Planning and Optimization

G6. Monte-Carlo Tree Search: Algorithms Part I

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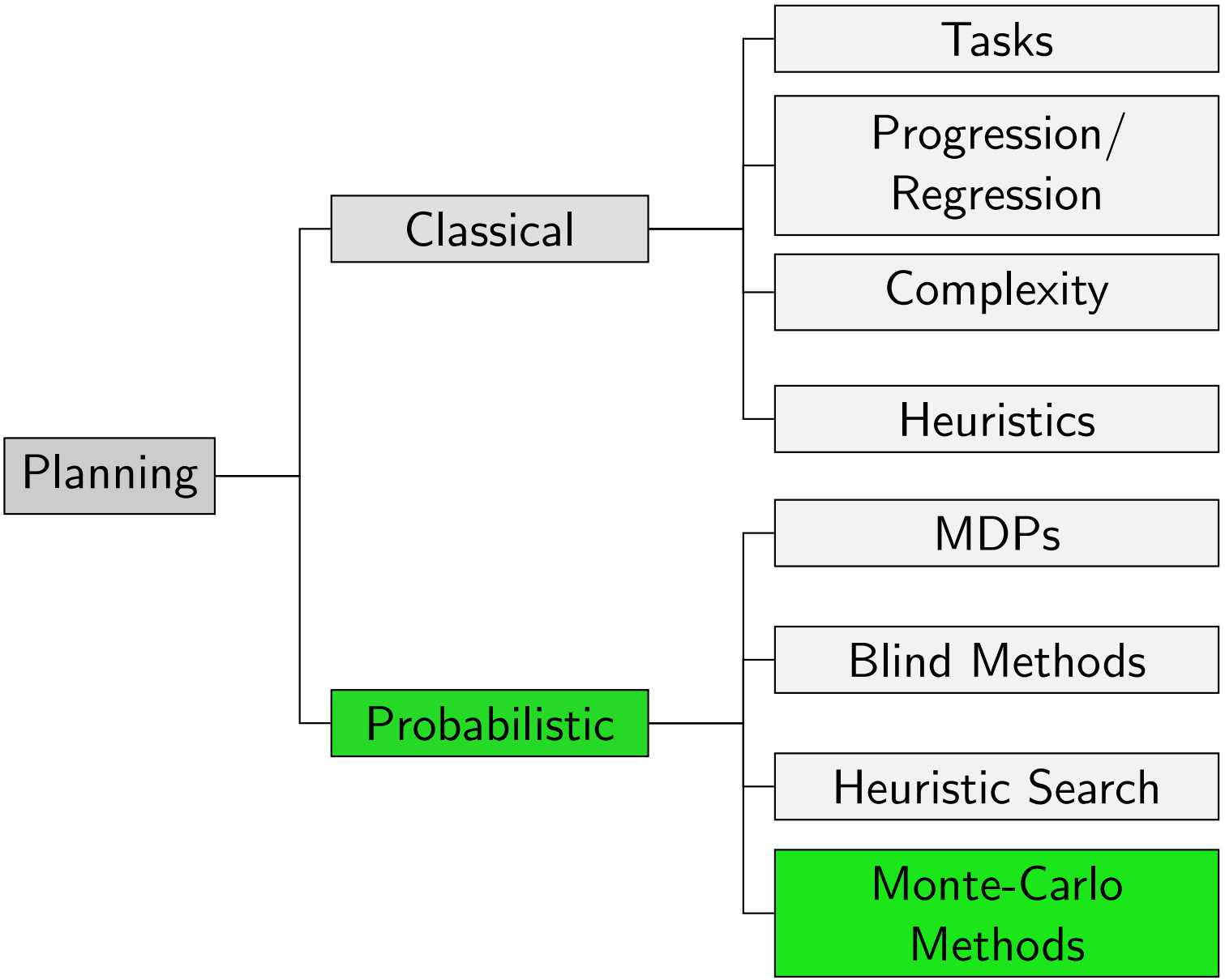
Content of this Course



