Planning and Optimization
A4. Planning Tasks

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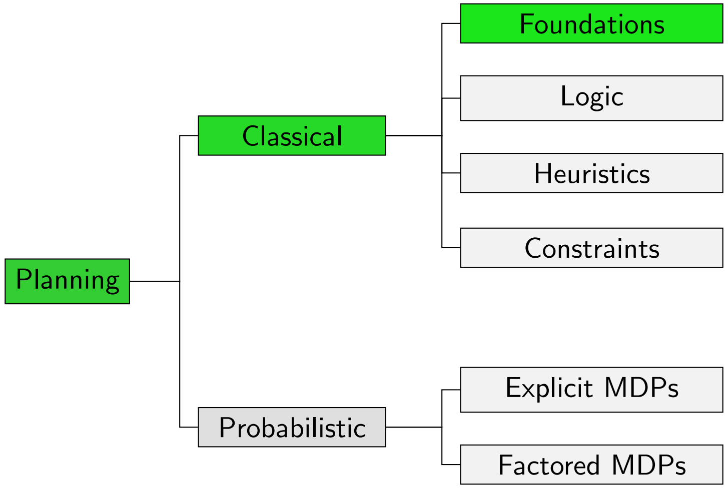
Content of this Course



A4.1 State Variables

State Variables

How to specify huge transition systems
without enumerating the states?

- represent different aspects of the world in terms of different state variables (Boolean or finite domain)
- individual state variables induce atomic propositions
  ⇝ a state is a valuation of state variables
- n Boolean state variables induce 2^n states
  ⇝ exponentially more compact than "flat" representations

Example: $O(n^2)$ Boolean variables or $O(n)$ finite-domain variables with domain size $O(n)$ suffice for blocks world with n blocks

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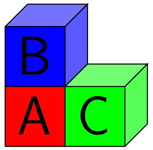
Blocks World State with Propositional Variables

Example

$$\begin{aligned}
s(\mathit{A\text{-}on\text{-}B}) &= \mathbf{F}\\
s(\mathit{A\text{-}on\text{-}C}) &= \mathbf{F}\\
s(\mathit{A\text{-}on\text{-}table}) &= \mathbf{T}\\
s(\mathit{B\text{-}on\text{-}A}) &= \mathbf{T}\\
s(\mathit{B\text{-}on\text{-}C}) &= \mathbf{F}\\
s(\mathit{B\text{-}on\text{-}table}) &= \mathbf{F}\\
s(\mathit{C\text{-}on\text{-}A}) &= \mathbf{F}\\
s(\mathit{C\text{-}on\text{-}B}) &= \mathbf{F}\\
s(\mathit{C\text{-}on\text{-}table}) &= \mathbf{T}
\end{aligned}$$



Note: it may be useful to add auxiliary state variables like *A-clear*.

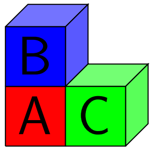
Blocks World State with Finite-Domain Variables

Example

Use three finite-domain state variables:

- *below-a*: $\{\mathrm{b}, \mathrm{c}, \mathrm{table}\}$
- *below-b*: $\{\mathrm{a}, \mathrm{c}, \mathrm{table}\}$
- *below-c*: $\{\mathrm{a}, \mathrm{b}, \mathrm{table}\}$

$$\begin{aligned}
s(\mathit{below\text{-}a}) &= \mathrm{table}\\
s(\mathit{below\text{-}b}) &= \mathrm{a}\\
s(\mathit{below\text{-}c}) &= \mathrm{table}
\end{aligned}$$



⇝ $3^3 = 27$ states

Note: it may be useful to add auxiliary state variables like *above-a*.

Propositional State Variables

Definition (Propositional State Variable)

A propositional state variable is a symbol X.

Let V be a finite set of propositional state variables.

A state s over V is a valuation for V, i.e.,
a truth assignment $s : V \to \{\mathbf{T}, \mathbf{F}\}$.

A formula over V is a propositional logic formula using V as the set of atomic propositions.

Propositional State Variables

Definition (Finite-Domain State Variable)

A finite-domain state variable is a symbol v with an associated domain $\text{dom}(v)$, which is a finite non-empty set of values.

Let V be a finite set of finite-domain state variables.

A state s over V is an assignment $s : V \to \bigcup_{v \in V} \text{dom}(v)$ such that $s(v) \in \text{dom}(v)$ for all $v \in V$.

A formula over V is a propositional logic formula whose atomic propositions are of the form $v = d$ where $v \in V$ and $d \in \text{dom}(v)$.

Slightly extending propositional logic, we treat states s over finite-domain variables as logical valuations where $s \models v = d$ iff $s(v) = d$.

