Planning and Optimization

C2. Delete Relaxation: Properties of Relaxed Planning Tasks

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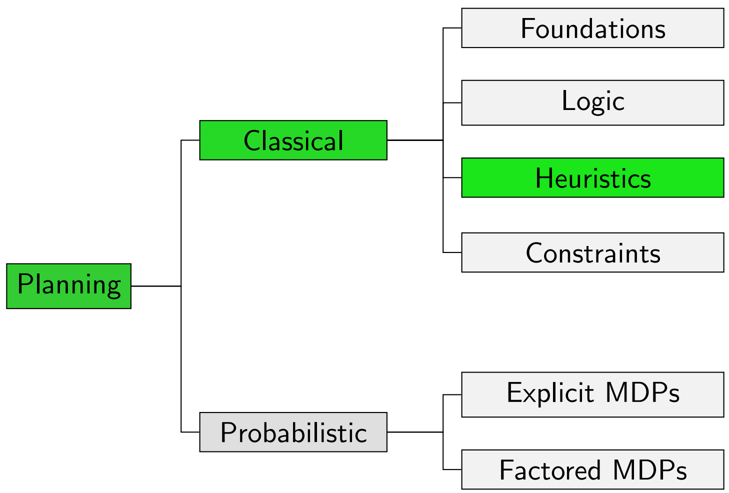

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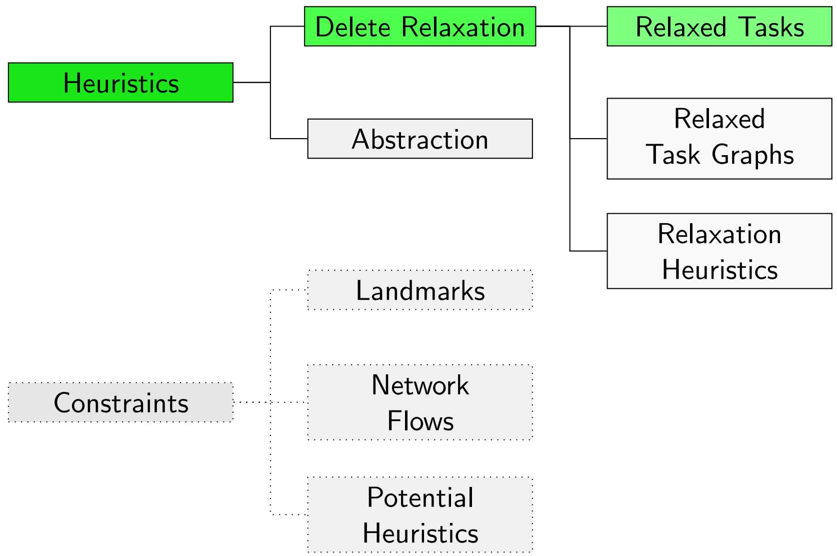

C2.1 The Domination Lemma

On-Set and Dominating States

Definition (On-Set)

The on-set of a valuation s is the set of propositional variables that are true in s, i.e., $on(s) = s^{-1}(\{\mathbf{T}\})$.

$\leadsto$ for states of propositional planning tasks: states can be viewed as sets of (true) state variables

Definition (Dominate)

A valuation s' dominates a valuation s if $on(s) \subseteq on(s')$.

$\leadsto$ all state variables true in s are also true in s'

Domination Lemma (1)

Lemma (Domination)

Let s and s' be valuations of a set of propositional variables V, and let χ be a propositional formula over V which does not contain negation symbols.

If $s \models \chi$ and s' dominates s, then $s' \models \chi$.

Proof.

Proof by induction over the structure of χ.

- Base case $\chi = \top$: then $s' \models \top$.
- Base case $\chi = \bot$: then $s \not\models \bot$.

...

Domination Lemma (2)

Proof (continued).

- Base case $\chi = v \in V$: if $s \models v$, then $v \in on(s)$. With $on(s) \subseteq on(s')$, we get $v \in on(s')$ and hence $s' \models v$.
- Inductive case $\chi = \chi_1 \land \chi_2$: by induction hypothesis, our claim holds for the proper subformulas χ_1 and χ_2 of χ.

$$\begin{aligned} s \models \chi &\Longrightarrow s \models \chi_1 \land \chi_2 \\ &\Longrightarrow s \models \chi_1 \text{ and } s \models \chi_2 \\ &\overset{\text{I.H. (twice)}}{\Longrightarrow} s' \models \chi_1 \text{ and } s' \models \chi_2 \\ &\Longrightarrow s' \models \chi_1 \land \chi_2 \\ &\Longrightarrow s' \models \chi. \end{aligned}$$

