Planning and Optimization

D8. Merge-and-Shrink: Algorithm and Heuristic Properties

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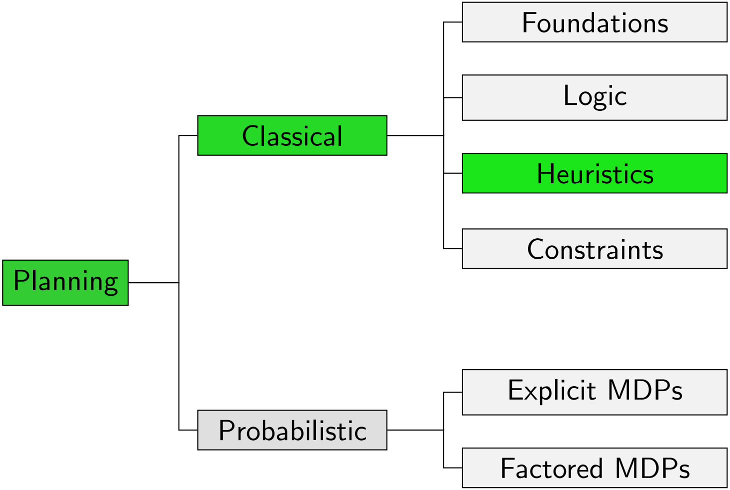
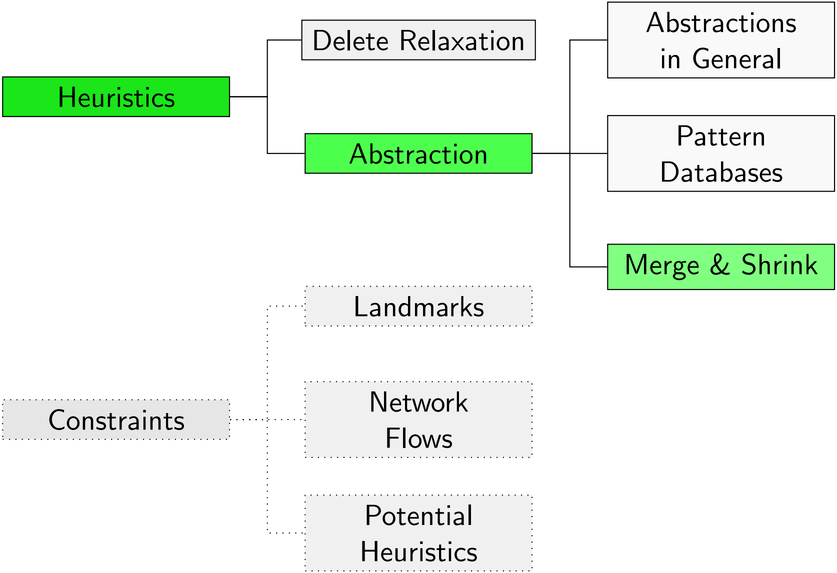
Content of this Course



Content of this Course: Heuristics



Generic Algorithm

Generic Merge-and-shrink Abstractions: Outline

Using the results of the previous chapter, we can develop a generic abstraction computation procedure that takes all state variables into account.

- Initialization: Compute the FTS consisting of all atomic projections.
- Loop: Repeatedly apply a transformation to the FTS.
  - Merging: Combine two factors by replacing them with their synchronized product.
  - Shrinking: If the factors are too large to merge, make one of them smaller by abstracting it further (applying an arbitrary abstraction to it).
- Termination: Stop when only one factor is left.

The final factor is then used for an abstraction heuristic.

