Planning and Optimization

B4. Practical Issues of Regression Search

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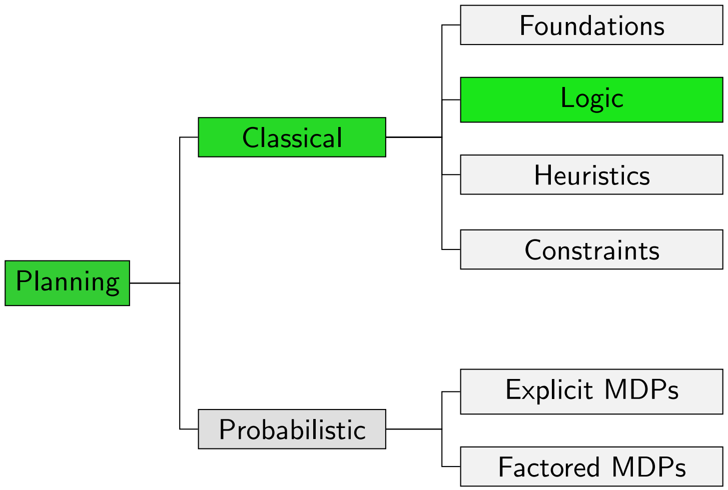
Content of this Course



Regression Search

regression search

- backward search from goal to initial state
- formulas represent sets of states
- regression computes possible predecessor states for a set of states and an operator.

B4.1 Unpromising Branches

Emptiness and Subsumption Testing

The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that $regr(\varphi, o)$ does not represent the empty set (which would mean that search is in a dead end). For example, $regr(p, \langle a, \neg p \rangle) \equiv a \wedge (\bot \vee (p \wedge \neg \top)) \equiv \bot$.
- Test that $regr(\varphi, o)$ does not represent a subset of φ (which would mean that the resulting search state is worse than φ and can be pruned). For example, $regr(a, \langle b, c \rangle) \equiv a \wedge b$.

Both of these problems are NP-complete.

B4.2 Formula Growth

Formula Growth

The formula $regr(regr(\dots regr(\varphi, o_n) \dots, o_2), o_1)$ may have size $O(|\varphi||o_1||o_2| \dots |o_{n-1}||o_n|)$, i.e., the product of the sizes of φ and the operators.
$\rightsquigarrow$ worst-case exponential size $\Omega(|\varphi|^n)$

Logical Simplifications

- $\bot \wedge \varphi \equiv \bot$, $\top \wedge \varphi \equiv \varphi$, $\bot \vee \varphi \equiv \varphi$, $\top \vee \varphi \equiv \top$
- $a \vee \varphi \equiv a \vee \varphi[\bot/a]$, $\neg a \vee \varphi \equiv \neg a \vee \varphi[\top/a]$, $a \wedge \varphi \equiv a \wedge \varphi[\top/a]$, $\neg a \wedge \varphi \equiv \neg a \wedge \varphi[\bot/a]$
- idempotence, absorption, commutativity, associativity, ...

Restricting Formula Growth in Search Trees

Problem very big formulas obtained by regression

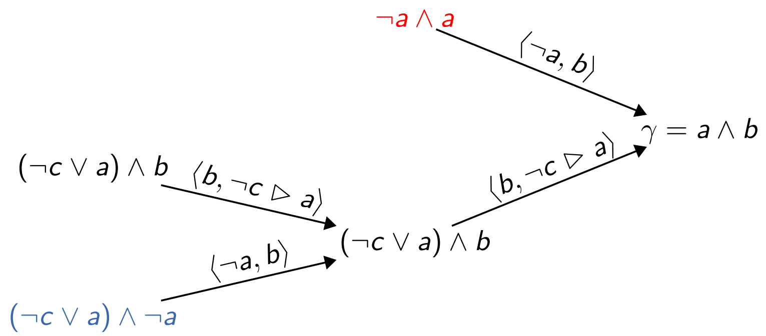
Cause disjunctivity in the (NNF) formulas
(formulas without disjunctions easily convertible to conjunctions $\ell_1 \land \cdots \land \ell_n$ where ℓ_i are literals and n is at most the number of state variables)

Idea split disjunctive formulas when generating search trees

Unrestricted Regression: Search Tree Example

Unrestricted regression: do not treat disjunctions specially

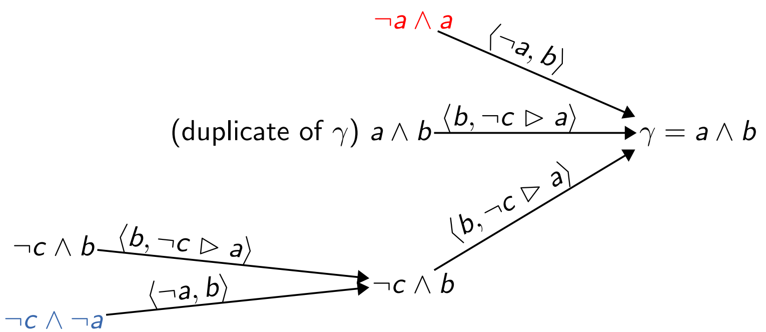
Goal $\gamma = a \land b$, initial state $I = \{a \mapsto \mathbf{F}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}\}$.



Full Splitting: Search Tree Example

Full splitting: always split all disjunctive formulas

Goal $\gamma = a \land b$, initial state $I = \{a \mapsto \mathbf{F}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}\}$.
$(\neg c \lor a) \land b$ in DNF: $(\neg c \land b) \lor (a \land b)$
$\rightsquigarrow$ split into $\neg c \land b$ and $a \land b$



General Splitting Strategies

Alternatives:

1. Do nothing (unrestricted regression).
2. Always eliminate all disjunctivity (full splitting).
3. Reduce disjunctivity if formula becomes too big.

Discussion:

- With unrestricted regression formulas may have sizes that are exponential in the number of state variables.
- With full splitting search tree can be exponentially bigger than without splitting.
- The third option lies between these two extremes.

B4.3 Summary

Summary

When applying regression in practice, we need to consider

- emptiness testing to prune dead-end search states
- subsumption testing to prune dominated search states
- logical simplifications and splitting to restrict formula growth