Planning and Optimization

E5. Cost Partitioning: Definition, Properties, and Abstractions

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November 19, 2018

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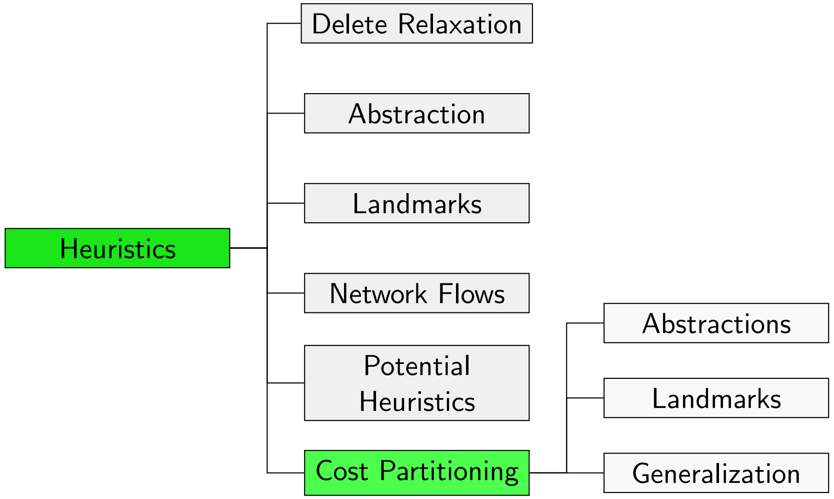
E5.4 Summary

Content of this Course: Heuristics



Heuristics

Delete Relaxation

Abstraction

Landmarks

Network Flows

Potential Heuristics

Cost Partitioning

Abstractions

Landmarks

Generalization

E5.1 Introduction

Exploiting Additivity

- Additivity allows to add up heuristic estimates admissibly. This gives better heuristic estimates than the maximum.
- For example, the canonical heuristic for PDBs sums up where addition is admissible (by an additivity criterion) and takes the maximum otherwise.
- Cost partitioning provides a more general additivity criterion, based on an adaption of the operator costs.

E5.2 Cost Partitioning

Cost Partitioning

Definition (Cost Partitioning)

Let Π be a planning task with operators O.

A cost partitioning for Π is a tuple $\langle cost_1, \dots, cost_n \rangle$, where

- $cost_i : O \to \mathbb{R}_0^+$ for $1 \leq i \leq n$ and
- $\sum_{i=1}^{n} cost_i(o) \leq cost(o)$ for all $o \in O$.

The cost partitioning induces a tuple $\langle \Pi_1, \dots, \Pi_n \rangle$ of planning tasks, where each Π_i is identical to Π except that the cost of each operator o is $cost_i(o)$.

Cost Partitioning: Admissibility (1)

Theorem (Sum of Solution Costs is Admissible)

Let Π be a planning task, $\langle cost_1, \dots, cost_n \rangle$ be a cost partitioning and $\langle \Pi_1, \dots, \Pi_n \rangle$ be the tuple of induced tasks.

Then the sum of the solution costs of the induced tasks is an admissible heuristic for Π, i.e., $\sum_{i=1}^{n} h^*_{\Pi_i} \leq h^*_{\Pi}$.

Cost Partitioning: Admissibility (2)

Proof of Theorem.

Let $\pi = \langle o_1, \dots, o_m \rangle$ be an optimal plan for state s of Π. Then

$$\begin{aligned}
\sum_{i=1}^{n} h^*_{\Pi_i}(s) &\le \sum_{i=1}^{n}\sum_{j=1}^{m} cost_i(o_j) && (\pi \text{ plan in each } \Pi_i)\\
&= \sum_{j=1}^{m}\sum_{i=1}^{n} cost_i(o_j) && (\text{comm./ass. of sum})\\
&\le \sum_{j=1}^{m} cost(o_j) && (\text{cost partitioning})\\
&= h^*_{\Pi}(s) && (\pi \text{ optimal plan in } \Pi)
\end{aligned}$$

□

Cost Partitioning Preserves Admissibility

In the rest of the chapter, we write h_Π to denote heuristic h evaluated on task Π.

Corollary (Sum of Admissible Estimates is Admissible)

Let Π be a planning task and let $\langle \Pi_1, \dots, \Pi_n \rangle$ be induced by a cost partitioning.

For admissible heuristics $h_1, \dots, h_n$, the sum $h(s) = \sum_{i=1}^{n} h_{i,\Pi_i}(s)$ is an admissible estimate for s in Π.

Cost Partitioning Preserves Consistency

Theorem (Cost Partitioning Preserves Consistency)

Let Π be a planning task and let $\langle \Pi_1, \dots, \Pi_n \rangle$ be induced by a cost partitioning $\langle cost_1, \dots, cost_n \rangle$.

If $h_1, \dots, h_n$ are consistent heuristics then $h = \sum_{i=1}^{n} h_{i,\Pi_i}$ is a consistent heuristic for Π.