Generic Algorithm Template

Generic Merge & Shrink Algorithm for planning task Π

$F := F(\Pi)$
while $|F| > 1$:
    select $type \in \{\text{merge}, \text{shrink}\}$
    if $type = \text{merge}$:
        select $\mathcal{T}_1, \mathcal{T}_2 \in F$
        $F := (F \setminus \{\mathcal{T}_1, \mathcal{T}_2\}) \cup \{\mathcal{T}_1 \otimes \mathcal{T}_2\}$
    if $type = \text{shrink}$:
        select $\mathcal{T} \in F$
        choose an abstraction mapping β on $\mathcal{T}$
        $F := (F \setminus \{\mathcal{T}\}) \cup \{\mathcal{T}^{\beta}\}$
return the remaining factor $\mathcal{T}^{\alpha}$ in F

Merge-and-Shrink Strategies

Choices to resolve to instantiate the template:

- When to merge, when to shrink?
  $\rightsquigarrow$ general strategy
- Which abstractions to merge?
  $\rightsquigarrow$ merging strategy
- Which abstraction to shrink, and how to shrink it (which β)?
  $\rightsquigarrow$ shrinking strategy

Choosing a Strategy

There are many possible ways to resolve these choices,
and we do not cover them in detail.

A typical general strategy:

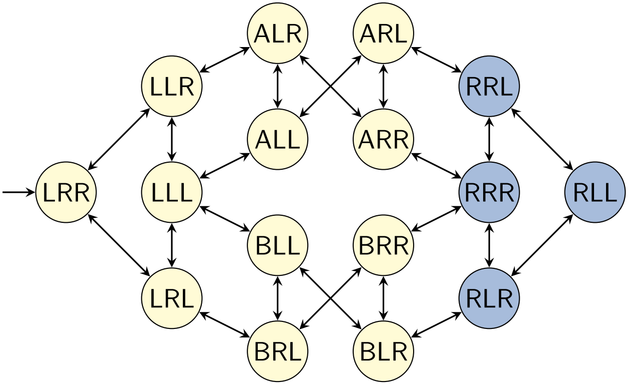
- define a limit N on the number of states allowed in each factor
- in each iteration, select two factors we would like to merge
- merge them if this does not exhaust the state number limit
- otherwise shrink one or both factors just enough
  to make a subsequent merge possible

Abstraction Mappings

- The pseudo-code as described only returns the final abstract transition system $\mathcal{T}^\alpha$.
- In practice, we also need the abstraction mapping α, so that we can map concrete states to abstract states when we need to evaluate heuristic values.
- We do not describe in detail how this can be done.
  - Key idea: keep track of which factors are merged, which factors are shrunk and how.
  - "Replay" these decisions to map a given concrete state s to the abstract state $\alpha(s)$.
- The run-time for such a heuristic look-up is $O(|V|)$ for a task with state variables V.

Example

Back to the Running Example

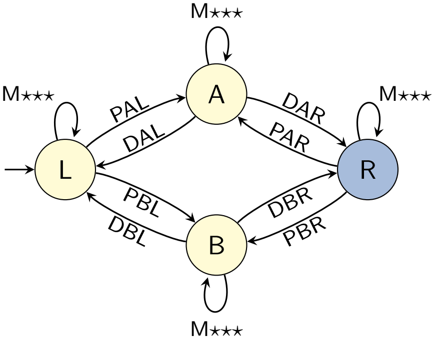


Logistics problem with one package, two trucks, two locations:

- state variable package: $\{L, R, A, B\}$
- state variable truck A: $\{L, R\}$
- state variable truck B: $\{L, R\}$

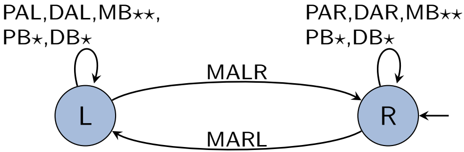
Initialization Step: Atomic Projection for Package

$\mathcal{T}^{\pi_{\{\text{package}\}}}$:



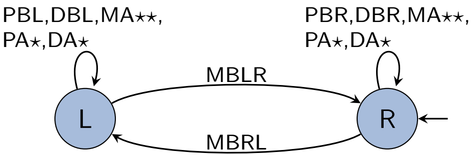
Initialization Step: Atomic Projection for Truck A

$\mathcal{T}^{\pi_{\{\text{truck A}\}}}$:



Initialization Step: Atomic Projection for Truck B

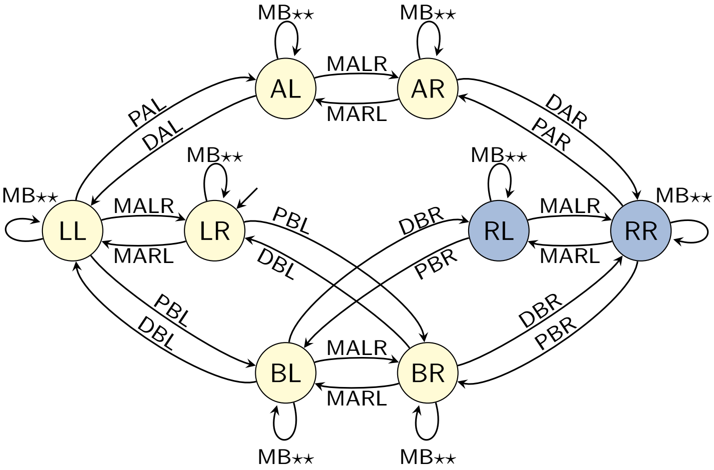
$\mathcal{T}^{\pi_{\{\text{truck B}\}}}$:



current FTS: $\{\mathcal{T}^{\pi_{\{\text{package}\}}}, \mathcal{T}^{\pi_{\{\text{truck A}\}}}, \mathcal{T}^{\pi_{\{\text{truck B}\}}}\}$

First Merge Step

$\mathcal{T}_1 := \mathcal{T}^{\pi_{\{\text{package}\}}} \otimes \mathcal{T}^{\pi_{\{\text{truck A}\}}}$:



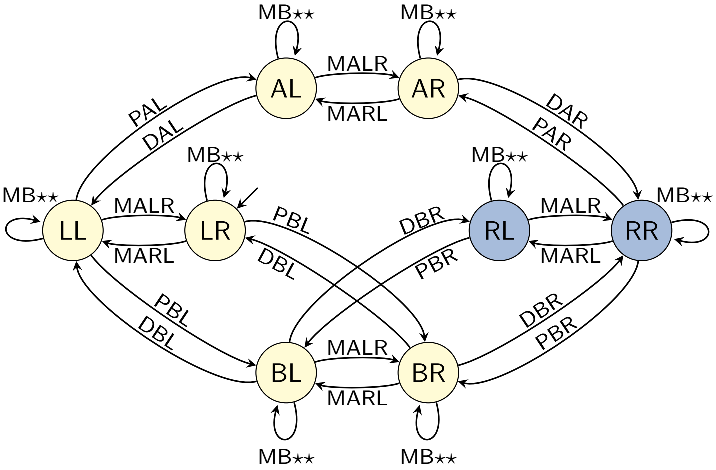
current FTS: $\{\mathcal{T}_1, \mathcal{T}^{\pi_{\{\text{truck B}\}}}\}$

Need to Shrink?

- With sufficient memory, we could now compute $\mathcal{T}_1 \otimes \mathcal{T}^{\pi_{\{\text{truck B}\}}}$ and recover the full transition system of the task.
- However, to illustrate the general idea,
  we assume that memory is too restricted:
  we may never create a factor with more than 8 states.
- To make the product fit the bound, we shrink $\mathcal{T}_1$ to 4 states.
  We can decide freely how exactly to abstract $\mathcal{T}_1$.
- In this example, we manually choose an abstraction
  that leads to a good result in the end. Making good shrinking decisions algorithmically is the job of the shrinking strategy.

First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



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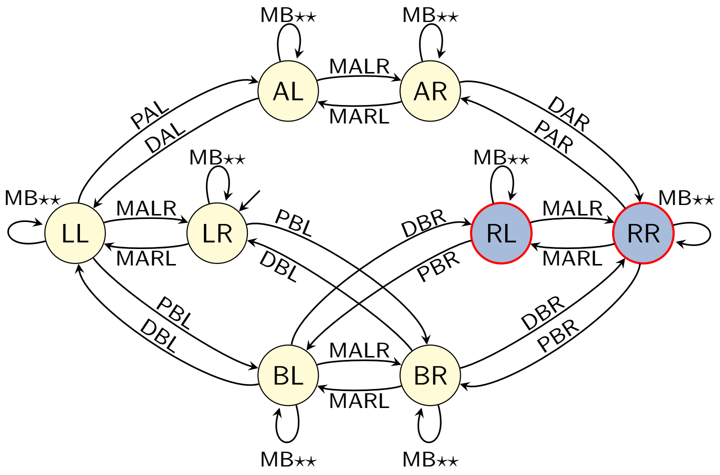

