

# Planning and Optimization

## E4. Flow & Potential Heuristics

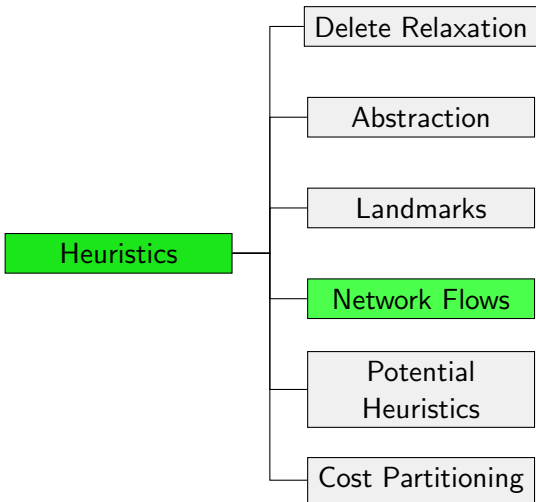
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# Introduction

# Content of this Course: Heuristics



# Reminder: SAS<sup>+</sup> Planning Tasks

For a SAS<sup>+</sup> planning task  $\Pi = \langle V, I, O, \gamma \rangle$ :

- $V$  is a set of **finite-domain state variables**,
- Each **atom** has the form  $v = d$  with  $v \in V, d \in \text{dom}(v)$ .
- Operator **preconditions** and the **goal** formula  $\gamma$  are **conjunctions of atoms**.
- Operator **effects** are **conjunctions of atomic effects**, i.e., they have the form  $v_1 := d_1 \wedge \dots \wedge v_n := d_n$ .

## Example Task (1)

- One package, two trucks, two locations
- Variables:
  - $pos-p$  with  $\text{dom}(pos-p) = \{loc_1, loc_2, t_1, t_2\}$
  - $pos-t-i$  with  $\text{dom}(pos-t-i) = \{loc_1, loc_2\}$  for  $i \in \{1, 2\}$
- The package is at location 1 and the trucks at location 2,
  - $I = \{pos-p \mapsto loc_1, pos-t-1 \mapsto loc_2, pos-t-2 \mapsto loc_2\}$
- The goal is to have the package at location 2 and truck 1 at location 1.
  - $\gamma = (pos-p = loc_2) \wedge (pos-t-1 = loc_1)$

## Example Task (2)

- Operators: for  $i, j, k \in \{1, 2\}$ :

$$\begin{aligned} \text{load}(t_i, \text{loc}_j) = \langle & \text{pos-}t\text{-}i = \text{loc}_j \wedge \text{pos-}p = \text{loc}_j, \\ & \text{pos-}p := t_i, 1 \rangle \end{aligned}$$

$$\begin{aligned} \text{unload}(t_i, \text{loc}_j) = \langle & \text{pos-}t\text{-}i = \text{loc}_j \wedge \text{pos-}p = t_i, \\ & \text{pos-}p := \text{loc}_j, 1 \rangle \end{aligned}$$

$$\begin{aligned} \text{drive}(t_i, \text{loc}_j, \text{loc}_k) = \langle & \text{pos-}t\text{-}i = \text{loc}_j, \\ & \text{pos-}t\text{-}i := \text{loc}_k, 1 \rangle \end{aligned}$$

## Example Task: Observations

Consider some atoms of the example task:

- $pos-p = loc_1$  initially true and must be false in the goal
  - ▷ at location 1 the package must be loaded one time more often than unloaded.
- $pos-p = loc_2$  initially false and must be true in the goal
  - ▷ at location 2 the package must be unloaded one time more often than loaded.
- $pos-p = t_1$  initially false and must be false in the goal
  - ▷ same number of load and unload actions for truck 1.

Can we derive a heuristic from this kind of information?

## Example: Flow Constraints

Let  $\pi$  be some arbitrary plan for the example task and let  $\#o$  denote the **number of occurrences** of operator  $o$  in  $\pi$ .