- Inductive case $\chi = \chi_1 \lor \chi_2$: analogous

$\square$

C2.2 The Relaxation Lemma

Add Sets and Delete Sets

Definition (Add Set and Delete Set for an Effect)

Consider a propositional planning task with state variables V.
Let e be an effect over V, and let s be a state over V.
The add set of e in s, written $addset(e, s)$,
and the delete set of e in s, written $delset(e, s)$,
are defined as the following sets of state variables:

$$addset(e, s) = \{v \in V \mid s \models effcond(v, e)\}$$
$$delset(e, s) = \{v \in V \mid s \models effcond(\neg v, e)\}$$

Note: For all states s and operators o applicable in s, we have $on(s[\![o]\!]) = (on(s) \setminus delset(eff(o), s)) \cup addset(eff(o), s)$.

Relaxation Lemma

For this and the following chapters on delete relaxation,
we assume implicitly that we are working with
propositional planning tasks in positive normal form.

Lemma (Relaxation)

Let s be a state, and let s' be a state that dominates s.

1. *If o is an operator applicable in s, then o^+ is applicable in s' and $s'[\![o^+]\!]$ dominates $s[\![o]\!]$.*
2. *If π is an operator sequence applicable in s, then π^+ is applicable in s' and $s'[\![\pi^+]\!]$ dominates $s[\![\pi]\!]$.*
3. *If additionally π leads to a goal state from state s, then π^+ leads to a goal state from state s'.*

Proof of Relaxation Lemma (1)

Proof.

Let V be the set of state variables.

Part 1: Because o is applicable in s, we have $s \models pre(o)$.

Because $pre(o)$ is negation-free and s' dominates s,
we get $s' \models pre(o)$ from the domination lemma.

Because $pre(o^+) = pre(o)$, this shows that o^+ is applicable in s'.

...

Proof of Relaxation Lemma (2)

Proof (continued).

To prove that $s'[\![o^+]\!]$ dominates $s[\![o]\!]$,
we first compare the relevant add sets:

$$\begin{aligned} \mathit{addset}(\mathit{eff}(o), s) &= \{v \in V \mid s \models \mathit{effcond}(v, \mathit{eff}(o))\} \\ &= \{v \in V \mid s \models \mathit{effcond}(v, \mathit{eff}(o^+))\} \quad (1) \\ &\subseteq \{v \in V \mid s' \models \mathit{effcond}(v, \mathit{eff}(o^+))\} \quad (2) \\ &= \mathit{addset}(\mathit{eff}(o^+), s'), \end{aligned}$$

where (1) uses $\mathit{effcond}(v, \mathit{eff}(o)) \equiv \mathit{effcond}(v, \mathit{eff}(o^+))$
and (2) uses the dominance lemma (note that effect conditions are negation-free for operators in positive normal form). ...

Proof of Relaxation Lemma (3)

Proof (continued).

We then get:

$$\begin{aligned} \mathit{on}(s[\![o]\!]) &= (\mathit{on}(s) \setminus \mathit{delset}(\mathit{eff}(o), s)) \cup \mathit{addset}(\mathit{eff}(o), s) \\ &\subseteq \mathit{on}(s) \cup \mathit{addset}(\mathit{eff}(o), s) \\ &\subseteq \mathit{on}(s') \cup \mathit{addset}(\mathit{eff}(o^+), s') \\ &= \mathit{on}(s'[\![o^+]\!]), \end{aligned}$$

and thus $s'[\![o^+]\!]$ dominates $s[\![o]\!]$.

This concludes the proof of Part 1. ...

Proof of Relaxation Lemma (4)

Proof (continued).

Part 2: by induction over $n = |\pi|$

Base case: $\pi = \langle\rangle$
The empty plan is trivially applicable in s', and
$s'[\![\langle\rangle^+]\!] = s'$ dominates $s[\![\langle\rangle]\!] = s$ by prerequisite.

Inductive case: $\pi = \langle o_1, \dots, o_{n+1}\rangle$
By the induction hypothesis, $\langle o_1^+, \dots, o_n^+\rangle$ is applicable in s',
and $t' = s'[\![\langle o_1^+, \dots, o_n^+\rangle]\!]$ dominates $t = s[\![\langle o_1, \dots, o_n\rangle]\!]$.
Also, o_{n+1} is applicable in t.

Using Part 1, o_{n+1}^+ is applicable in t' and $s'[\![\pi^+]\!] = t'[\![o_{n+1}^+]\!]$ dominates $s[\![\pi]\!] = t[\![o_{n+1}]\!]$.

This concludes the proof of Part 2. ...