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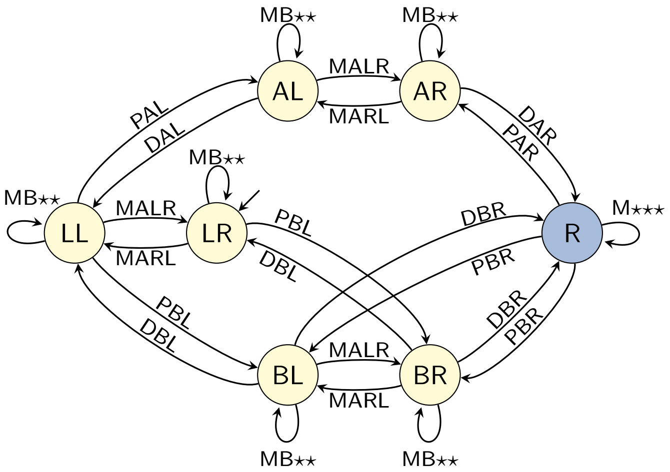

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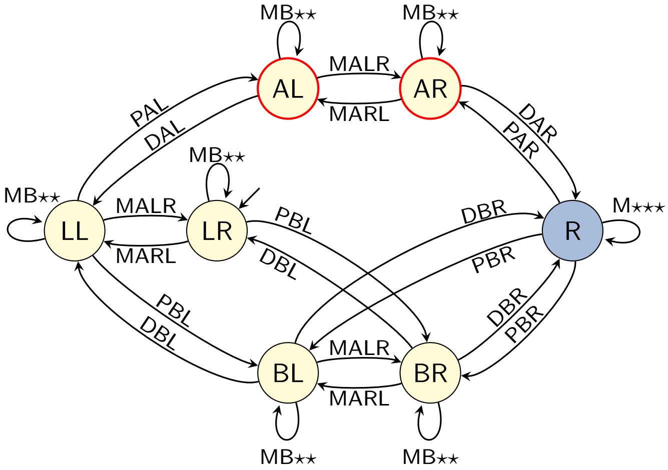

First Shrink Step

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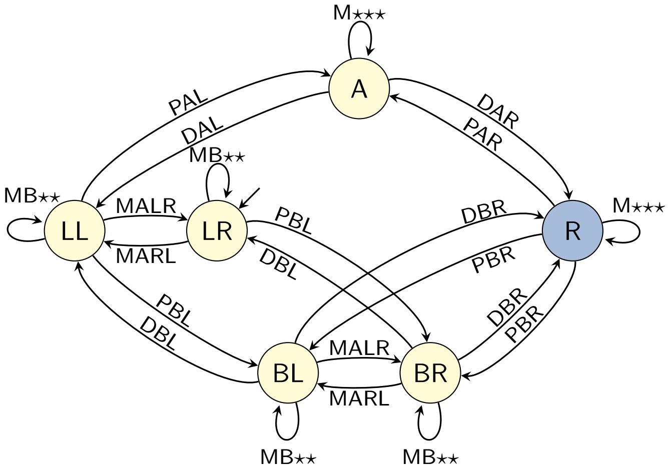
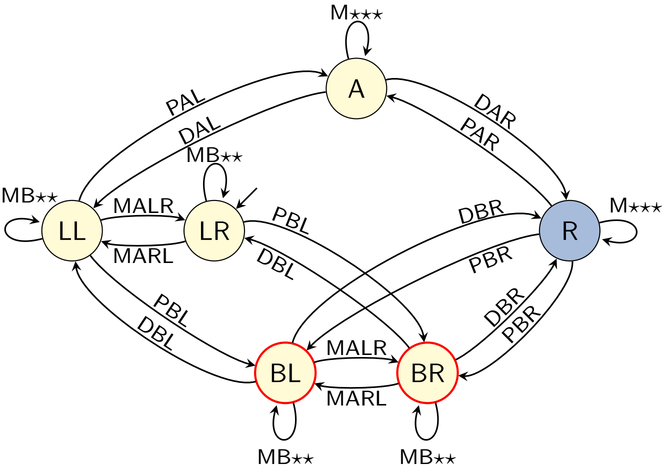