Then the following holds:

- $pos-p = loc_1$  initially true and must be false in the goal
  - ▷ at location 1 the package must be loaded one time more often than unloaded.

$$\begin{aligned} \#load(t_1, loc_1) + \#load(t_2, loc_1) = \\ 1 + \#unload(t_1, loc_1) + \#unload(t_2, loc_1) \end{aligned}$$

- $pos-p = t_1$  initially false and must be false in the goal
  - ▷ same number of load and unload actions for truck 1.

$$\begin{aligned} \#unload(t_1, loc_1) + \#unload(t_1, loc_2) = \\ \#load(t_1, loc_1) + \#load(t_1, loc_2) \end{aligned}$$



## Network Flow Heuristics: General Idea

- Formulate **flow constraints** for each atom.
- These are satisfied by **every plan** of the task.
- The cost of a plan is  $\sum_{o \in O} cost(o) \#o$
- The objective value of an integer program that minimizes this cost subject to the flow constraints is a lower bound on the plan cost (i.e., an admissible heuristic estimate).
- As solving the IP is NP-hard, we solve the LP relaxation instead.

How do we get the flow constraints?

## How to Derive Flow Constraints?

- The constraints formulate how often an atom can be produced or consumed.
- “Produced” (resp. “consumed”) means that the atom is false (resp. true) before an operator application and true (resp. false) in the successor state.
- For general  $SAS^+$  operators, this depends on the state where the operator is applied: effect  $v := d$  only produces  $v = d$  if the operator is applied in a state  $s$  with  $s(v) \neq d$ .
- For general  $SAS^+$  tasks, the goal does not have to specify a value for every variable.
- All this makes the definition of flow constraints somewhat involved and in general such constraints are inequalities.

Good news: easy for tasks in transition normal form

# Transition Normal Form

# Variables Occurring in Conditions and Effects

- Many algorithmic problems for SAS<sup>+</sup> planning tasks become simpler when we can make two further restrictions.
- These are related to the **variables** that **occur** in conditions and effects of the task.

## Definition ( $\text{vars}(\varphi)$ , $\text{vars}(e)$ )

For a logical formula  $\varphi$  over finite-domain variables  $V$ ,  $\text{vars}(\varphi)$  denotes the set of finite-domain variables occurring in  $\varphi$ .

For an effect  $e$  over finite-domain variables  $V$ ,  $\text{vars}(e)$  denotes the set of finite-domain variables occurring in  $e$ .

# Transition Normal Form

## Definition (Transition Normal Form)

A SAS<sup>+</sup> planning task  $\Pi = \langle V, I, O, \gamma \rangle$

is in **transition normal form (TNF)** if

- for all  $o \in O$ ,  $\text{vars}(\text{pre}(o)) = \text{vars}(\text{eff}(o))$ , and
- $\text{vars}(\gamma) = V$ .

In words, an **operator** in TNF must mention the same variables in the precondition and effect, and a **goal** in TNF must mention all variables (= specify exactly one goal state).

## Converting Operators to TNF: Violations

There are two ways in which an operator  $o$  can violate TNF:

- There exists a variable  $v \in \text{vars}(\text{pre}(o)) \setminus \text{vars}(\text{eff}(o))$ .
- There exists a variable  $v \in \text{vars}(\text{eff}(o)) \setminus \text{vars}(\text{pre}(o))$ .

The **first case** is easy to address: if  $v = d$  is a precondition with no effect on  $v$ , just add the effect  $v := d$ .

The **second case** is more difficult: if we have the effect  $v := d$  but no precondition on  $v$ , how can we add a precondition on  $v$  without changing the meaning of the operator?

# Converting Operators to TNF: Multiplying Out

## Solution 1: multiplying out

- ① While there exists an operator  $o$  and a variable  $v \in \text{vars}(\text{eff}(o))$  with  $v \notin \text{vars}(\text{pre}(o))$ :
  - For each  $d \in \text{dom}(v)$ , add a new operator that is like  $o$  but with the additional precondition  $v = d$ .
  - Remove the original operator.
- ② Repeat the previous step until no more such variables exist.

## Problem:

- If an operator  $o$  has  $n$  such variables, each with  $k$  values in its domain, this introduces  $k^n$  variants of  $o$ .
- Hence, this is an **exponential** transformation.