Motivation

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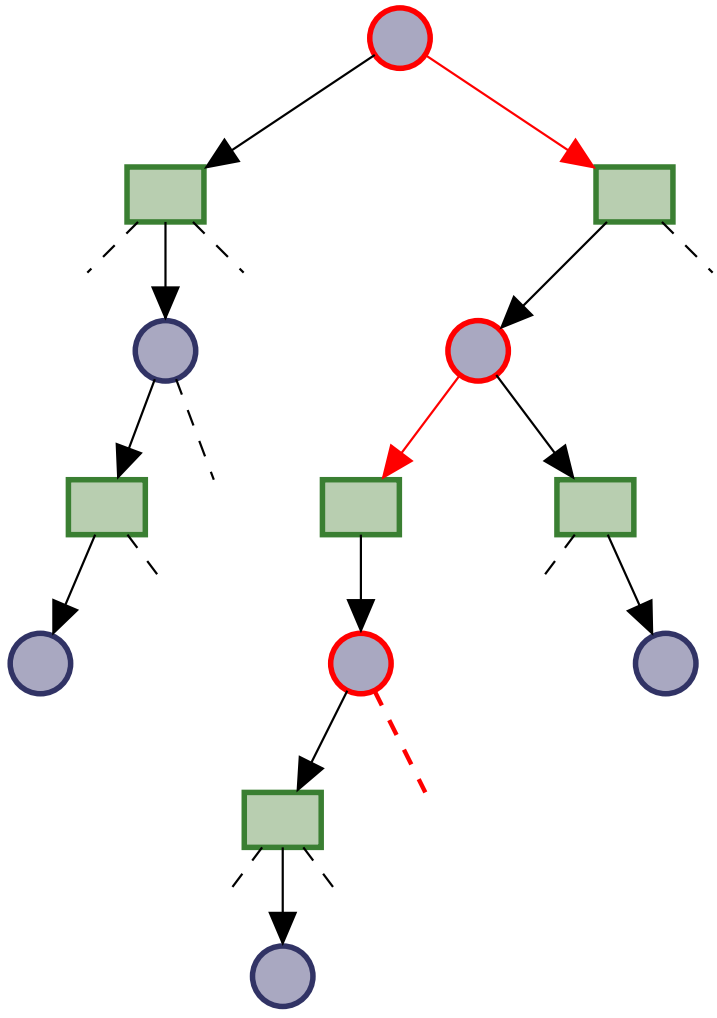
- Monte-Carlo Tree Search is a framework of algorithms
- Concrete MCTS algorithms are specified in terms of:
  - tree policy
  - default policy
- For most tasks, a well-suited MCTS configuration exists
- But: for each task, many MCTS configurations ill-suited
- And: every MCTS configuration that works well in one problem performs poorly in another problem

