Planning and Optimization

G2. Heuristic Search: AO* & LAO* Part II

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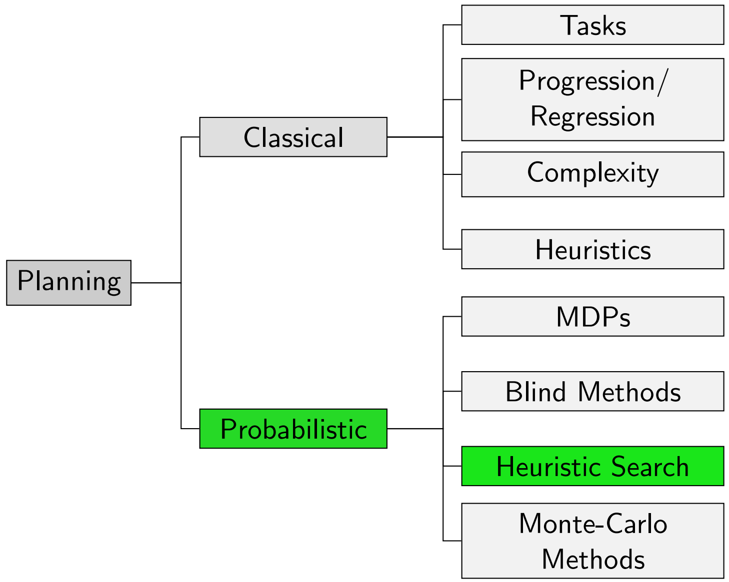
Content of this Course



G2.1 AO*

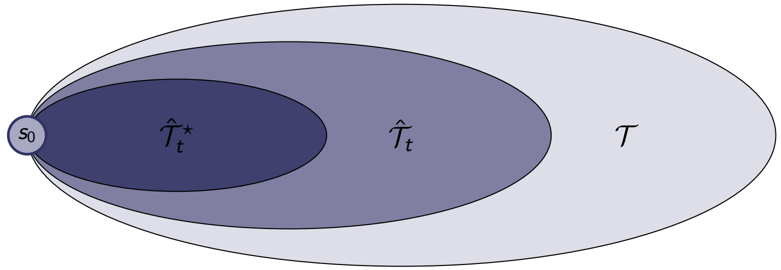
From A* with Backward Induction to AO*

- A* with backward induction already very similar to AO*
- Support for uncertain outcomes missing
- We focus on SSPs in these slides
- Adaption to FH-MDPs simple
  Careful: admissible heuristic in reward setting must not underestimate true reward
- Still two steps ahead:
  - restrict to acyclic probabilistic tasks ⇒ AO*
  - allow general probabilistic tasks ⇒ LAO*

Transition Systems

AO* distinguishes three transition systems:

- The acyclic SSP $\mathcal{T} = \langle S, L, c, T, s_0, S^\star \rangle$
  ⇒ given implicitly
- The explicated graph $\hat{\mathcal{T}}_t = \langle \hat{S}_t, L, c, \hat{T}_t, s_0, S^\star \rangle$
  ⇒ the part of $\mathcal{T}$ explicitly considered during search
- The partial solution graph $\hat{\mathcal{T}}^\star_t = \langle \hat{S}^\star_t, L, c, \hat{T}^\star_t, s_0, S^\star \rangle$
  ⇒ The part of $\hat{\mathcal{T}}_t$ that contains best solution



Explicated Graph

- Expanding a state s at time step t explicates all outcomes $s' \in \text{succ}(s, \ell)$ for all $\ell \in L(s)$ by adding them to explicated graph:
  $$\hat{\mathcal{T}}_t = \langle \hat{S}_{t-1} \cup \text{succ}(s), L, c, \hat{T}_t, s_0, S^\star \},$$
  where $\hat{T}_t = \hat{T}_{t-1}$ except that $\hat{T}_t(s, \ell, s') = T(s, \ell, s')$ for all $\ell \in L(s)$ and $s' \in \text{succ}(s, \ell)$
- Explicated states are annotated with state-value estimate $\hat{V}_t(s)$ that describes estimated expected cost to goal at step t
- When state s' is explicated and $s' \notin \hat{S}_{t-1}$, its state-value estimate is initialized to $\hat{V}_t(s') := h(s')$
- We call leaf states of $\hat{\mathcal{T}}_t$ fringe states

