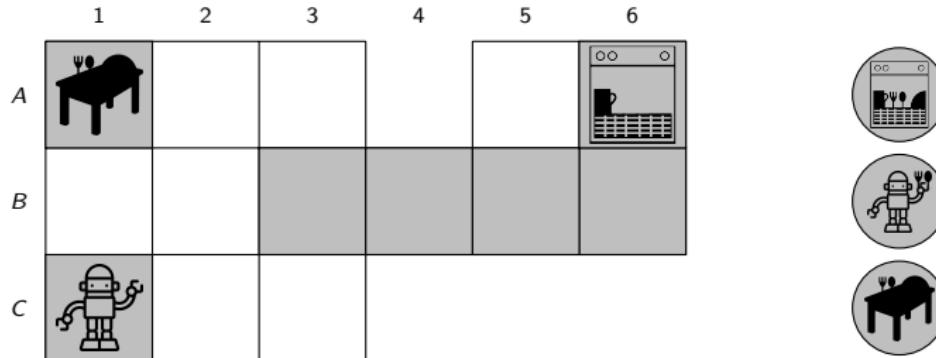
Which values do *robot-at* and *dishes-at* take in every solution?



- $\text{robot-at} = \text{C1}$, $\text{dishes-at} = \text{Table}$ (initial state)
- $\text{robot-at} = \text{B6}$, $\text{dishes-at} = \text{Dishwasher}$ (goal state)

Fact Landmarks: Example

Which values do *robot-at* and *dishes-at* take in every solution?



- $\text{robot-at} = C1$, $\text{dishes-at} = \text{Table}$ (initial state)
- $\text{robot-at} = B6$, $\text{dishes-at} = \text{Dishwasher}$ (goal state)
- $\text{robot-at} = A1, \text{robot-at} = B3, \text{robot-at} = B4, \text{robot-at} = B5, \text{robot-at} = A6$, $\text{dishes-at} = \text{Robot}$

Constraints

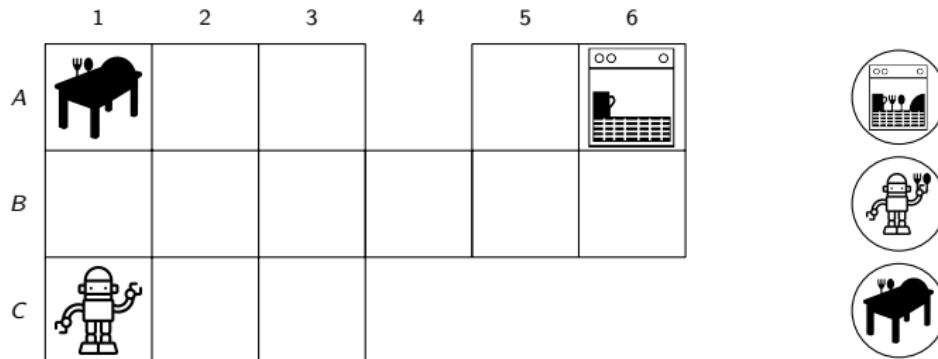
Some heuristics exploit **constraints** that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some value in at least one visited state.
(a **fact landmark** constraint)
- an action must be applied.
(an **action landmark** constraint)

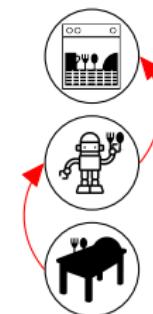
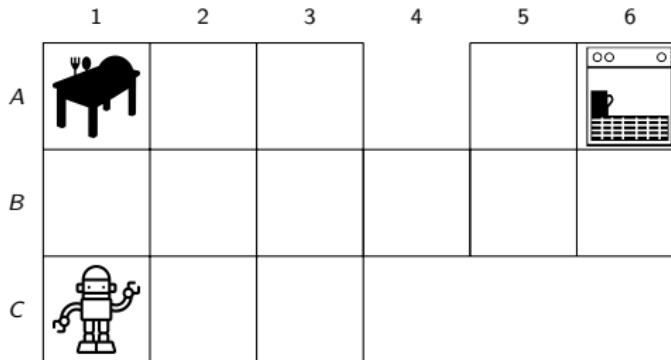
Action Landmarks: Example

Which actions must be applied in every solution?