$\Rightarrow$ no dominating MCTS configuration

$\Rightarrow$ we present and analyze different tree and default policies

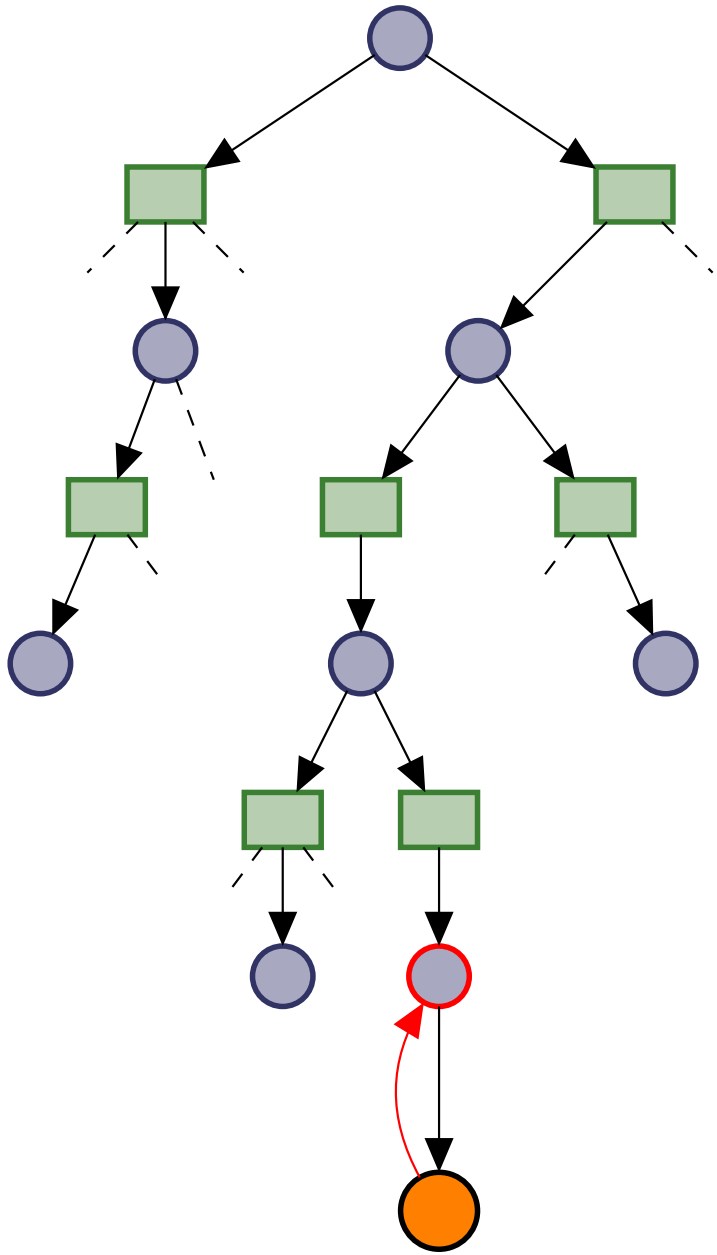
Tree Policy: Recap

- Tree policy used to traverse explicated tree, starting at root
- Assigns probability distribution over actions to each decision node
- May access information from current search tree
- Comparable to evaluation function in best-first search
- Tree policy more general: evaluation function determined upon node generation, while tree policy dynamic in each trial



Default Policy: Recap

- Default policy used to simulate run, starting at recently added decision node
- Assigns probability distribution over actions to each state
- Independent from current search tree
- Same role in MCTS as heuristic in heuristic search
- Heuristic more general: default policy is a specific kind of heuristic



Default Policy

Default Policy

Default Policy for state s of SSP $\mathcal{T}$

cost $= 0$
while $s \notin S_\star$:
    sample action a from default policy $\pi(\cdot \mid s)$
    cost $:=$ cost $+ c(a)$
    $s :\sim \text{succ}(s, a)$
return cost

- Default policy used to simulate run
- Role of default policy comparable to role of heuristic
- Heuristic value is accumulated cost of simulated run under default policy

Default Policy as Heuristic: Properties

Is a default policy

- goal-aware?
- safe?
- admissible?
- consistent?

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Default Policy Realizations

- Early work on MCTS proposed random walk default policy:

$$\pi(a \mid s) = \begin{cases} \frac{1}{|L(s)|} & \text{if } a \in L(s) \\ 0 & \text{otherwise} \end{cases}$$

- Random walks are proper
- Poor guidance, and due to high variance even misguidance
  $\Rightarrow$ Variant: run multiple random walks and use average
- Computation expensive if probability to reach goal is low
  $\Rightarrow$ Variant: apply heuristic after finite number of steps
- Predominant alternative: domain-dependent solutions
  e.g., neural networks of AlphaGo variants

Asymptotic Optimality

Optimal Search

- Optimal heuristic search algorithms AO* and RTDP use greedy policy
- with admissible heuristic
- and full Bellman backups to guarantee optimality
- MCTS uses Monte-Carlo backups
- and default policy is not admissible

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$\Rightarrow$ MCTS requires different way to guarantee optimality

Asymptotic Optimality

Asymptotic Optimality

An MCTS algorithm is asymptotically optimal if $\hat{Q}^k(c)$ converges to the optimal action-value $Q_\star(s(c), a(c))$ for all $c \in \text{succ}(d_0)$ when the number of trials k approaches infinity ($k \to \infty$).

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Note: this definition does not catch all MCTS configurations that are asymptotically optimal
(e.g., if all $\hat{Q}^k(c)$ converge to $\ell \cdot Q_\star(s(c), a(c))$ for some $\ell \in \mathbb{R}^+$).

Asymptotically Optimal Tree Policy

An MCTS algorithm is asymptotically optimal if

1. its tree policy explores forever:
   - the (infinite) sum of the probabilities that a decision node is visited must diverge
   - $\Rightarrow$ every search node is explicated eventually and visited infinitely often
2. its tree policy is greedy in the limit:
   - probability that optimal action is selected converges to 1
   - $\Rightarrow$ in the limit, backups based on iterations where only an optimal policy is followed dominate suboptimal backups
3. its default policy initializes decision nodes with finite values
   - $\Rightarrow$ require proper default policy and dead end free SSP

Example: Uniform Tree Policy

Example

Consider the random tree policy for decision node d where:

$$\pi(a \mid d) = \begin{cases} \frac{1}{|L(s(d))|} & \text{if } a \in L(s(d)) \\ 0 & \text{otherwise} \end{cases}$$

The random tree policy explores forever:
a decision node at depth h is visited with probability at least $(\frac{1}{|L|} \cdot \underline{p})^h$, where $\underline{p} := \min_{\{s,a,s' \mid T(s,a,s')>0\}} T(s,a,s')$ and hence

$$\sum_{i=1}^{k} (\frac{1}{|L|} \cdot \underline{p})^h = k \cdot (\frac{1}{|L|} \cdot \underline{p})^h \to \infty \text{ when } k \to \infty$$

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The random tree policy is not greedy in the limit:
the probability that an optimal action a is selected in node d is

$$1 - \sum_{\{a' \in L(d(s)) | a' \text{ suboptimal}\}} \frac{1}{|L(s(d))|} \not\to 1 \text{ when } k \to \infty.$$

$\rightsquigarrow$ Random tree policy not asymptotically optimal

Example: Greedy Tree Policy

Example

Consider the greedy tree policy for decision node d where:

$$\pi(a \mid d) = \begin{cases} \frac{1}{|L_\star^k(d)|} & \text{if } a \in L_\star^k(d) \\ 0 & \text{otherwise,} \end{cases}$$

with $L_\star^k(d) = \{a(c) \in L(s(d)) \mid c \in \arg\min_{c' \in \text{children}(d)} \hat{Q}^k(c')\}$.

