Planning and Optimization

E6. Cost Partitioning: Landmarks and Generalization

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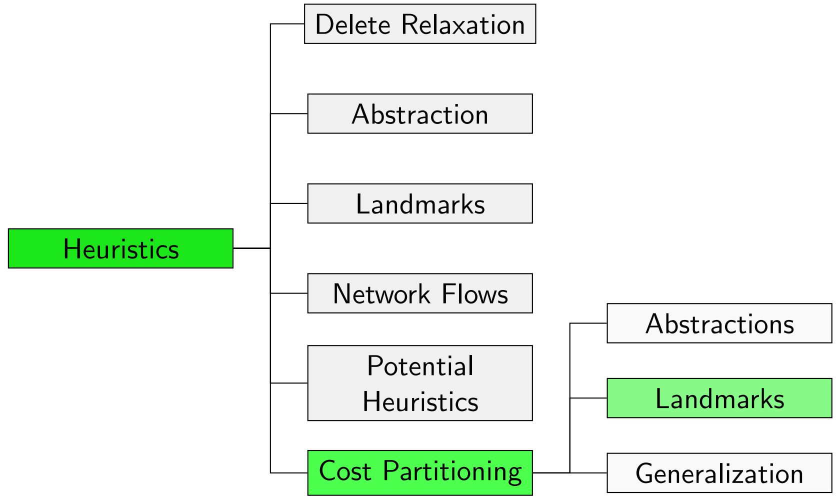
E6.1 Cost Partitioning for Landmarks

Content of this Course: Heuristics



Reminder: Disjunctive Action Landmarks

Disjunctive action landmark

- Set of operators
- Every plan uses at least one of them
- Landmark cost = cost of cheapest operator

Reminder: Cost Partitioning Heuristic for Landmarks

We have already seen a landmark heuristic based on cost partitioning:

Definition (Uniform Cost Partitioning Heuristic for Landmarks)

Let $\mathcal{L}$ be a set of disjunctive action landmarks.

The uniform cost partitioning heuristic $h^{\mathrm{UCP}}(\mathcal{L})$ is defined as

$$h^{\mathrm{UCP}}(\mathcal{L}) = \sum_{L \in \mathcal{L}} \min_{o \in L} c'(o) \text{ with}$$

$$c'(o) = cost(o)/|\{L \in \mathcal{L} \mid o \in L\}|.$$

Reminder: Proof Back Then

Theorem (Uniform Cost Partitioning Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s of Π. Then $h^{\mathrm{UCP}}(\mathcal{L})$ is an admissible heuristic estimate for s.

Proof.

Let $\pi = \langle o_1, \dots, o_n \rangle$ be an optimal plan for s. For $L \in \mathcal{L}$ define a new cost function $cost_L$ as $cost_L(o) = c'(o)$ if $o \in L$ and $cost_L(o) = 0$ otherwise. Let Π_L be a modified version of Π, where for all operators o the cost is replaced with $cost_L(o)$.
$(\dots)$ □

$\sum_{L \in \mathcal{L}} cost_L(o) = \sum_{L \in \mathcal{L}: o \in L} cost(o)/|\{L \in \mathcal{L} \mid o \in L\}| = cost(o)$

Heuristic is Based on Cost Partitioning

- For disj. action landmark L of state s in task Π', let $h_{L,\Pi'}(s)$ be the cost of L in Π'.
- Consider set $\{L_1, \dots, L_n\}$ of disj. action landmarks for state s of task Π.
- Use cost partitioning $\langle cost_{L_1}, \dots, cost_{L_n} \rangle$, where

$$cost_{L_i}(o) = \begin{cases} cost(o)/|\{L \in \mathcal{L} \mid o \in L\}| & \text{if } o \in L_i \\ 0 & \text{otherwise} \end{cases}$$

- Let $\langle \Pi_{L_1}, \dots, \Pi_{L_n} \rangle$ be the tuple of induced tasks.
- $h(s) = \sum_{i=1}^{n} h_{L_i, \Pi_{L_i}}(s)$ is an admissible estimate for s in Π.
- h is uniform cost partitioning heuristic for landmarks.

