Planning and Optimization

E1. Landmarks: MHS & Uniform Cost Partitioning Heuristic

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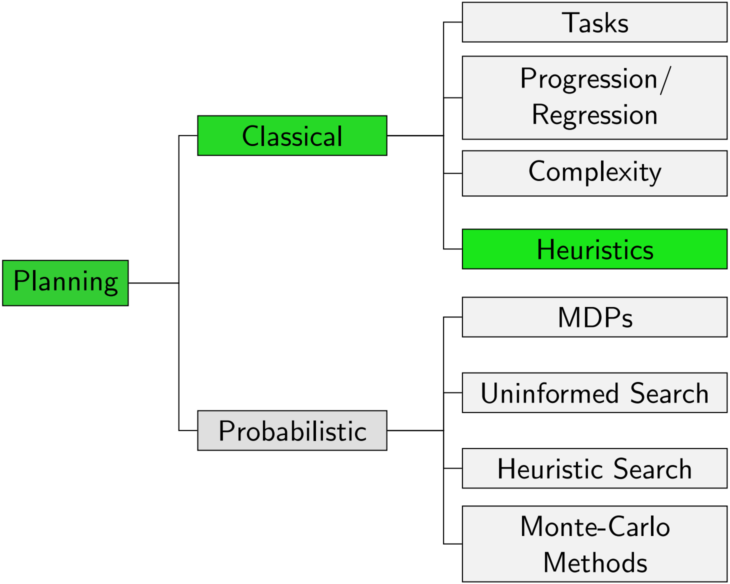
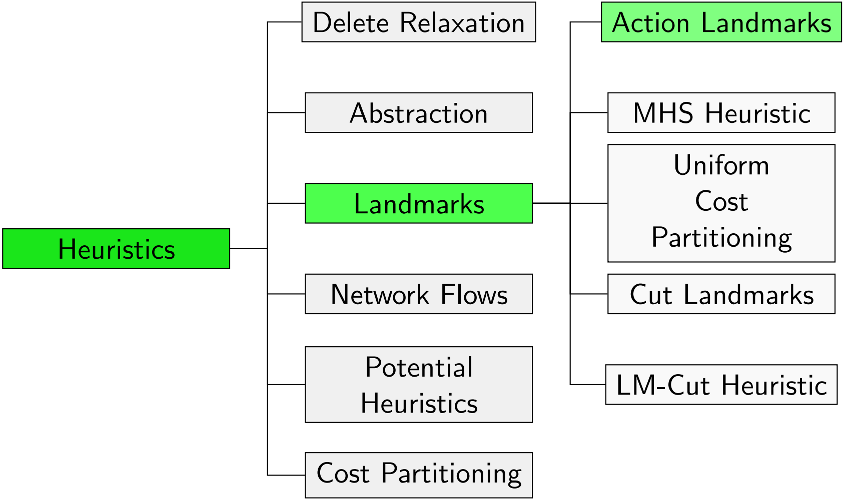
Landmarks

Content of this Course



Content of this Course: Heuristics



Landmarks

Basic Idea: Something that must happen in every solution

For example

- some operator must be applied
- some atom must be true
- some formula must be true

$\rightarrow$ Derive heuristic estimate from this kind of information.

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We will only consider disjunctive action landmarks.

Disjunctive Action Landmarks

Definition (Disjunctive Action Landmark)

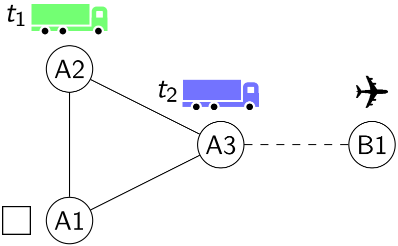
Let s be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.

A disjunctive action landmark for s is a set of operators $L \subseteq O$ such that every plan for s ($=$ label path from s to a goal state) contains an operator from L.

The cost of landmark L is $cost(L) = \min_{o \in L} cost(o)$.