# Converting Operators to TNF: Auxiliary Values

## Solution 2: auxiliary values

- 1 For every variable  $v$ , add a new **auxiliary value**  $u$  to its domain.
- 2 For every variable  $v$  and value  $d \in \text{dom}(v) \setminus \{u\}$ , add a new operator to change the value of  $v$  from  $d$  to  $u$  at no cost:  $\langle v = d, v := u, 0 \rangle$ .
- 3 For all operators  $o$  and all variables  $v \in \text{vars}(\text{eff}(o)) \setminus \text{vars}(\text{pre}(o))$ , add the precondition  $v = u$  to  $\text{pre}(o)$ .

## Properties:

- Transformation can be computed in linear time.
- Due to the auxiliary values, there are new states and transitions in the induced transition system, but all **path costs** between **original states** remain the same.



# Converting Goals to TNF

- The auxiliary value idea can also be used to convert the goal  $\gamma$  to TNF.
- For every variable  $v \notin \text{vars}(\gamma)$ , add the condition  $v = u$  to  $\gamma$ .

With these ideas, every SAS<sup>+</sup> planning task can be converted into transition normal form in linear time.

# TNF for Example Task (1)

The example task is not in transition normal form:

- Load and unload operators have preconditions on the position of some truck but no effect on this variable.
- The goal does not specify a value for variable *pos-t-2*.

## TNF for Example Task (2)

Operators in transition normal form: for  $i, j, k \in \{1, 2\}$ :

$$\text{load}(t_i, \text{loc}_j) = \langle \text{pos-}t\text{-}i = \text{loc}_j \wedge \text{pos-}p = \text{loc}_j, \\ \text{pos-}p := t_i \wedge \text{pos-}t\text{-}i := \text{loc}_j, 1 \rangle$$

$$\text{unload}(t_i, \text{loc}_j) = \langle \text{pos-}t\text{-}i = \text{loc}_j \wedge \text{pos-}p = t_i, \\ \text{pos-}p := \text{loc}_j \wedge \text{pos-}t\text{-}i := \text{loc}_j, 1 \rangle$$

$$\text{drive}(t_i, \text{loc}_j, \text{loc}_k) = \langle \text{pos-}t\text{-}i = \text{loc}_j, \\ \text{pos-}t\text{-}i := \text{loc}_k, 1 \rangle$$

## TNF for Example Task (3)

To bring the goal in normal form,

- add an additional value  $\mathbf{u}$  to  $\text{dom}(pos-t-2)$

- add zero-cost operators

$$o_1 = \langle pos-t-2 = loc_1, pos-t-2 := \mathbf{u}, 0 \rangle \text{ and}$$

$$o_2 = \langle pos-t-2 = loc_2, pos-t-2 := \mathbf{u}, 0 \rangle$$

- Add  $pos-t-2 = \mathbf{u}$  to the goal:

$$\gamma = (pos-p = loc_2) \wedge (pos-t-1 = loc_1) \wedge (pos-t-2 = \mathbf{u})$$

# Flow Heuristic

# Notation

- In  $SAS^+$  tasks, states are variable assignments, conditions are conjunctions over atoms, and effects are conjunctions of atomic effects.
- In the following, we use a **unifying notation** to express that an atom is true in a state/entailed by a condition/made true by an effect.
- For **state**  $s$ , we write  $(v = d) \in s$  to express that  $s(v) = d$ .
- For a **conjunction of atoms**  $\varphi$ , we write  $(v = d) \in \varphi$  to express that  $\varphi$  has a conjunct  $v = d$  (or alternatively  $\varphi \models v = d$ ).
- For **effect**  $e$ , we write  $(v = d) \in e$  to express that  $e$  contains the atomic effect  $v := d$ .

# Flow Constraints (1)

A flow constraint for an atom relates how often it can be produced to how often it can be consumed.

Let  $o$  be an operator in transition normal form. Then:

- $o$  **produces** atom  $a$  iff  $a \in \text{eff}(o)$  and  $a \notin \text{pre}(o)$ .
- $o$  **consumes** atom  $a$  iff  $a \in \text{pre}(o)$  and  $a \notin \text{eff}(o)$ .
- Otherwise  $o$  is **neutral** wrt. atom  $a$ .

↪ State-independent

## Flow Constraints (2)

A flow constraint for an atom relates how often it can be produced to how often it can be consumed.