Action Landmarks: Example

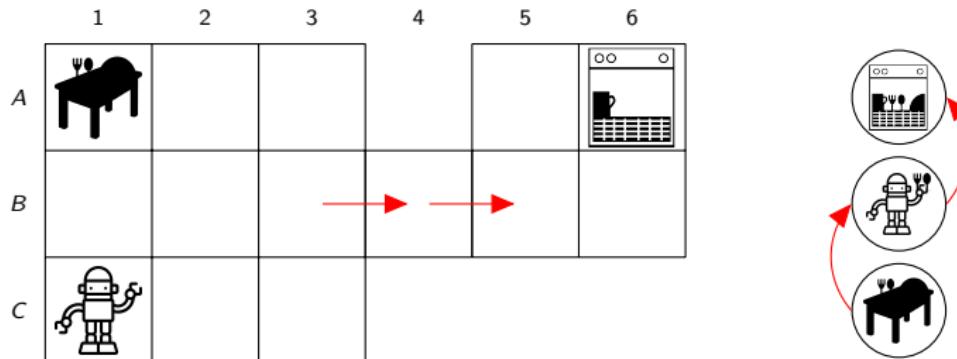
Which actions must be applied in every solution?



- pickup
- load

Action Landmarks: Example

Which actions must be applied in every solution?



- pickup
- load
- move-B3-B4
- move-B4-B5

Constraints

Some heuristics exploit **constraints** that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some **value** in at least one visited state.
(a **fact landmark** constraint)
- an action must be applied.
(an **action landmark** constraint)

Constraints

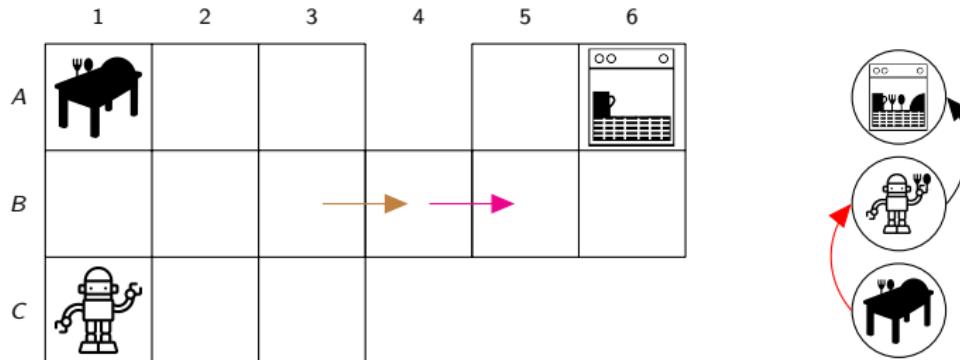
Some heuristics exploit **constraints** that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some **value** in at least one visited state.
(a **fact landmark** constraint)
- at least one action from a set of actions must be applied.
(a **disjunctive action landmark** constraint)

Disjunctive Action Landmarks: Example

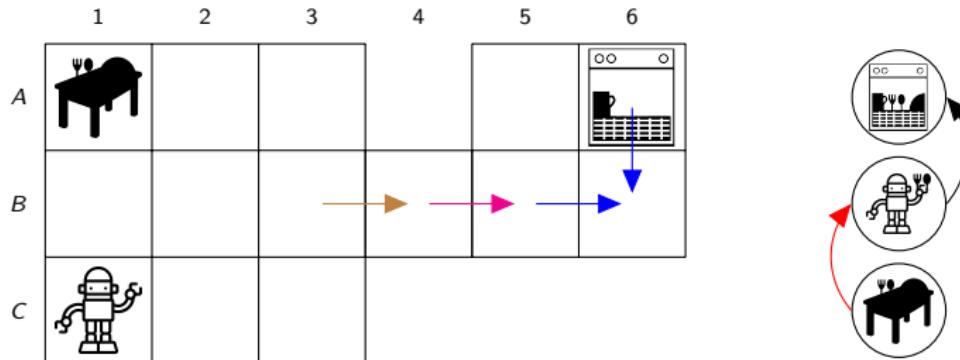
Which set of actions is such that at least one must be applied?



- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}

Disjunctive Action Landmarks: Example

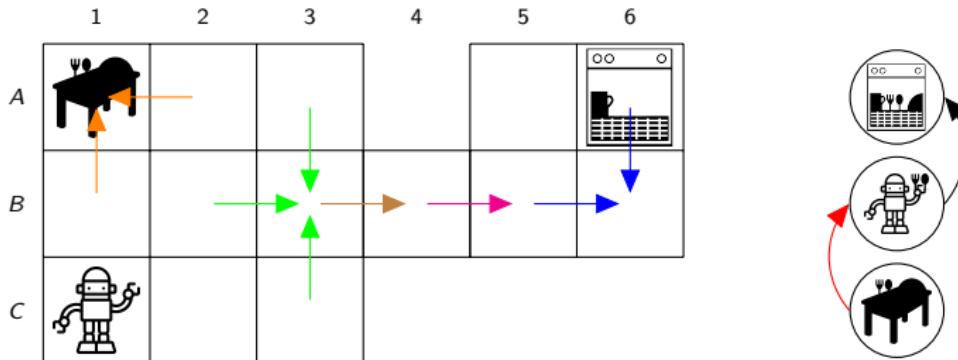
Which set of actions is such that at least one must be applied?



- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}

Disjunctive Action Landmarks: Example

Which set of actions is such that at least one must be applied?



- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}
- {move-A3-B3, move-B2-B3, move-C3-B3}
- {move-B1-A1, move-A2-A1}
- ...

Constraints

Some heuristics exploit **constraints** that describe something that holds in every solution of the task.

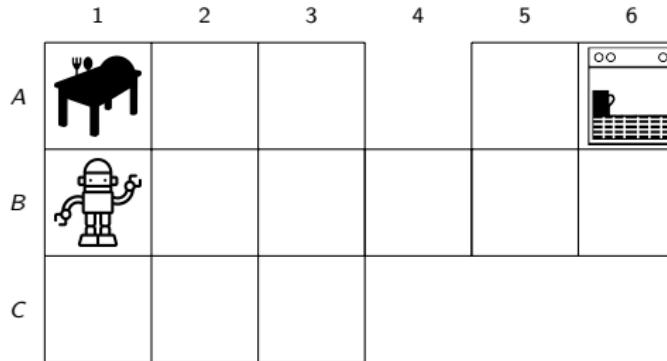
For instance, every solution is such that

- a variable takes some value in at least one visited state.
(a **fact landmark** constraint)
- at least one action from a set of actions must be applied.
(a **disjunctive action landmark** constraint)
- fact consumption and production is “balanced” .
(a **network flow** constraint)

Network Flow: Example

Consider the fact $\text{robot-at} = B2$.

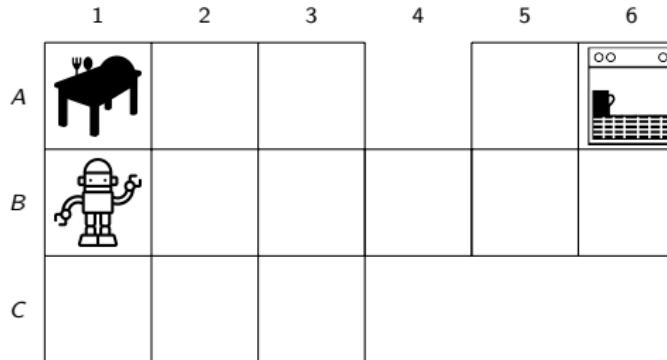
How often are actions used that enter this cell?



Network Flow: Example

Consider the fact $\text{robot-at} = B2$.

How often are actions used that enter this cell?



Answer: as often as actions that leave this cell

If Count_o denotes how often operator o is applied, we have:

$$\begin{aligned}\text{Count}_{\text{move-A1-B1}} + \text{Count}_{\text{move-B2-B1}} + \text{Count}_{\text{move-C1-B1}} = \\ \text{Count}_{\text{move-B1-A1}} + \text{Count}_{\text{move-B1-B2}} + \text{Count}_{\text{move-B1-C1}}\end{aligned}$$

Multiple Heuristics

Combining Admissible Heuristics Admissibly

Major ideas to combine heuristics admissibly:

- maximize
- canoncial heuristic (for abstractions)
- **minimum hitting set** (for landmarks)
- **cost partitioning**
- **operator counting**

Often computed as solution to a **(integer) linear program**.

Combining Heuristics Admissibly: Example

Example

Consider an FDR planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma \rangle$ with $V = \{v_1, v_2, v_3\}$ with $\text{dom}(v_1) = \{A, B\}$ and $\text{dom}(v_2) = \text{dom}(v_3) = \{A, B, C\}$, $I = \{v_1 \mapsto A, v_2 \mapsto A, v_3 \mapsto A\}$,

$$o_1 = \langle v_1 = A, v_1 := B, 1 \rangle$$

$$o_2 = \langle v_2 = A \wedge v_3 = A, v_2 := B \wedge v_3 := B, 1 \rangle$$

$$o_3 = \langle v_2 = B, v_2 := C, 1 \rangle$$

$$o_4 = \langle v_3 = B, v_3 := C, 1 \rangle$$

and $\gamma = (v_1 = B) \wedge (v_2 = C) \wedge (v_3 = C)$.

Let \mathcal{C} be the pattern collection that contains all atomic projections. What is the canonical heuristic function $h^{\mathcal{C}}$?

Combining Heuristics Admissibly: Example

Example

Consider an FDR planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma \rangle$ with $V = \{v_1, v_2, v_3\}$ with $\text{dom}(v_1) = \{A, B\}$ and $\text{dom}(v_2) = \text{dom}(v_3) = \{A, B, C\}$, $I = \{v_1 \mapsto A, v_2 \mapsto A, v_3 \mapsto A\}$,

$$o_1 = \langle v_1 = A, v_1 := B, 1 \rangle$$

$$o_2 = \langle v_2 = A \wedge v_3 = A, v_2 := B \wedge v_3 := B, 1 \rangle$$

$$o_3 = \langle v_2 = B, v_2 := C, 1 \rangle$$

$$o_4 = \langle v_3 = B, v_3 := C, 1 \rangle$$

and $\gamma = (v_1 = B) \wedge (v_2 = C) \wedge (v_3 = C)$.