State Variables: Either/Or

- State variables are the basis of compact descriptions of transition systems.
- For a given transition system, we will either use propositional or finite-domain state variables. We will not mix them.
- However, finite-domain variables can have any finite domain including the domain $\{\mathbf{T}, \mathbf{F}\}$, so are in some sense a proper generalization of propositional state variables.

From State Variables to Succinct Transition Systems

Problem:

- How to succinctly represent transitions and goal states?

Idea: Use formulas to describe sets of states

- states: all assignments to the state variables
- goal states: defined by a formula
- transitions: defined by operators (see following section)

A4.2 Operators

Syntax of Operators

Definition (Operator)

An operator o over state variables V is an object
with three properties:

- a precondition $pre(o)$, a formula over V
- an effect $eff(o)$ over V, defined on the following slides
- a cost $cost(o) \in \mathbb{R}_0^+$

Notes:

- Operators are also called actions.
- Operators are often written as triples $\langle pre(o), eff(o), cost(o)\rangle$.
- This can be abbreviated to pairs $\langle pre(o), eff(o)\rangle$ when the cost of the operator is irrelevant.

Operators: Intuition

Intuition for operators o:

- The operator precondition describes the set of states in which a transition labeled with o can be taken.
- The operator effect describes how taking such a transition changes the state.
- The operator cost describes the cost of taking a transition labeled with o.

Syntax of Effects

Definition (Effect)

Effects over state variables V are inductively defined as follows:

- If $v \in V$ is a propositional state variable, then v and $\neg v$ are effects (atomic effect).
- If $v \in V$ is a finite-domain state variable and $d \in \mathrm{dom}(v)$, then $v := d$ is an effect (atomic effect).
- If $e_1, \dots, e_n$ are effects, then $(e_1 \wedge \dots \wedge e_n)$ is an effect (conjunctive effect).
  The special case with $n = 0$ is the empty effect $\top$.
- If χ is a formula over V and e is an effect, then $(\chi \rhd e)$ is an effect (conditional effect).

Parentheses can be omitted when this does not cause ambiguity.

Effects: Intuition

Intuition for effects:

- Atomic effects can be understood as assignments that update the value of a state variable.
  - For propositional state variables, v means "$v := \mathbf{T}$" and $\neg v$ means "$v := \mathbf{F}$".
- A conjunctive effect $e = (e_1 \wedge \dots \wedge e_n)$ means that all subeffects $e_1, \dots, e_n$ take place simultaneously.
- A conditional effect $e = (\chi \rhd e')$ means that subeffect e' takes place iff χ is true in the state where e takes place.

Semantics of Effects

Definition (Effect Condition for an Effect)

Let e be an atomic effect.
The effect condition $\mathit{effcond}(e, e')$ under which e triggers given the effect e' is a propositional formula defined as follows:

- $\mathit{effcond}(e, e) = \top$
- $\mathit{effcond}(e, e') = \bot$ for atomic effects $e' \neq e$
- $\mathit{effcond}(e, (e_1 \wedge \cdots \wedge e_n)) = \mathit{effcond}(e, e_1) \vee \cdots \vee \mathit{effcond}(e, e_n)$
- $\mathit{effcond}(e, (\chi \rhd e')) = \chi \wedge \mathit{effcond}(e, e')$

Intuition: $\mathit{effcond}(e, e')$ represents the condition that must be true in the current state for the effect e' to lead to the atomic effect e

Semantics of Operators: Propositional Case

Definition (Applicable, Resulting State)

Let V be a set of propositional state variables.
Let s be a state over V, and let o be an operator over V.
Operator o is applicable in s if $s \models \mathit{pre}(o)$.

If o is applicable in s, the resulting state of applying o in s, written $s[\![o]\!]$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \wedge \neg \mathit{effcond}(v, e) \\ s(v) & \text{if } s \not\models \mathit{effcond}(v, e) \vee \mathit{effcond}(\neg v, e) \end{cases}$$

where $e = \mathit{eff}(o)$.

Add-after-Delete Semantics

Note:

- The definition implies that if a variable is simultaneously "added" (set to **T**) and "deleted" (set to **F**), the value **T** takes precedence.
- This is called add-after-delete semantics.
- This detail of semantics is somewhat arbitrary, but has proven useful in applications.
- For finite-domain variables, there are no distinguished values like "true" and "false", and a different semantics is used.

Conflicting Effects and Consistency Condition

- What should an effect of the form $v := a \wedge v := b$ mean?
- For finite-domain representations, the accepted semantics is to make this illegal, i.e., to make an operator inapplicable if it would lead to conflicting effects.

Definition (Consistency Condition)

Let e be an effect over finite-domain state variables V.