- Greedy tree policy is greedy in the limit
- Greedy tree policy does not explore forever

$\rightsquigarrow$ Greedy tree policy not asymptotically optimal

Tree Policy: Objective

To satisfy both requirements, MCTS tree policies have two contradictory objectives:

- explore parts of the search space that have not been investigated thoroughly
- exploit knowledge about good actions to focus search on promising areas of the search space

central challenge: balance exploration and exploitation

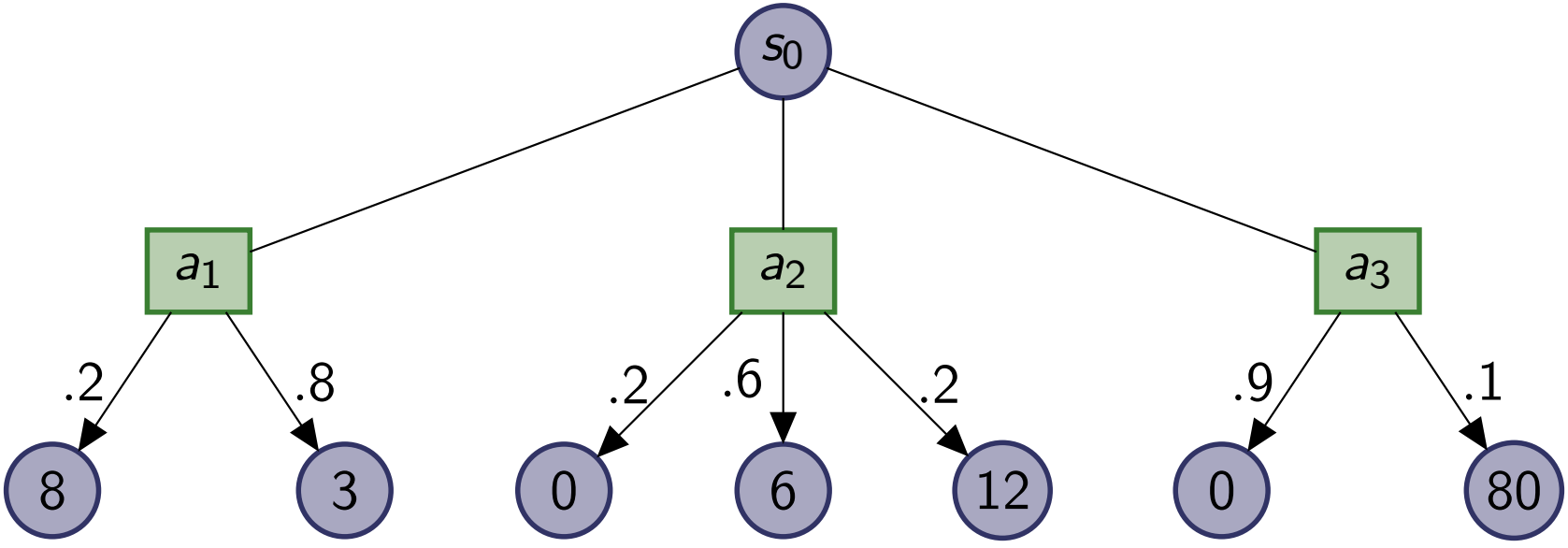
$\Rightarrow$ borrow ideas from related multi-armed bandit problem