Let \mathcal{C} be the pattern collection that contains all atomic projections. What is the canonical heuristic function $h^{\mathcal{C}}$?

Answer: Let $h_i := h^{v_i}$. Then $h^{\mathcal{C}} = \max \{h_1 + h_2, h_1 + h_3\}$.

Reminder: Orthogonality and Additivity

Why can we add h_1 and h_2 (h_1 and h_3) admissibly?

Theorem (Additivity for Orthogonal Abstractions)

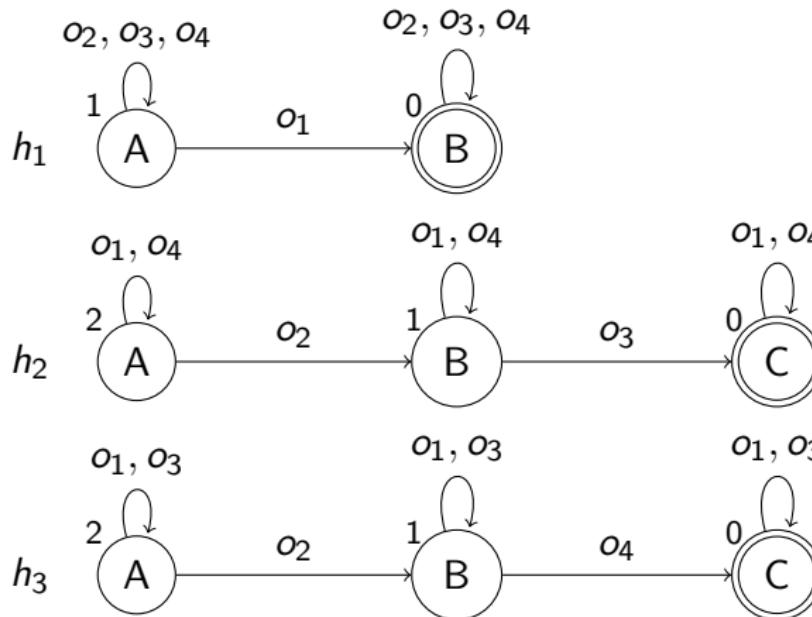
Let $h^{\alpha_1}, \dots, h^{\alpha_n}$ be abstraction heuristics of the same transition system such that α_i and α_j are orthogonal for all $i \neq j$.

Then $\sum_{i=1}^n h^{\alpha_i}$ is a safe, goal-aware, admissible and consistent heuristic for Π .

Consistency proof exploits that **every concrete transition** induces state-changing transition in **at most one abstraction**.

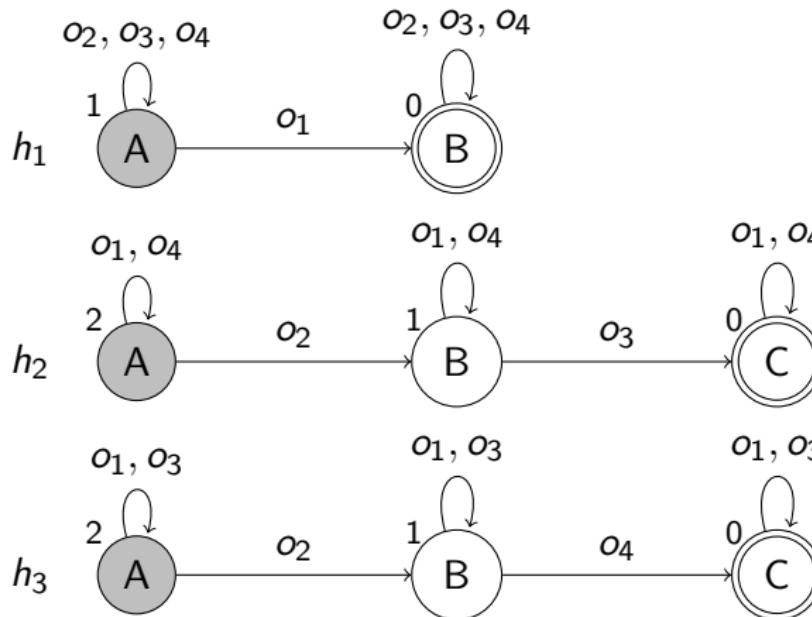
Combining Heuristics Admissibly: Example

Let $h = h_1 + h_2 + h_3$. Where is consistency violated?