The constraint depends on the current state  $s$  and the goal  $\gamma$ .  
If  $\gamma$  mentions all variables (as in TNF), the following holds:

- If  $a \in s$  and  $a \in \gamma$  then atom  $a$  must be equally often produced and consumed.
- Analogously for  $a \notin s$  and  $a \notin \gamma$ .
- If  $a \in s$  and  $a \notin \gamma$  then  $a$  must be consumed one time more often than it is produced.
- If  $a \notin s$  and  $a \in \gamma$  then  $a$  must be produced one time more often than it is consumed.



# Iverson Bracket

The dependency on the current state and the goal can concisely be expressed with Iverson brackets:

## Definition (Iverson Bracket)

Let  $P$  be a logical proposition (= some statement that can be evaluated to true or false). Then

$$[P] = \begin{cases} 1 & \text{if } P \text{ is true} \\ 0 & \text{if } P \text{ is false.} \end{cases}$$

**Example:**  $[2 \neq 3] = 1$

## Flow Constraints (3)

### Definition (Flow Constraint)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a task in transition normal form.  
The **flow constraint** for atom  $a$  in state  $s$  is

$$[a \in s] + \sum_{o \in O: a \in \text{eff}(o)} \text{Count}_o = [a \in \gamma] + \sum_{o \in O: a \in \text{pre}(o)} \text{Count}_o$$

- $\text{Count}_o$  is an LP variable for the number of occurrences of operator  $o$ .
- Neutral operators either appear on both sides or on none.

# Flow Heuristic

## Definition (Flow Heuristic)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a SAS<sup>+</sup> task in transition normal form and let  $A = \{(v = d) \mid v \in V, d \in \text{dom}(v)\}$  be the set of atoms of  $\Pi$ .

The **flow heuristic**  $h^{\text{flow}}(s)$  is the objective value of the following LP or  $\infty$  if the LP is infeasible:

minimize  $\sum_{o \in O} \text{cost}(o) \cdot \text{Count}_o$  subject to

$$[a \in s] + \sum_{o \in O: a \in \text{eff}(o)} \text{Count}_o = [a \in \gamma] + \sum_{o \in O: a \in \text{pre}(o)} \text{Count}_o \text{ for all } a \in A$$

$$\text{Count}_o \geq 0 \text{ for all } o \in O$$

# Flow Heuristic on Example Task

↪ Blackboard

# Flow Heuristic: Properties (1)

## Theorem

*The flow heuristic  $h^{\text{flow}}$  is goal-aware, safe, consistent and admissible.*

## Proof Sketch.

It suffices to prove goal-awareness and consistency.

**Goal-awareness:** If  $s \models \gamma$  then  $\text{Count}_o = 0$  for all  $o \in O$  is feasible and the objective function has value 0. As  $\text{Count}_o \geq 0$  for all variables and operator costs are nonnegative, the objective value cannot be smaller. . . .

## Flow Heuristic: Properties (2)

### Proof Sketch (continued).

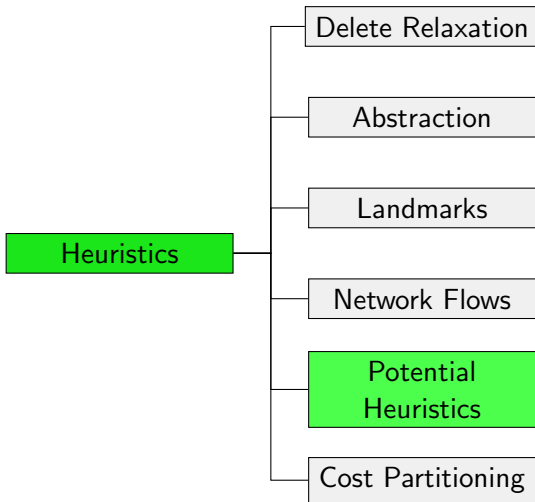
**Consistency:** Let  $o$  be an operator that is applicable in state  $s$  and let  $s' = s[o]$ .

Increasing  $\text{Count}_o$  by one in an optimal feasible assignment for the LP for state  $s'$  yields a feasible assignment for the LP for state  $s$ , where the objective function is  $h^{\text{flow}}(s') + \text{cost}(o)$ .