Multi-armed Bandit Problem

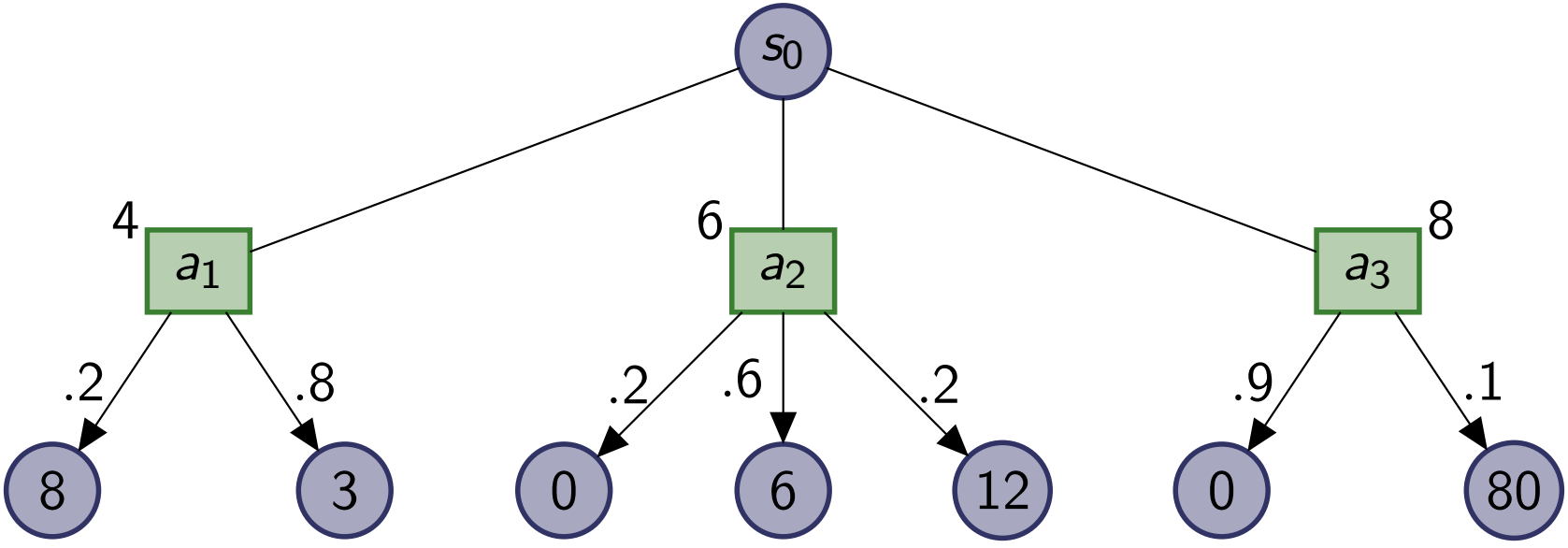
Multi-armed Bandit Problem

- Most commonly used tree policies are inspired from research on the multi-armed bandit problem (MAB)
- MAB is a learning scenario (model not revealed to agent)
- agent repeatedly faces the same decision:
  to pull one of several arms of a slot machine
- pulling an arm yields stochastic reward
  Note: In this section, we consider rewards rather than costs
- can be modeled as MDP