Combining Heuristics Admissibly: Example

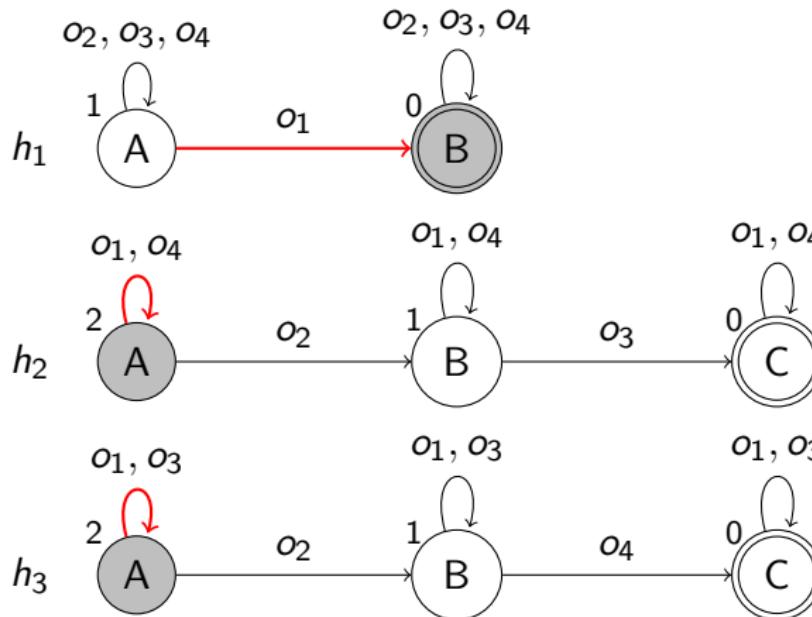
Let $h = h_1 + h_2 + h_3$. Where is consistency violated?



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

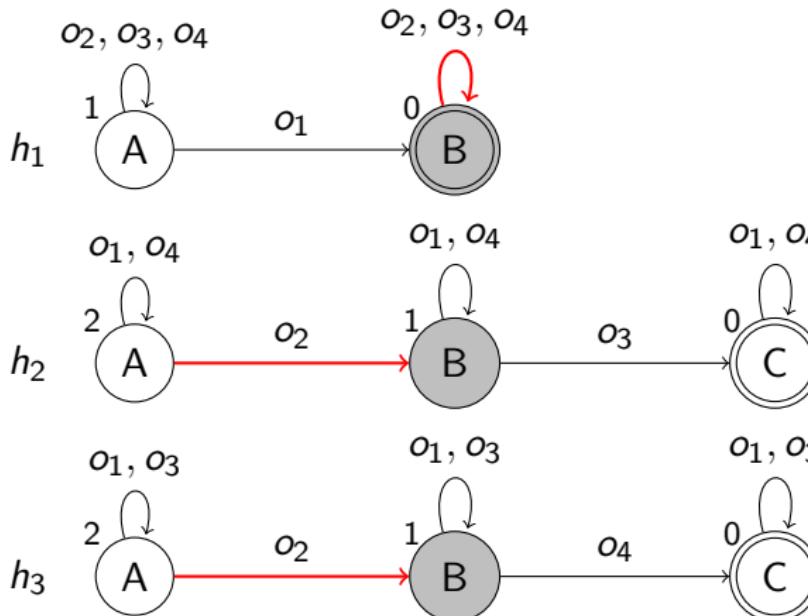
Let $h = h_1 + h_2 + h_3$. Where is consistency violated?



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

Let $h = h_1 + h_2 + h_3$. Where is consistency violated?



Here:

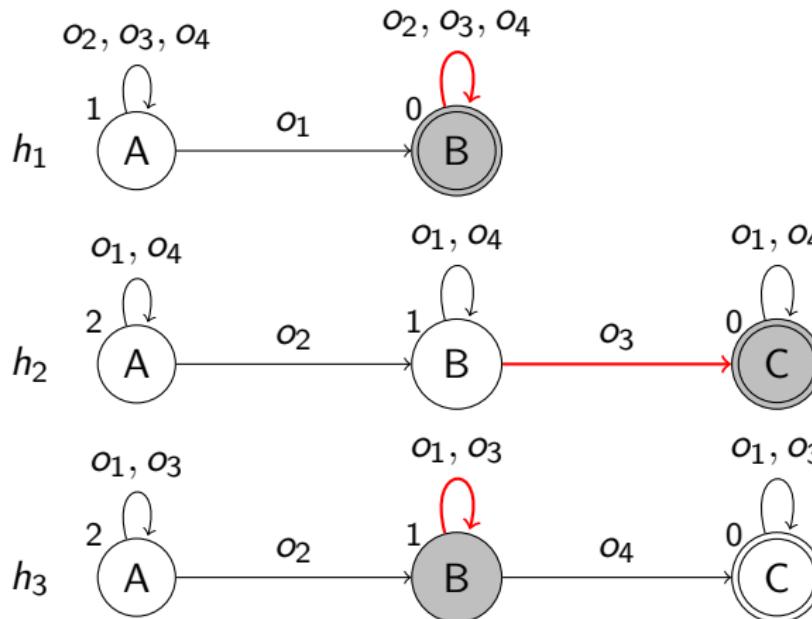
$$h(BAA) = 4$$
$$h(BBB) = 2$$

h_2 and h_3
not additive
because of o_2

Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

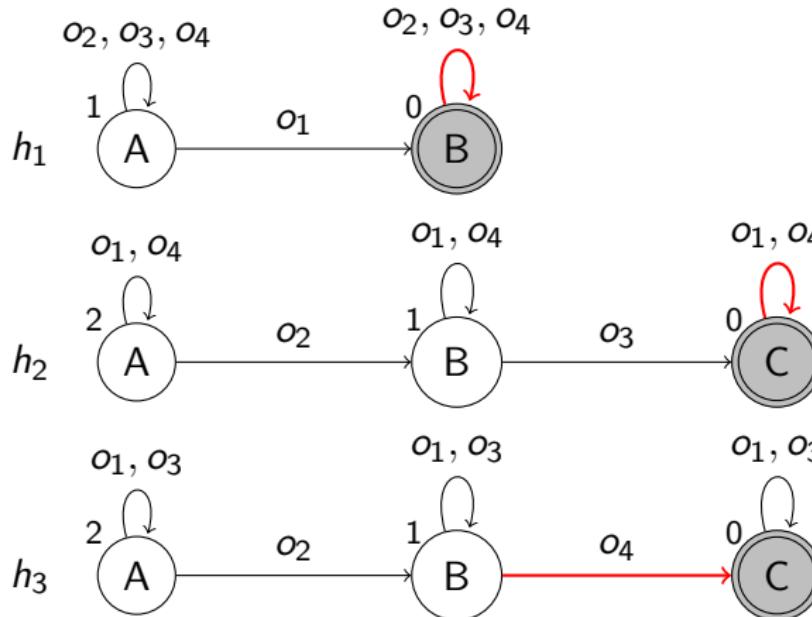
Let $h = h_1 + h_2 + h_3$. Where is consistency violated?



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

Let $h = h_1 + h_2 + h_3$. Where is consistency violated?



Consider solution $\langle o_1, o_2, o_3, \textcolor{red}{o}_4 \rangle$

Inconsistency of h_2 and h_3

The reason that h_2 and h_3 are not additive is because the cost of o_2 is considered in both.

Is there anything we can do about this?

Inconsistency of h_2 and h_3

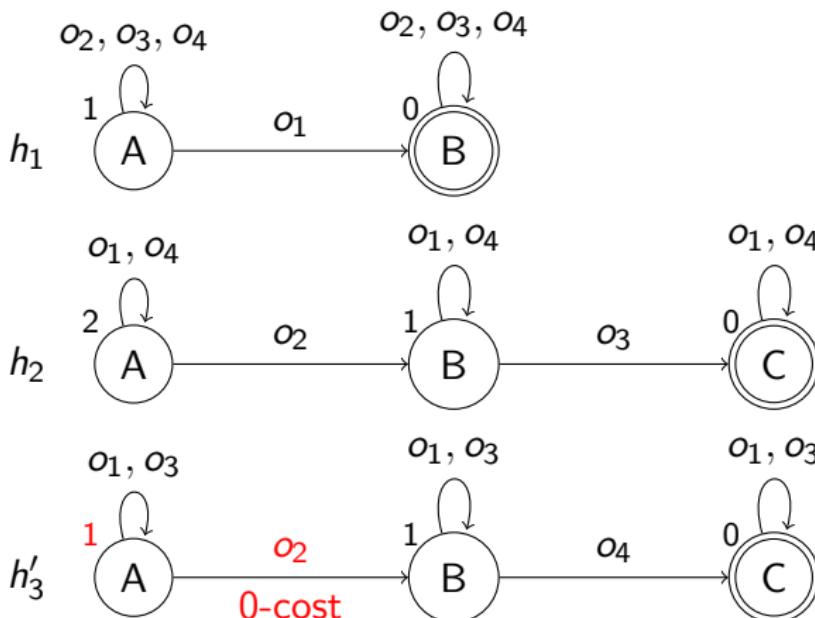
The reason that h_2 and h_3 are not additive is because the cost of o_2 is considered in both.

Is there anything we can do about this?

Solution: We can ignore the cost of o_2 in one heuristic by setting its cost to 0 (e.g., $cost_3(o_2) = 0$).