Proof.

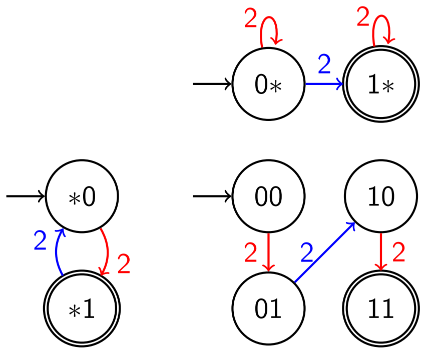
Let o be an operator that is applicable in state s.

$$\begin{aligned}
h(s) = \sum_{i=1}^{n} h_{i,\Pi_i}(s) &\le \sum_{i=1}^{n}(cost_i(o) + h_{i,\Pi_i}(s[\![o]\!]))\\
&= \sum_{i=1}^{n} cost_i(o) + \sum_{i=1}^{n} h_{i,\Pi_i}(s[\![o]\!]) \le cost(o) + h(s[\![o]\!])
\end{aligned}$$

□

Cost Partitioning: Example

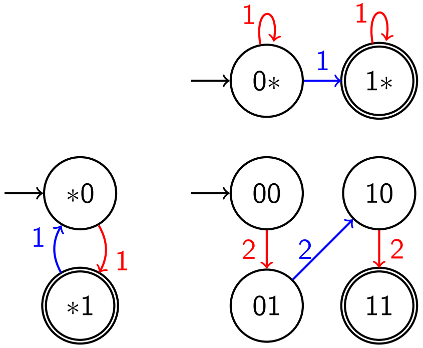
Example (No Cost Partitioning)



Heuristic value: $\max\{2, 2\} = 2$

Cost Partitioning: Example

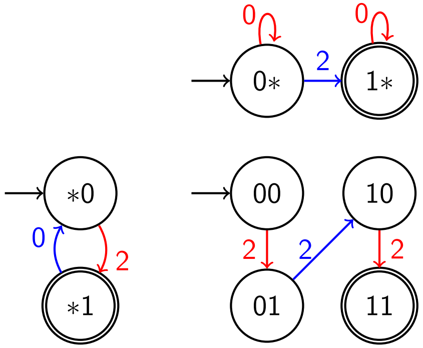
Example (Cost Partitioning 1)



Heuristic value: $1 + 1 = 2$

Cost Partitioning: Example

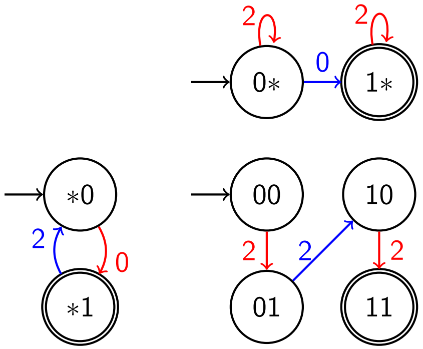
Example (Cost Partitioning 2)



Heuristic value: $2 + 2 = 4$

Cost Partitioning: Example

Example (Cost Partitioning 3)



Heuristic value: $0 + 0 = 0$

Cost Partitioning: Quality

- $h(s) = h_{1,\Pi_1}(s) + \cdots + h_{n,\Pi_n}(s)$ can be better or worse than any $h_{i,\Pi}(s)$ $\rightarrow$ depending on cost partitioning
- strategies for defining cost-functions
  - uniform: $cost_i(o) = cost(o)/n$
  - zero-one: full operator cost in one copy, zero in all others
  - ...

Can we find an optimal cost partitioning?

Optimal Cost Partitioning

Optimal Cost Partitioning with LPs

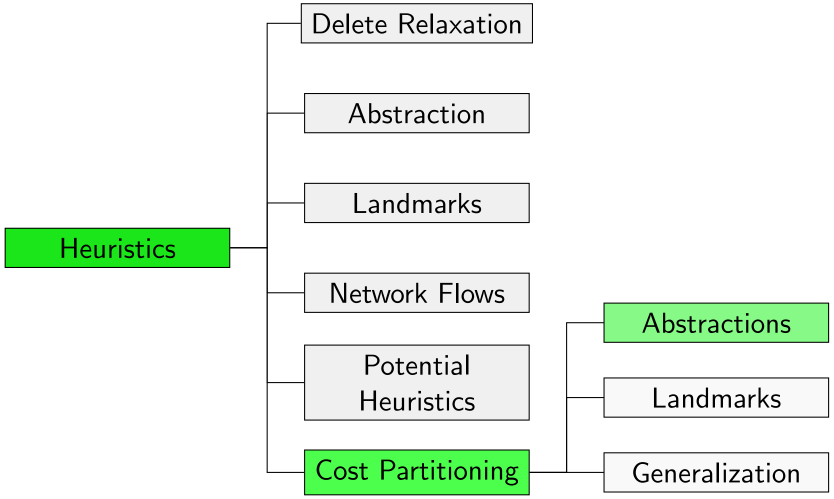
- Use variables for cost of each operator in each task copy
- Express heuristic values with linear constraints
- Maximize sum of heuristic values subject to these constraints