Multi-armed Bandit Problem

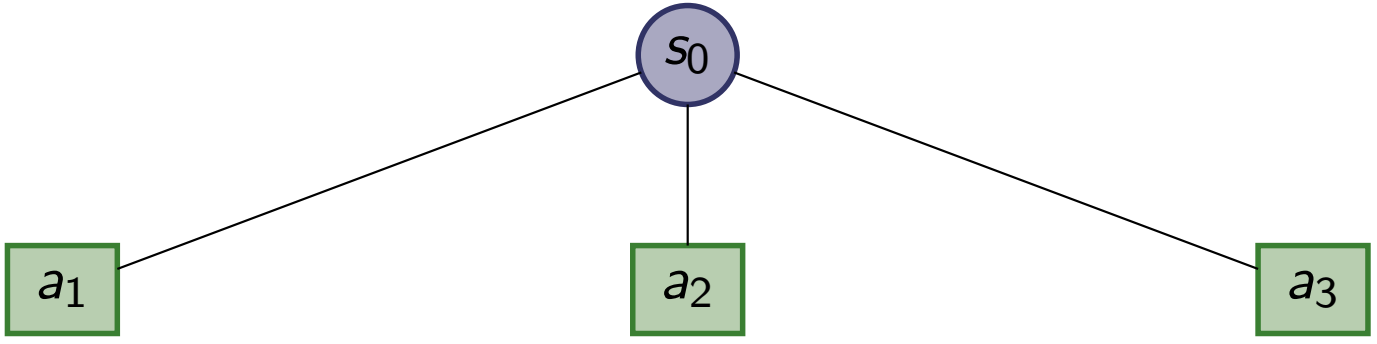


Multi-armed Bandit Problem: Planning Scenario



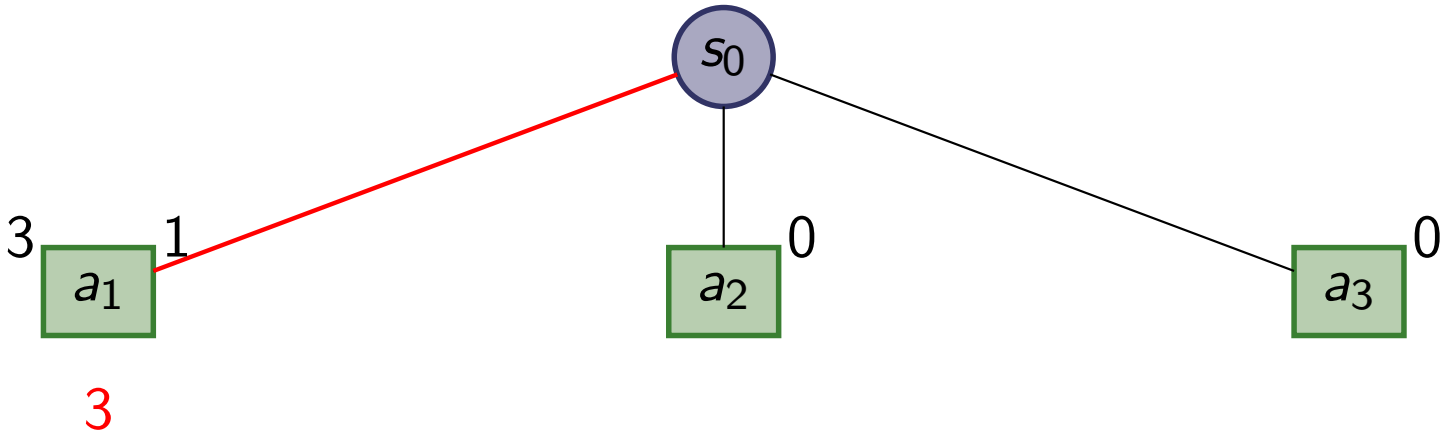
- Compute $Q_\star(a)$ for $a \in \{a_1, a_2, a_3\}$
- Pull arm $\arg\max_{a \in \{a_1, a_2, a_3\}} Q_\star(a) = a_3$ forever
- Expected accumulated reward after k trials is $8 \cdot k$

Multi-armed Bandit Problem: Learning Scenario



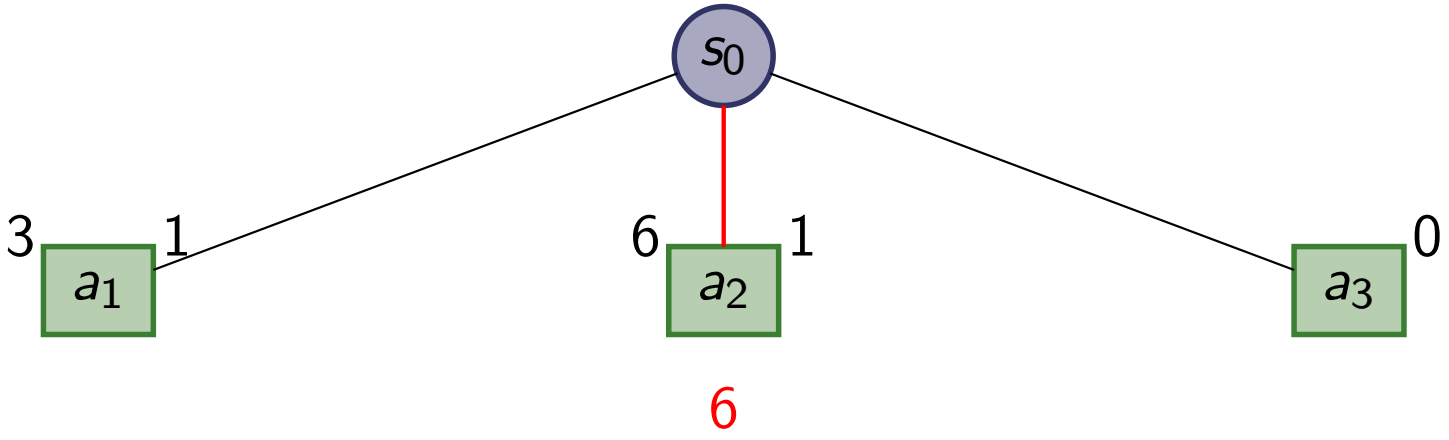
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
-