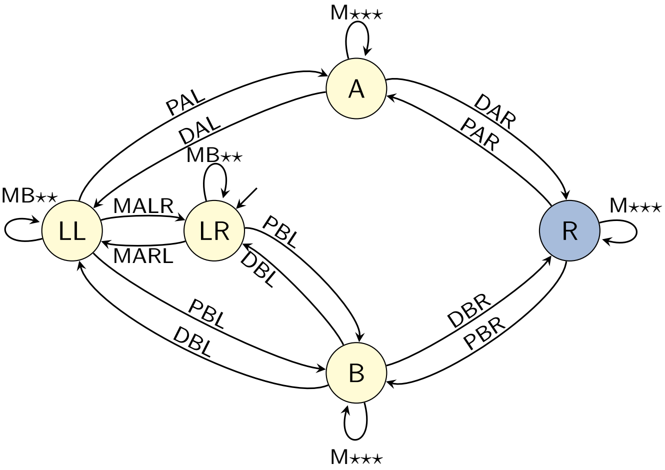
First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



First Shrink Step

$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



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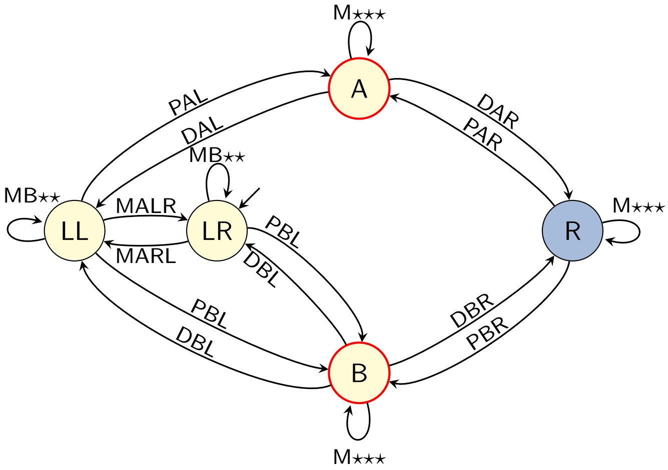
$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



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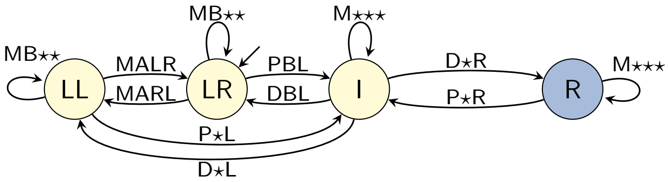
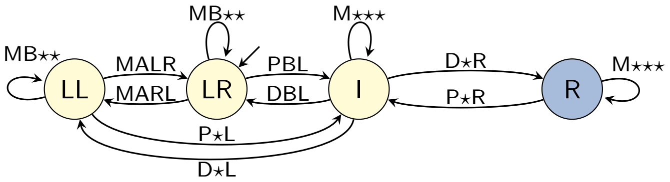


First Shrink Step

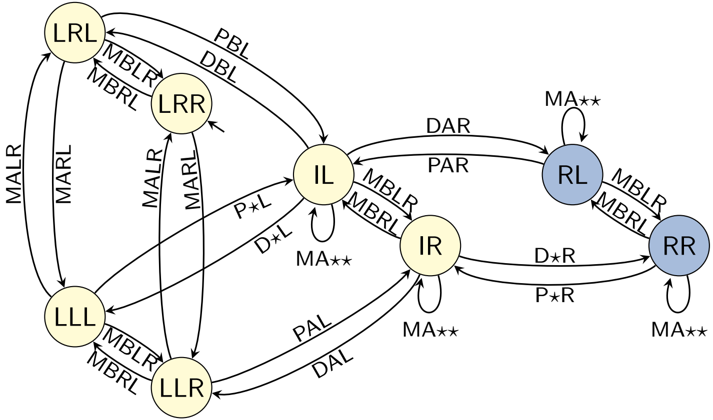
$\mathcal{T}_2 :=$ some abstraction of $\mathcal{T}_1$