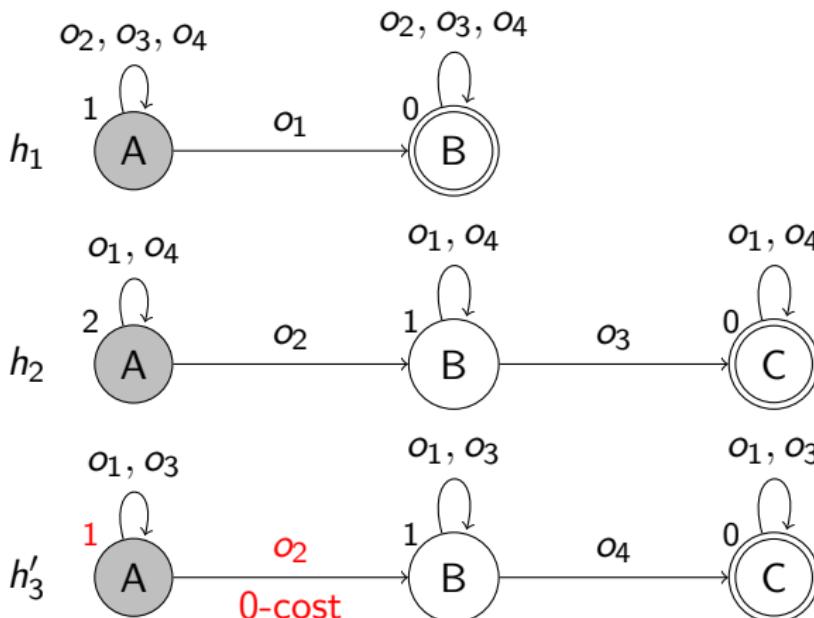
Combining Heuristics Admissibly: Example

Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



Combining Heuristics Admissibly: Example

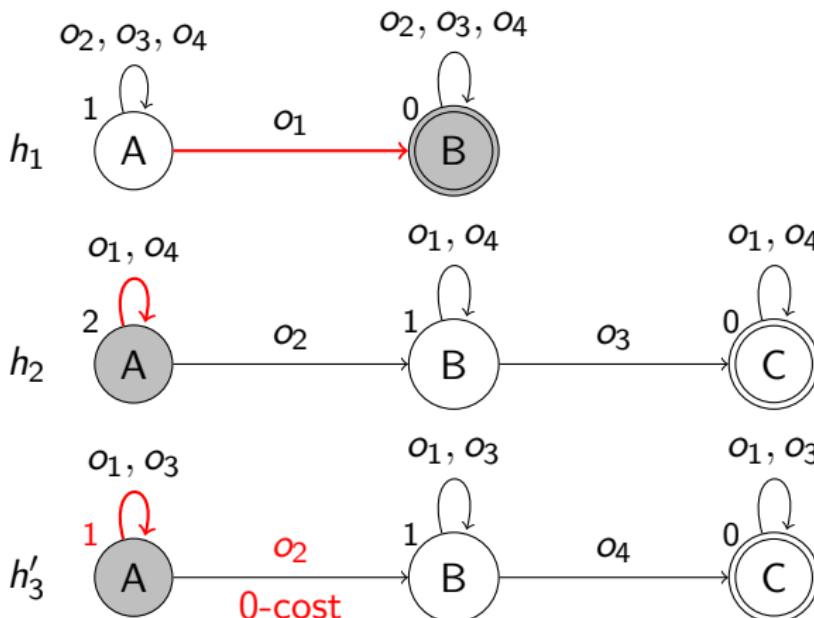
Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

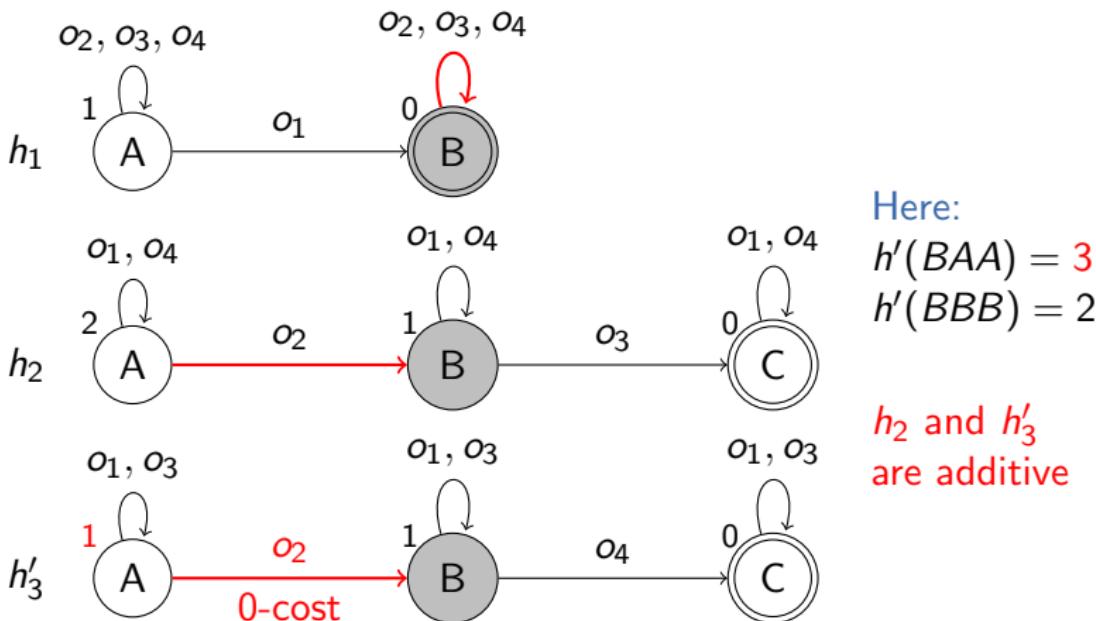
Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