Proof of Relaxation Lemma (5)

Proof (continued).

Part 3: Let γ be the goal formula.

From Part 2, we obtain that $t' = s'[\![\pi^+]\!]$ dominates $t = s[\![\pi]\!]$.
By prerequisite, t is a goal state and hence $t \models \gamma$.

Because the task is in positive normal form, γ is negation-free, and hence $t' \models \gamma$ because of the domination lemma.

Therefore, t' is a goal state. □

C2.3 Further Properties

Further Properties of Delete Relaxation

- The relaxation lemma is the main technical result that we will use to study delete relaxation.
- Next, we derive some further properties of delete relaxation that will be useful for us.
- Two of these are direct consequences of the relaxation lemma.

Consequences of the Relaxation Lemma (1)

Corollary (Relaxation Preserves Plans and Leads to Dominance)

Let π be an operator sequence that is applicable in state s.
Then π^+ is applicable in s and $s[\![\pi^+]\!]$ dominates $s[\![\pi]\!]$.
If π is a plan for Π, then π^+ is a plan for Π^+.

Proof.
Apply relaxation lemma with $s' = s$. □

- $\leadsto$ Relaxations of plans are relaxed plans.
- $\leadsto$ Delete relaxation is no harder to solve than original task.
- $\leadsto$ Optimal relaxed plans are never more expensive than optimal plans for original tasks.

Consequences of the Relaxation Lemma (2)

Corollary (Relaxation Preserves Dominance)

Let s be a state, let s' be a state that dominates s,
and let π^+ be a relaxed operator sequence applicable in s.
Then π^+ is applicable in s' and $s'[\![\pi^+]\!]$ dominates $s[\![\pi^+]\!]$.

Proof.
Apply relaxation lemma with π^+ for π,
noting that $(\pi^+)^+ = \pi^+$. □

- $\leadsto$ If there is a relaxed plan starting from state s, the same plan can be used starting from a dominating state s'.
- $\leadsto$ Dominating states are always "better" in relaxed tasks.

Monotonicity of Relaxed Planning Tasks

Lemma (Monotonicity)

Let s be a state in which relaxed operator o^+ is applicable. Then $s[\![o^+]\!]$ dominates s.

Proof.

Since relaxed operators only have positive effects, we have $on(s) \subseteq on(s) \cup addset(eff(o^+), s) = on(s[\![o^+]\!])$. □

$\rightsquigarrow$ Together with our previous results, this means that making a transition in a relaxed planning task never hurts.

Finding Relaxed Plans

Using the theory we developed, we are now ready to study the problem of finding plans for relaxed planning tasks.

C2.4 Greedy Algorithm

Greedy Algorithm for Relaxed Planning Tasks

The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

Greedy Planning Algorithm for $\langle V, I, O^+, \gamma\rangle$

$s := I$
$\pi^+ := \langle\rangle$
loop forever:
  if $s \models \gamma$:
    return π^+
  else if there is an operator $o^+ \in O^+$ applicable in s with $s[\![o^+]\!] \neq s$:
    Append such an operator o^+ to π^+.
    $s := s[\![o^+]\!]$
  else:
    return unsolvable

Correctness of the Greedy Algorithm

The algorithm is sound:
- If it returns a plan, this is indeed a correct solution.
- If it returns "unsolvable", the task is indeed unsolvable
  - Upon termination, there clearly is no relaxed plan from s.
  - By iterated application of the monotonicity lemma, s dominates I.
  - By the relaxation lemma, there is no solution from I.

What about completeness (termination) and runtime?
- Each iteration of the loop adds at least one atom to $on(s)$.
- This guarantees termination after at most $|V|$ iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.
  - A good implementation runs in $O(\|\Pi\|)$.

Using the Greedy Algorithm as a Heuristic

We can apply the greedy algorithm within heuristic search:
- When evaluating a state s in progression search, solve relaxation of planning task with initial state s.
- When evaluating a subgoal φ in regression search, solve relaxation of planning task with goal φ.
- Set $h(s)$ to the cost of the generated relaxed plan.

Is this an admissible heuristic?
- Yes if the relaxed plans are optimal (due to the plan preservation corollary).
- However, usually they are not, because our greedy relaxed planning algorithm is very poor.

(What about safety? Goal-awareness? Consistency?)

C2.5 Summary

Summary

- Delete relaxation is a simplification in the sense that it is never harder to solve a relaxed task than the original one.
- Delete-relaxed tasks have a domination property: it is always beneficial to make more state variables true.
- Because of their monotonicity property, delete-relaxed tasks can be solved in polynomial time by a greedy algorithm.
- However, the solution quality of this algorithm is poor.