current FTS: $\{\mathcal{T}_2, \mathcal{T}^{\pi_{\{\text{truck B}\}}}\}$

Second Merge Step

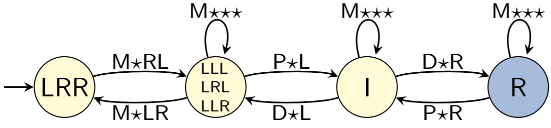
$\mathcal{T}_3 := \mathcal{T}_2 \otimes \mathcal{T}^{\pi_{\{\text{truck B}\}}}$:



current FTS: $\{\mathcal{T}_3\}$

Another Shrink Step?

- At this point, merge-and-shrink construction stops.
  The distances in the final factor define the heuristic function.
- If there were further state variables to integrate,
  we would shrink again, e.g., leading to the following
  abstraction (again with four states):



- We get a heuristic value of 3 for the initial state,
  better than any PDB heuristic that is a proper abstraction.
- The example generalizes to arbitrarily many trucks,
  even if we stick to the fixed size limit of 8.

Heuristic Properties

Properties of Merge-and-Shrink Heuristics

To understand merge-and-shrink abstractions better,
we are interested in the properties of the resulting heuristic:

- Is it admissible ($h^\alpha(s) \leq h^*(s)$ for all states s)?
- Is it consistent ($h^\alpha(s) \leq c(o) + h^\alpha(t)$ for all trans. $s \xrightarrow{o} t$)?
- Is it perfect ($h^\alpha(s) = h^*(s)$ for all states s)?

Because merge-and-shrink is a generic procedure,
the answers may depend on how exactly we instantiate it:

- size limits
- merge strategy
- shrink strategy

Merge-and-Shrink as Sequence of Transformations

- Consider a run of the merge-and-shrink construction algorithm with n iterations of the main loop.
- Let F_i $(0 \leq i \leq n)$ be the FTS F after i loop iterations.
- Let $\mathcal{T}_i$ $(0 \leq i \leq n)$ be the transition system represented by F_i, i.e., $\mathcal{T}_i = \bigotimes F_i$.
- In particular, $F_0 = F(\Pi)$ and $F_n = \{\mathcal{T}_n\}$.
- For SAS$^+$ tasks Π, we also know $\mathcal{T}_0 = \mathcal{T}(\Pi)$.

For a formal study, it is useful to view merge-and-shrink construction as a sequence of transformations from $\mathcal{T}_i$ to $\mathcal{T}_{i+1}$.

Transformations

Definition (Transformation)

Let $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$ and $\mathcal{T}' = \langle S', L, c, T', s_0', S_\star' \rangle$ be transition systems with the same labels and costs.
Let $\sigma : S \to S'$ map the states of $\mathcal{T}$ to the states of $\mathcal{T}'$.

The triple $\tau = \langle \mathcal{T}, \sigma, \mathcal{T}' \rangle$ is called a transformation from $\mathcal{T}$ to $\mathcal{T}'$.
We also write it as $\mathcal{T} \xrightarrow{\sigma} \mathcal{T}'$.

The transformation τ induces the heuristic h^τ for $\mathcal{T}$ defined as $h^\tau(s) = h^*_{\mathcal{T}'}(\sigma(s))$.

Example: If α is an abstraction mapping for transition system $\mathcal{T}$, then $\mathcal{T} \xrightarrow{\alpha} \mathcal{T}^\alpha$ is a transformation.

Special Transformations

- A transformation $\tau = \mathcal{T} \xrightarrow{\sigma} \mathcal{T}'$ is called conservative if it corresponds to an abstraction, i.e., if $\tau = \mathcal{T} \xrightarrow{\alpha} \mathcal{T}^{\alpha}$ for some abstraction mapping α.
- A transformation $\tau = \mathcal{T} \xrightarrow{\sigma} \mathcal{T}'$ is called exact if it induces the perfect heuristic, i.e., if $h^{\tau}(s) = h^{*}(s)$ for all states s of $\mathcal{T}$.

Merge transformations are always conservative and exact.

Shrink transformations are always conservative.