Optimal Cost Partitioning for Landmarks

Can we find a better cost partitioning?

- Use again LP that covers heuristic computation and cost partitioning.
- LP variable Cost_L for cost of landmark L in induced task (corresponds to $h_{L_i,\Pi_{L_i}}$)
- Explicit variables for cost partitioning not necessary. Use implicitly $cost_L(o) = \text{Cost}_L$ for all $o \in L$ and 0 otherwise.

Optimal Cost Partitioning for Landmarks: LP

Variables
Cost_L for each disj. action landmark $L \in \mathcal{L}$

Objective
Maximize $\sum_{L \in \mathcal{L}} \text{Cost}_L$

Subject to

$$\sum_{L \in \mathcal{L}: o \in L} \text{Cost}_L \leq cost(o) \quad \text{for all operators } o$$

$$\text{Cost}_L \geq 0 \quad \text{for all landmarks } L \in \mathcal{L}$$

Optimal Cost Partitioning for Landmarks (Dual view)

Variables
Applied_o for each operator o

Objective
Minimize $\sum_o \text{Applied}_o \cdot cost(o)$

Subject to

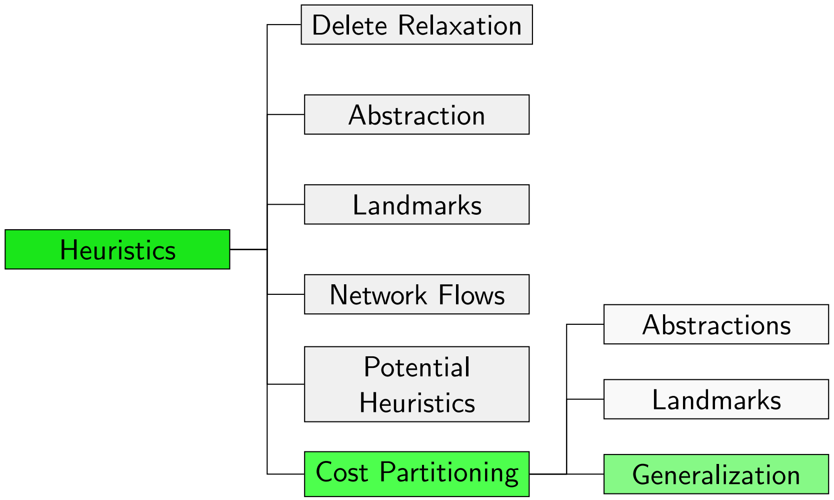
$$\sum_{o \in L} \text{Applied}_o \geq 1 \text{ for all landmarks } L$$

$$\text{Applied}_o \geq 0 \text{ for all operators } o$$

Minimize "plan cost" with all landmarks satisfied.

E6.2 General Cost Partitioning

Content of this Course: Heuristics



General Cost Partitioning

Cost functions usually non-negative

- We tacitly also required this for task copies
- Makes intuitively sense: original costs are non-negative
- But: not necessary for cost-partitioning!

General Cost Partitioning

Definition (General Cost Partitioning)

Let Π be a planning task with operators O.

A general cost partitioning for Π is a tuple $\langle cost_1, \dots, cost_n \rangle$, where

- $cost_i : O \to \mathbb{R}$ for $1 \leq i \leq n$ and
- $\sum_{i=1}^{n} cost_i(o) \leq cost(o)$ for all $o \in O$.

General Cost Partitioning: Admissibility

Theorem (Sum of Solution Costs is Admissible)

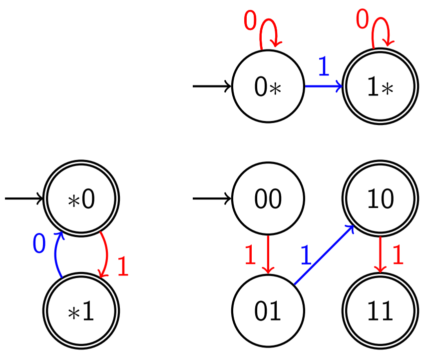
Let Π be a planning task, $\langle cost_1, \dots, cost_n \rangle$ be a general cost partitioning and $\langle \Pi_1, \dots, \Pi_n \rangle$ be the tuple of induced tasks.