Partial Solution Graph

- The partial solution graph $\hat{\mathcal{T}}^\star_t$ is the subgraph of $\hat{\mathcal{T}}_t$ that is spanned by the smallest set of states $\hat{S}^\star_t$ that satisfies:
  - $s_0 \in \hat{S}^\star_t$
  - if $s \in \hat{S}^\star_t$, $s' \in \hat{S}_t$ and $\hat{T}_t(s, a_{\hat{V}_t}(s), s') > 0$, then s' in $\hat{S}^\star_t$
- The partial solution graph forms a partial acyclic policy defined in the initial state s_0 and all non-leaf states that can be reached by its execution
- Leaf states that can be reached by the policy described by the partial solution graph are the states in the greedy fringe

Bellman backups

- AO* does not maintain static open list
- State-value estimates determine partial solution graph
- Partial solution graph determines which state is a candidate for expansion
  Different strategies to select among candidates exist
- (Some) state-value estimates are updated in time step t by Bellman backups:

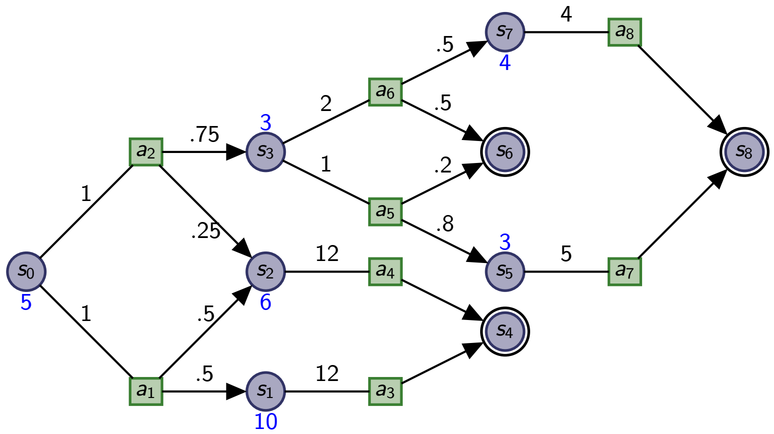
$$\hat{V}_t(s) = \min_{l \in L} c(l) + \sum_{s' \in \hat{S}_t} \hat{T}_t(s, l, s') \cdot \hat{V}_t(s')$$

AO*

AO* for acyclic SSP $\mathcal{T}$

explicate s_0
while there is a greedy fringe state not in $S_\star$:
  select a greedy fringe state $s \notin S_\star$
  expand s
  perform Bellman backups of states in $\hat{\mathcal{T}}^\star_{t-1}$ in reverse order
return $\hat{\mathcal{T}}^\star_t$

AO*: Example (Blackboard)



$h(s) = 0$ for goal states, otherwise in blue above or below s

Theoretical properties

Theorem

Using an admissible heuristic, AO converges to an optimal solution without (necessarily) explicating all states.*

Proof omitted.

G2.2 LAO*

LAO*

- A* with backward induction finds sequential solutions (a plan) in classical planning tasks
- AO* finds acyclic solutions with branches (an acyclic policy) in acyclic SSPs
- LAO* is the generalization of AO* to cyclic solutions in cyclic SSPs

LAO*

- From plans to acyclic policies, we only changed backup procedure from backward induction to Bellman backups
- When solutions may be cyclic, we cannot perform updates in reverse order

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- When solutions may be cyclic, we cannot perform updates in reverse order
- Bellman backups are essentially acyclic version of value iteration
- replacing Bellman backups with value iteration is LAO* variant
- the original algorithm of Hansen & Zilberstein (1998) uses policy iteration instead

LAO*

LAO* for SSP $\mathcal{T}$

explicate s_0
while there is a greedy fringe state not in $S_\star$:
  select a greedy fringe state $s \notin S_\star$
  expand s
  perform policy iteration in $\hat{\mathcal{T}}_t$
return $\hat{\mathcal{T}}_t^\star$

LAO*: Optimizations

Several optimizations for LAO* have been proposed:

- Use value iteration instead of policy iteration
- Terminate VI when the partial solution graph changes
- Expand all states in greedy fringe before backup
- Order states (arbitrarily within cycles) and use backward induction for updates

⇒ last two combine to famous variant iLAO*

Theoretical properties

Theorem

Using an admissible heuristic, LAO converges to an optimal solution without (necessarily) explicating all states.*

Proof omitted.

G2.3 Summary

Summary

- AO* finds optimal solutions for acyclic SSPs
- LAO* finds optimal solutions for SSPs
- Both algorithms differ from A* with backward induction in way backups are performed
- Unlike previous optimal algorithms, both are able to find optimal solution without explicating all states