Composing Transformations

Merge-and-shrink performs many transformations in sequence.
We can formalize this with a notion of composition:

- Given $\tau = \mathcal{T} \xrightarrow{\sigma} \mathcal{T}'$ and $\tau' = \mathcal{T}' \xrightarrow{\sigma'} \mathcal{T}''$,
  their composition $\tau'' = \tau' \circ \tau$ is defined as $\tau'' = \mathcal{T} \xrightarrow{\sigma' \circ \sigma} \mathcal{T}''$.
- If τ and τ' are conservative, then $\tau' \circ \tau$ is conservative.
- If τ and τ' are exact, then $\tau' \circ \tau$ is exact.

Properties of Merge-and-Shrink Heuristics

We can conclude the following properties of merge-and-shrink heuristics for SAS$^+$ tasks:

- The heuristic is always admissible and consistent (because it is induced by a a composition of conservative transformations and therefore an abstraction).
- If all shrink transformation used are exact, the heuristic is perfect (because it is induced by a composition of exact transformations).

Further Topics and Literature

Further Topics in Merge and Shrink

Further topics in merge-and-shrink abstraction:

- how to keep track of the abstraction mapping
- efficient implementation
- concrete merge strategies
  - often focus on goal variables and causal connectivity (similar to hill-climbing for pattern selection)
  - sometimes based on mutexes or symmetries
- concrete shrink strategies
  - especially: h-preserving, f-preserving, bisimulation-based
  - (some) bisimulation-based shrinking strategies are exact
- other transformations besides merging and shrinking
  - especially: pruning and label reduction

Literature (1)

References on merge-and-shrink abstractions:

Klaus Dräger, Bernd Finkbeiner and Andreas Podelski.
Directed Model Checking with Distance-Preserving Abstractions.
Proc. SPIN 2006, pp. 19–34, 2006.
Introduces merge-and-shrink abstractions (for model checking) and DFP merging strategy.

Malte Helmert, Patrik Haslum and Jörg Hoffmann.
Flexible Abstraction Heuristics for Optimal Sequential Planning.
Proc. ICAPS 2007, pp. 176–183, 2007.
Introduces merge-and-shrink abstractions for planning.

Literature (2)

- Raz Nissim, Jörg Hoffmann and Malte Helmert.
  Computing Perfect Heuristics in Polynomial Time:
  On Bisimulation and Merge-and-Shrink Abstractions
  in Optimal Planning.
  Proc. IJCAI 2011, pp. 1983–1990, 2011.
  Introduces bisimulation-based shrinking.

- Malte Helmert, Patrik Haslum, Jörg Hoffmann
  and Raz Nissim.
  Merge-and-Shrink Abstraction: A Method
  for Generating Lower Bounds in Factored State Spaces.
  Journal of the ACM 61 (3), pp. 16:1–63, 2014.
  Detailed journal version of the previous two publications.

Literature (3)

Silvan Sievers, Martin Wehrle and Malte Helmert.
Generalized Label Reduction for Merge-and-Shrink Heuristics.
Proc. AAAI 2014, pp. 2358–2366, 2014.
Introduces modern version of label reduction.
(There was a more complicated version before.)

Gaojian Fan, Martin Müller and Robert Holte.
Non-linear merging strategies for merge-and-shrink based on variable interactions.
Proc. SoCS 2014, pp. 53–61, 2014.
Introduces UMC and MIASM merging strategies

Summary

Summary (1)

- Merge-and-shrink abstractions are constructed by iteratively transforming the factored transition system of a planning task.
- Merge transformations combine two factors into their synchronized product.
- Shrink transformations reduce the size of a factor by abstracting it.

Summary (2)

- Projections of SAS^+ tasks correspond to merges of atomic factors.
- By also including shrinking, merge-and-shrink abstractions generalize projections: they can reflect all state variables, but in a potentially lossy way.

Summary (3)

- Merge-and-shrink abstractions can be analyzed by viewing them as a sequence of transformations.
- We only use conservative transformations, and hence merge-and-shrink heuristics for SAS$^+$ tasks are admissible and consistent.
- Merge-and-shrink heuristics for SAS$^+$ tasks that only use exact transformations are perfect.