The consistency condition for e, $\mathit{consist}(e)$ is defined as

$$\bigwedge_{v \in V} \bigwedge_{d, d' \in \mathrm{dom}(v), d \neq d'} \neg(\mathit{effcond}(v := d, e) \wedge \mathit{effcond}(v := d', e)).$$

Semantics of Operators: Finite-Domain Case

Definition (Applicable, Resulting State)

Let V be a set of finite-domain state variables.
Let s be a state over V, and let o be an operator over V.
Operator o is applicable in s if $s \models \textit{pre}(o) \land \textit{consist}(\textit{eff}(o))$.

If o is applicable in s, the resulting state of applying o in s, written $s\llbracket o \rrbracket$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} d & \text{if } s \models \textit{effcond}(v := d, \textit{eff}(o)) \text{ for some } d \in \text{dom}(v) \\ s(v) & \text{otherwise} \end{cases}$$

Applying Operators: Example

Example

Consider the operator $o = \langle a, \neg a \land (\neg c \rhd \neg b)\rangle$
and the state $s = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.

The operator o is applicable in s because $s \models a$.

Effect conditions of $\textit{eff}(o)$:

$$\begin{aligned} \textit{effcond}(a, \textit{eff}(o)) &= \textit{effcond}(a, \neg a \land (\neg c \rhd \neg b)) \\ &= \textit{effcond}(a, \neg a) \lor \textit{effcond}(a, \neg c \rhd \neg b) \\ &= \bot \lor (\neg c \land \textit{effcond}(a, \neg b)) \\ &= \bot \lor (\neg c \land \bot) \\ &\equiv \bot \quad \rightsquigarrow \text{ false in state } s \end{aligned}$$

Applying Operators: Example

Example

Consider the operator $o = \langle a, \neg a \land (\neg c \rhd \neg b)\rangle$
and the state $s = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.

The operator o is applicable in s because $s \models a$.

Effect conditions of $\textit{eff}(o)$:

$$\begin{aligned} \textit{effcond}(\neg a, \textit{eff}(o)) &= \textit{effcond}(\neg a, \neg a \land (\neg c \rhd \neg b)) \\ &= \textit{effcond}(\neg a, \neg a) \lor \textit{effcond}(\neg a, \neg c \rhd \neg b) \\ &= \top \lor \textit{effcond}(\neg a, \neg c \rhd \neg b) \\ &\equiv \top \quad \rightsquigarrow \text{ true in state } s \end{aligned}$$

Applying Operators: Example

Example

Consider the operator $o = \langle a, \neg a \land (\neg c \rhd \neg b)\rangle$
and the state $s = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.

The operator o is applicable in s because $s \models a$.

Effect conditions of $\textit{eff}(o)$:

$$\begin{aligned} \textit{effcond}(b, \textit{eff}(o)) &= \textit{effcond}(b, \neg a \land (\neg c \rhd \neg b)) \\ &= \textit{effcond}(b, \neg a) \lor \textit{effcond}(b, \neg c \rhd \neg b) \\ &= \bot \lor (\neg c \land \textit{effcond}(b, \neg b)) \\ &= \bot \lor (\neg c \land \bot) \\ &\equiv \bot \quad \rightsquigarrow \text{ false in state } s \end{aligned}$$

Applying Operators: Example

Example

Consider the operator $o = \langle a, \neg a \wedge (\neg c \rhd \neg b) \rangle$ and the state $s = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.

The operator o is applicable in s because $s \models a$.

Effect conditions of $\mathit{eff}(o)$:

$$\begin{aligned} \mathit{effcond}(\neg b, \mathit{eff}(o)) &= \mathit{effcond}(\neg b, \neg a \wedge (\neg c \rhd \neg b)) \\ &= \mathit{effcond}(\neg b, \neg a) \vee \mathit{effcond}(\neg b, \neg c \rhd \neg b) \\ &= \bot \vee (\neg c \wedge \mathit{effcond}(\neg b, \neg b)) \\ &= \bot \vee (\neg c \wedge \top) \\ &\equiv \neg c \quad \rightsquigarrow \text{ false in state } s \end{aligned}$$

Applying Operators: Example

Example

Consider the operator $o = \langle a, \neg a \wedge (\neg c \rhd \neg b) \rangle$ and the state $s = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.

The operator o is applicable in s because $s \models a$.

Effect conditions of $\mathit{eff}(o)$:

$$\begin{aligned} \mathit{effcond}(c, \mathit{eff}(o)) &\equiv \bot \quad \rightsquigarrow \text{ false in state } s \\ \mathit{effcond}(\neg c, \mathit{eff}(o)) &\equiv \bot \quad \rightsquigarrow \text{ false in state } s \\ \mathit{effcond}(d, \mathit{eff}(o)) &\equiv \bot \quad \rightsquigarrow \text{ false in state } s \\ \mathit{effcond}(\neg d, \mathit{eff}(o)) &\equiv \bot \quad \rightsquigarrow \text{ false in state } s \end{aligned}$$

The resulting state of applying o in s is the state $\{a \mapsto \mathbf{F}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.