Multi-armed Bandit Problem: Learning Scenario



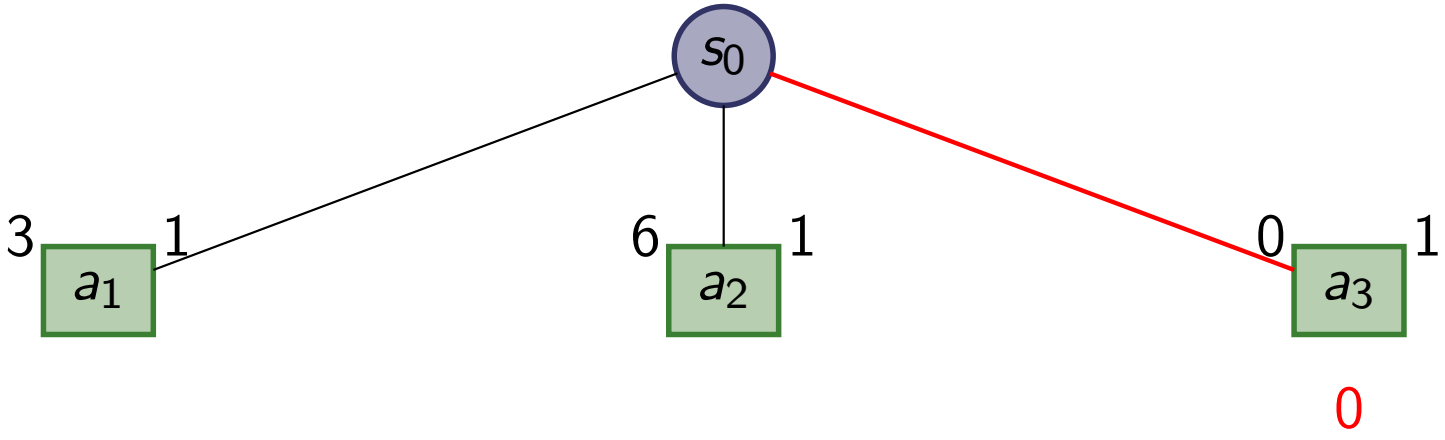
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 1 trial is 3

Multi-armed Bandit Problem: Learning Scenario



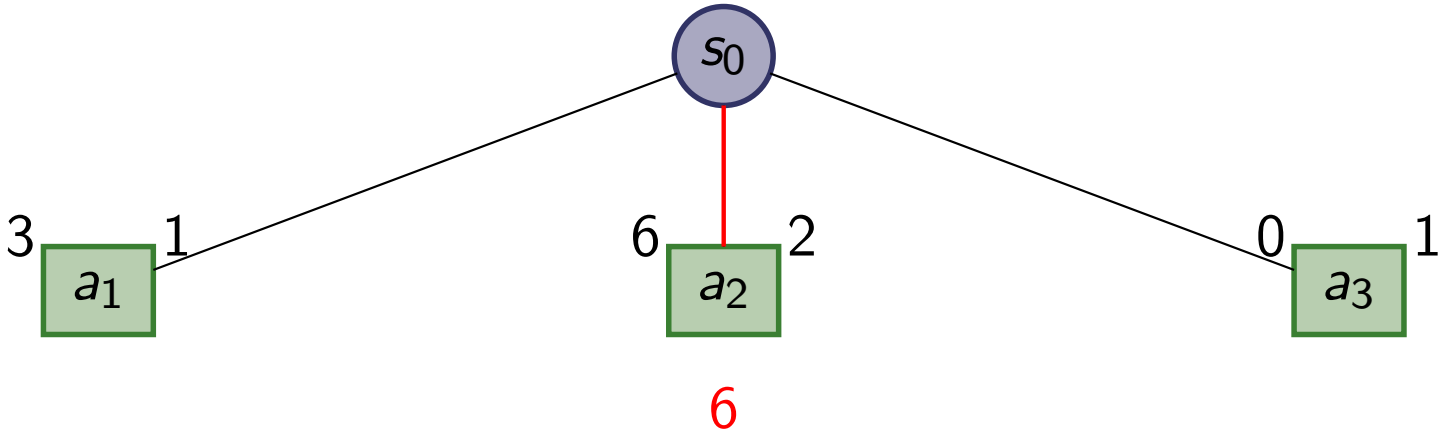
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 2 trials is $3 + 6 = 9$