This is an upper bound on  $h^{\text{flow}}(s)$ , so in total  $h^{\text{flow}}(s) \leq h^{\text{flow}}(s') + \text{cost}(o)$ . □

# Potential Heuristics

# Content of this Course: Heuristics





# Potential Heuristics

## Potential Heuristics: Idea

Heuristic design as an optimization problem:

- Define simple numerical **state features**  $f_1, \dots, f_n$ .
- Consider heuristics that are **linear combinations** of features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (**potentials**)  $w_i \in \mathbb{R}$

- Find potentials for which  $h$  is admissible and well-informed.

Motivation:

- **declarative approach** to heuristic design
- heuristic **very fast to compute** if features are

# Features

## Definition (feature)

A (state) **feature** for a planning task is a numerical function defined on the states of the task:  $f : S \rightarrow \mathbb{R}$ .

# Features

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A (state) **feature** for a planning task is a numerical function defined on the states of the task:  $f : S \rightarrow \mathbb{R}$ .

**Atomic features** test if some atom is true in a state:

## Definition (atomic feature)

Let  $v = d$  be an atom of a FDR planning task.

The **atomic feature**  $f_{v=d}$  is defined as:

$$f_{v=d}(s) = [(v = d) \in s]$$

# Potential Heuristics

## Definition (potential heuristic)

A **potential heuristic** for a set of features  $\mathcal{F} = \{f_1, \dots, f_n\}$  is a heuristic function  $h$  defined as a **linear combination** of the features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (**potentials**)  $w_i \in \mathbb{R}$ .

↪ cf. **evaluation functions** for board games like Chess

- We only consider **atomic** potential heuristics, which are based on the set of all atomic features.
- **Example** for a task with state variables  $v_1$  and  $v_2$  and  $\text{dom}(v_1) = \text{dom}(v_2) = \{d_1, d_2, d_3\}$ :

$$h(s) = 3f_{v_1=d_1} + \frac{1}{2}f_{v_1=d_2} - 2f_{v_1=d_3} + 2.5f_{v_2=d_1}$$

# How to Set the Weights?

We want to find **good** atomic potential heuristics:

- admissible
- consistent
- well-informed

How to achieve this? **Linear programming to the rescue!**

# Admissible and Consistent Potential Heuristics

Constraints on potentials **characterize** (= are necessary and sufficient for) admissible and consistent atomic potential heuristics:

## Goal-awareness

$$\sum_{\text{goal atoms } a} w_a = 0$$

## Consistency

$$\sum_{\substack{a \text{ consumed} \\ \text{by } o}} w_a - \sum_{\substack{a \text{ produced} \\ \text{by } o}} w_a \leq \text{cost}(o) \quad \text{for all operators } o$$

## Remarks:

- assumes transition normal form (not a limitation)
- goal-aware and consistent = admissible and consistent

# Well-Informed Potential Heuristics

How to find a **well-informed** potential heuristic?

↪ encode **quality metric** in the **objective function**  
and use LP solver to find a heuristic maximizing it

Examples:

- maximize **heuristic value of a given state** (e.g., initial state)
- maximize average heuristic value of **all states**  
(including unreachable ones)
- maximize average heuristic value of some **sample states**
- minimize **estimated search effort**

# Potential and Flow Heuristic

## Theorem

For state  $s$ , let  $h^{\max\text{pot}}(s)$  denote the *maximal* heuristic value of all admissible and consistent atomic potential heuristics in  $s$ .

Then  $h^{\max\text{pot}}(s) = h^{\text{flow}}(s)$ .

**Proof idea:** compare dual of  $h^{\text{flow}}(s)$  LP to potential heuristic constraints optimized for state  $s$ .

If we optimize the potentials for a given state then for this state it equals the flow heuristic.



# Summary

# Summary

- A flow constraint for an atom describes how the number of producing operator applications is linked to the number of consuming operator applications.
- The flow heuristic computes a lower bound on the cost of each operator sequence that satisfies these constraints for all atoms.
- The flow heuristic only considers the number of occurrences of each operator, but ignores their order.
- Potential heuristics can be used as **fast admissible approximations** of  $h^{\text{flow}}$ .