Example Operators: Blocks World

Example (Blocks World Operators)

To model blocks world operators conveniently, we use auxiliary state variables *A-clear*, *B-clear*, and *C-clear* to express that there is nothing on top of a given block.

Then blocks world operators can be modeled as:

- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}table} \wedge \mathit{B\text{-}clear},\ \mathit{A\text{-}on\text{-}B} \wedge \neg \mathit{A\text{-}on\text{-}table} \wedge \neg \mathit{B\text{-}clear} \rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}table} \wedge \mathit{C\text{-}clear},\ \mathit{A\text{-}on\text{-}C} \wedge \neg \mathit{A\text{-}on\text{-}table} \wedge \neg \mathit{C\text{-}clear} \rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}B},\ \mathit{A\text{-}on\text{-}table} \wedge \neg \mathit{A\text{-}on\text{-}B} \wedge \mathit{B\text{-}clear} \rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}C},\ \mathit{A\text{-}on\text{-}table} \wedge \neg \mathit{A\text{-}on\text{-}C} \wedge \mathit{C\text{-}clear} \rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}B} \wedge \mathit{C\text{-}clear},\ \mathit{A\text{-}on\text{-}C} \wedge \neg \mathit{A\text{-}on\text{-}B} \wedge \mathit{B\text{-}clear} \wedge \neg \mathit{C\text{-}clear} \rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}C} \wedge \mathit{B\text{-}clear},\ \mathit{A\text{-}on\text{-}B} \wedge \neg \mathit{A\text{-}on\text{-}C} \wedge \mathit{C\text{-}clear} \wedge \neg \mathit{B\text{-}clear} \rangle$
- ...

Example Operator: 4-Bit Counter

Example (Incrementing a 4-Bit Counter)

Operator to increment a 4-bit number $b_3 b_2 b_1 b_0$ represented by 4 state variables $b_0, \ldots, b_3$:

precondition:

$$\neg b_0 \vee \neg b_1 \vee \neg b_2 \vee \neg b_3$$

effect:

$$\begin{gathered} (\neg b_0 \rhd b_0) \wedge \\ ((\neg b_1 \wedge b_0) \rhd (b_1 \wedge \neg b_0)) \wedge \\ ((\neg b_2 \wedge b_1 \wedge b_0) \rhd (b_2 \wedge \neg b_1 \wedge \neg b_0)) \wedge \\ ((\neg b_3 \wedge b_2 \wedge b_1 \wedge b_0) \rhd (b_3 \wedge \neg b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{gathered}$$

A4.3 Planning Tasks

Planning Tasks

Definition (Planning Task)

A planning task is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- V is a finite set of state variables,
- I is a valuation over V called the initial state,
- O is a finite set of operators over V, and
- γ is a formula over V called the goal.

V must either consist only of propositional or only of finite-domain state variables.

In the first case, Π is called a propositional planning task, otherwise an FDR planning task (finite-domain representation).

Note: Whenever we just say planning task (without "propositional" or "FDR"), both kinds of tasks are allowed.

Mapping Planning Tasks to Transition Systems

Definition (Transition System Induced by a Planning Task)

The planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- S is the set of all states over V,
- L is the set of operators O,
- $c(o) = \mathit{cost}(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

Planning Tasks: Terminology

- Terminology for transitions systems is also applied to the planning tasks Π that induce them.
- For example, when we speak of the states of Π, we mean the states of $\mathcal{T}(\Pi)$.
- A sequence of operators that forms a solution of $\mathcal{T}(\Pi)$ is called a plan of Π.

Satisficing and Optimal Planning

By planning, we mean the following two algorithmic problems:

Definition (Satisficing Planning)

Given: a planning task Π

Output: a plan for Π, or **unsolvable** if no plan for Π exists

Definition (Optimal Planning)

Given: a planning task Π

Output: a plan for Π with minimal cost among all plans for Π, or **unsolvable** if no plan for Π exists

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A4.4 Summary

Summary

- Planning tasks compactly represent transition systems and are suitable as inputs for planning algorithms.
- They are based on concepts from propositional logic, enhanced to model state change.
- Planning tasks can be propositional or finite-domain.
- States of planning tasks are assignments to its state variables.
- Operators of propositional planning tasks describe in which situations (precondition), how (effect) and at which cost the state of the world can be changed.
- In satisficing planning, we must find a solution for a planning task (or show that no solution exists).
- In optimal planning, we must additionally guarantee that generated solutions are of minimal cost.