Then the sum of the solution costs of the induced tasks is an admissible heuristic for Π, i.e., $\sum_{i=1}^{n} h^*_{\Pi_i} \leq h^*_{\Pi}$.

(Proof omitted.)

General Cost Partitioning: Example

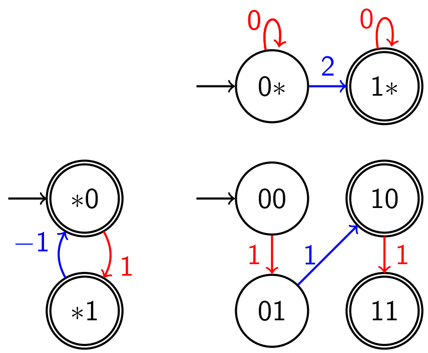
Example



Heuristic value: $0 + 1 = 1$

General Cost Partitioning: Example

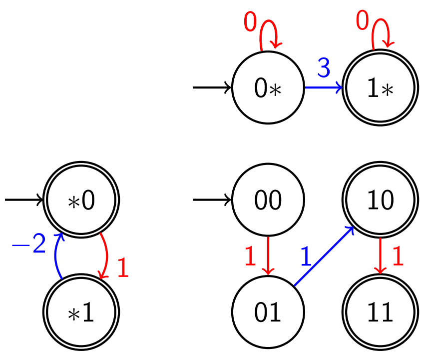
Example



Heuristic value: $0 + 2 = 2$

General Cost Partitioning: Example

Example



Heuristic value: $-\infty + 3 = -\infty$

LP for Shortest Path in State Space with Negative Costs

Variables

Distance$_s$ for each state s,
GoalDist

Objective

Maximize GoalDist

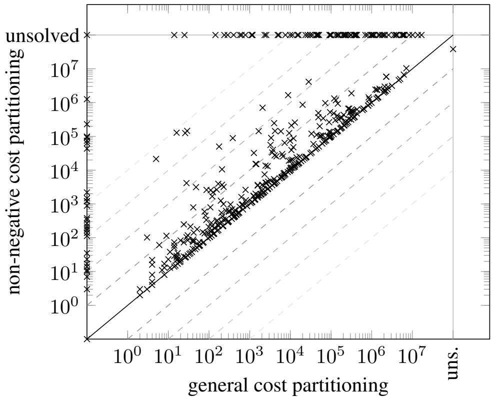
Subject to

$\text{Distance}_{s_I} = 0$	for the initial state s_I
$\text{Distance}_{s'} \leq \text{Distance}_s + cost(o)$	for all alive transitions $s \xrightarrow{o} s'$
$\text{GoalDist} \leq \text{Distance}_{s_\star}$	for all goal states $s_\star$

alive: on any path from initial state to goal state

Modification also correct (but unnecessary) for non-negative costs

Experimental Results



Expansions with A* (excluding last f-layer) for optimal cost partitioning of all projections to single variables.
[Pommerening et al., AAAI 2015]

General Cost Partitioning: Remarks

- More powerful than non-negative cost partitioning
- Optimal general cost partitioning:
  omit constraints to non-negative cost variables
  - optimal cost partitioning maximizes objective value
  - removing constraints can only increase heuristic value
- Optimal general cost partitioning is never worse than an optimal non-negative cost partitioning.

E6.3 Summary

Summary

- We can compute an optimal cost partitioning for a given set of disjunctive action landmarks in polynomial time.
- In constrast to standard (non-negative) cost partitioning, general cost partitioning allows negative operators costs.
- General cost partitioning has the same relevant properties as non-negative cost partitioning but is more powerful.