Multi-armed Bandit Problem: Learning Scenario



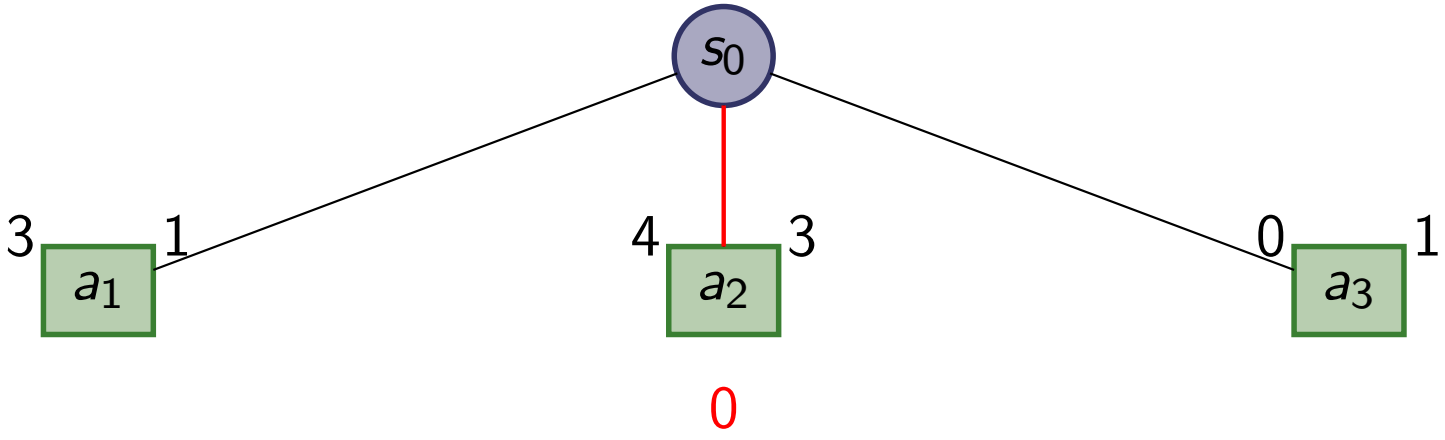
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 3 trials is $3 + 6 + 0 = 9$

Multi-armed Bandit Problem: Learning Scenario



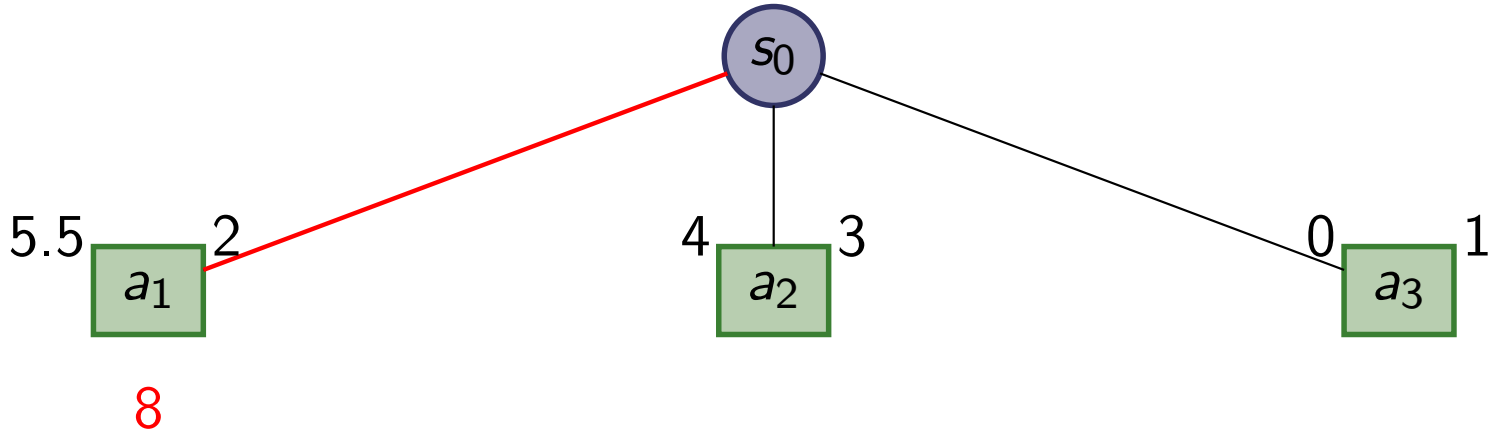
- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 4 trials is $3 + 6 + 0 + 6 = 15$

Multi-armed Bandit Problem: Learning Scenario



- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 5 trials is $3 + 6 + 0 + 6 + 0 = 15$

Multi-armed Bandit Problem: Learning Scenario



- Pull arms following policy to explore or exploit
- Update $\hat{Q}$ and N based on observations
- Accumulated reward after 6 trials is $3+6+0+6+0+8=23$

Policy Quality

- Since model unknown to MAB agent, it cannot achieve accumulated reward of $k \cdot V_\star$ with $V_\star := \max_a Q_\star(a)$ in k trials
- Quality of MAB policy π measured in terms of regret, i.e., the difference between $k \cdot V_\star$ and expected reward of π in k trials
- Regret cannot grow slower than logarithmic in number of trials

Connection between MCTS Tree Policy and MAB

$\rightsquigarrow$ Blackboard

Summary

Summary

- Default policies simulate run under policy
- Default policy not admissible
- MCTS requires different idea to achieve optimality than heuristic search:
  tree policy must be greedy in the limit and explore forever
- Central challenge of tree policies:
  balance exploration and exploitation
- Each decision of MCTS tree policy can be viewed as multi-armed bandit problem