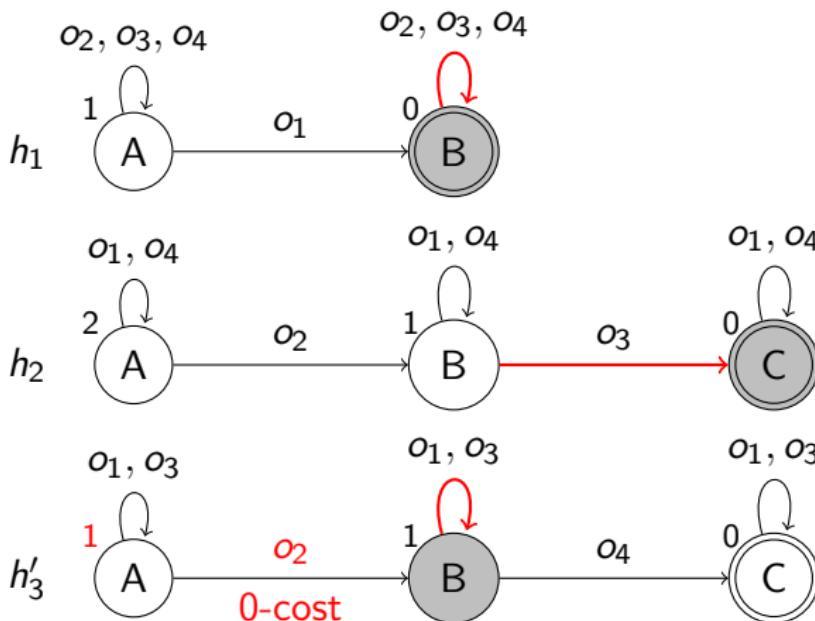
Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

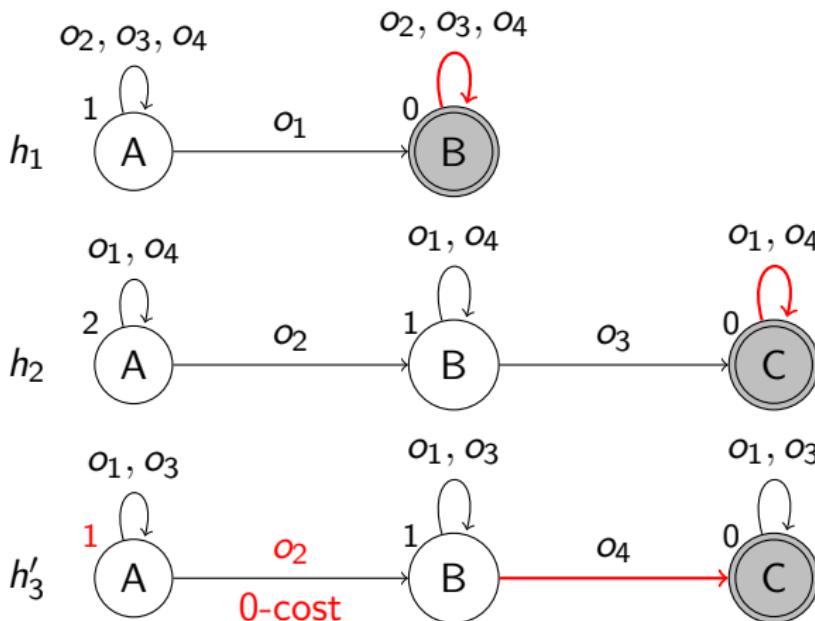
Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Combining Heuristics Admissibly: Example

Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



Consider solution $\langle o_1, o_2, o_3, o_4 \rangle$

Cost partitioning

Using the cost of every operator only in one heuristic is called a **zero-one cost partitioning**.

Cost partitioning

Using the cost of every operator only in one heuristic is called a **zero-one cost partitioning**.

More generally, heuristics are additive if all operator costs are distributed in a way that the sum of the individual costs is no larger than the cost of the operator.

This can also be expressed as a constraint,
the **cost partitioning constraint**:

$$\sum_{i=1}^n cost_i(o) \leq cost(o) \text{ for all } o \in O$$

(more details later)

Summary

Summary

- Landmarks and network flows are **constraints** that describe something that holds in every solution of the task.
- Heuristics can be combined admissibly if the **cost partitioning constraint** is satisfied.