LPs known for

- abstraction heuristics
- landmark heuristic

E5.3 Optimal Cost Partitioning for Abstractions

Content of this Course: Heuristics



Optimal Cost Partitioning for Abstractions

Abstractions

- Simplified versions of the planning task, e.g. projections
- Cost of optimal abstract plan is admissible estimate

How to express the heuristic value as linear constraints?
⇝ Shortest path problem in abstract transition system

LP for Shortest Path in State Space

Variables

Distance_s for each state s,
GoalDist

Objective

Maximize GoalDist

Subject to

$$\begin{aligned} \text{Distance}_{s_I} &= 0 && \text{for the initial state } s_I \\ \text{Distance}_{s'} &\leq \text{Distance}_s + \mathit{cost}(o) && \text{for all transitions } s \xrightarrow{o} s' \\ \text{GoalDist} &\leq \text{Distance}_{s_\star} && \text{for all goal states } s_\star \end{aligned}$$

Optimal Cost Partitioning for Abstractions I

Variables

For each abstraction α:

- Distance^α_s for each abstract state s,
- cost^α_o for each operator o,
- GoalDist^α

Objective

Maximize $\sum_\alpha \text{GoalDist}^\alpha$

...

Optimal Cost Partitioning for Abstractions II

Subject to

for all operators o

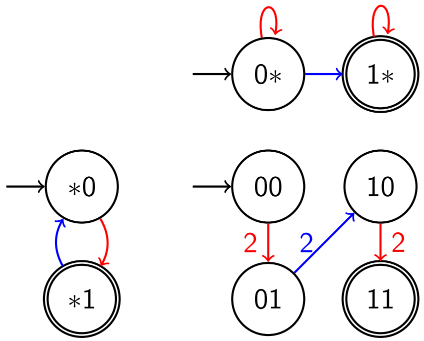
$$\begin{aligned} \textstyle\sum_\alpha \text{Cost}^\alpha_o &\leq \mathit{cost}(o) && \\ \text{Cost}^\alpha_o &\geq 0 && \text{for all abstractions } \alpha \end{aligned}$$

and for all abstractions α

$$\begin{aligned} \text{Distance}^\alpha_{s_I} &= 0 && \text{for the abstract initial state } s_I \\ \text{Distance}^\alpha_{s'} &\leq \text{Distance}^\alpha_s + \text{Cost}^\alpha_o && \text{for all transition } s \xrightarrow{o} s' \\ \text{GoalDist}^\alpha &\leq \text{Distance}^\alpha_{s_\star} && \text{for all abstract goal states } s_\star \end{aligned}$$

Example (1)

Example



Example (2)

Maximize $\text{GoalDist}^1 + \text{GoalDist}^2$ subject to

$$
\begin{aligned}
\text{Cost}^1_{\text{red}} + \text{Cost}^2_{\text{red}} &\le 2\\
\text{Cost}^1_{\text{blue}} + \text{Cost}^2_{\text{blue}} &\le 2\\
\text{Cost}^1_{\text{red}} &\ge 0\\
\text{Cost}^2_{\text{red}} &\ge 0\\
\text{Cost}^1_{\text{blue}} &\ge 0\\
\text{Cost}^2_{\text{blue}} &\ge 0 \qquad \dots
\end{aligned}
$$

Example (3)

... and ...

$$
\begin{aligned}
\text{Distance}^1_0 &= 0\\
\text{Distance}^1_0 &\le \text{Distance}^1_0 + \text{Cost}^1_{\text{red}}\\
\text{Distance}^1_1 &\le \text{Distance}^1_0 + \text{Cost}^1_{\text{blue}}\\
\text{Distance}^1_1 &\le \text{Distance}^1_1 + \text{Cost}^1_{\text{red}}\\
\text{GoalDist}^1 &\le \text{Distance}^1_1
\end{aligned}
$$

$$
\begin{aligned}
\text{Distance}^2_0 &= 0\\
\text{Distance}^2_1 &\le \text{Distance}^2_0 + \text{Cost}^2_{\text{red}}\\
\text{Distance}^2_0 &\le \text{Distance}^2_1 + \text{Cost}^2_{\text{blue}}\\
\text{GoalDist}^2 &\le \text{Distance}^2_1
\end{aligned}
$$

Caution

A word of warning

- optimization for every state gives best-possible cost partitioning
- but takes time

Better heuristic guidance often does not outweigh the overhead.

E5.4 Summary

Summary

- Cost partitioning allows to admissibly add up estimates of several heuristics.
- This can be better or worse than the best individual heuristic on the original problem, depending on the cost partitioning.
- For some heuristic classes, we know how to determine an optimal cost partitioning, using linear programming.
- Although solving a linear program is possible in polynomial time, the better heuristic guidance often does not outweigh the overhead.