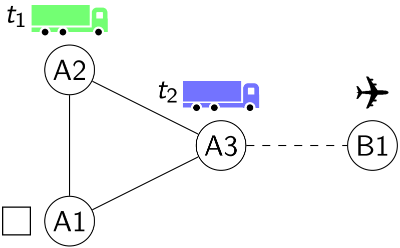
Example Task

- Two trucks, one airplane
- Airplane can fly between locations A3 and B1
- Trucks can drive arbitrarily between locations A1, A2, and A3
- Package to be transported from A1 to B1
- Operators
  - Load(v, ℓ) and Unload(v, ℓ) for vehicle v and location ℓ
  - Drive(t, ℓ, ℓ') for truck t and locations ℓ, ℓ'
  - Fly(ℓ, ℓ') for locations ℓ, ℓ'



Example: Disjunctive Action Landmarks

$L_1 = \{Load(Truck1, A1), Load(Truck2, A1)\}$ and
$L_2 = \{Fly(B1, A3)\}$ are disjunctive action landmarks.



What other disjunctive action landmarks are there?

Remarks

- Not every landmark is informative.
  - For example: If the initial state is not already a goal state then the set of all operators is a disjunctive action landmark.
- Deciding whether a given operator set is a disjunctive action landmark is as hard as the plan existence problem.

Exploiting Disjunctive Action Landmarks

How can we exploit a given set $\mathcal{L}$ of disjunctive action landmarks?

- Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ not admissible!
- Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ usually very weak heuristic
- better: hitting sets or cost partitioning

Minimum Hitting Set Heuristic

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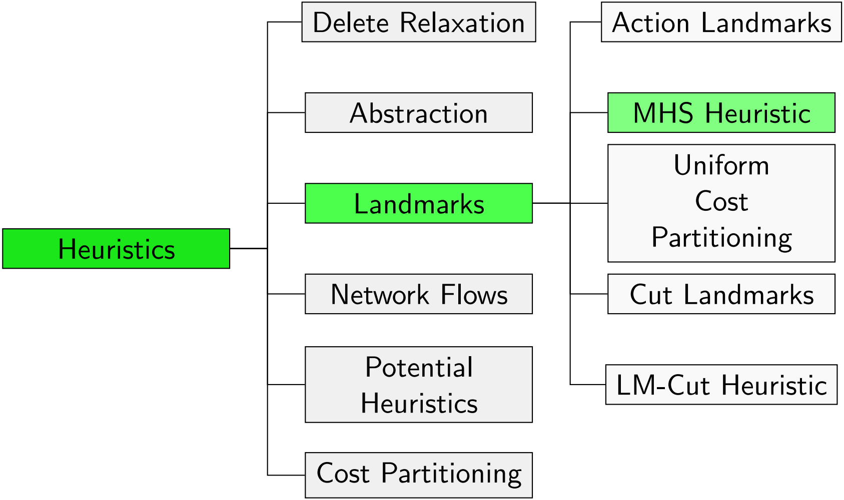

Hitting Sets

Definition (Hitting Set)

Let X be a set, $\mathcal{F} = \{F_1, \dots, F_n\} \subseteq 2^X$ be a family of subsets of X and $c : X \to \mathbb{R}_0^+$ be a cost function for X.

A hitting set is a subset $H \subseteq X$ that "hits" all subsets in $\mathcal{F}$, i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The cost of H is $\sum_{x \in H} c(x)$.

A minimum hitting set (MHS) is a hitting set with minimal cost.

MHS is a "classical" NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3, \;\; c(o_2) = 4, \;\; c(o_3) = 5, \;\; c(o_4) = 0$

minimum hitting set:

Example: Hitting Sets

Example

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$

$c(o_1) = 3, \ \ c(o_2) = 4, \ \ c(o_3) = 5, \ \ c(o_4) = 0$

minimum hitting set: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Disjunctive Action Landmarks

Idea: disjunctive action landmarks are interpreted as instance of minimum hitting set

Definition (Hitting Set Heuristic)

Let $\mathcal{L}$ be a set of disjunctive action landmarks. The hitting set heuristic $h^{\mathrm{MHS}}(\mathcal{L})$ is defined as the cost of a minimum hitting set for $\mathcal{L}$ with $c(o) = cost(o)$.

Proposition (Hitting Set Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s. Then $h^{MHS}(\mathcal{L})$ is an admissible estimate for s.

Why?

Hitting Set Heuristic: Discussion

- The hitting set heuristic is the best possible heuristic that only uses the given information. . .
- . . . but is NP-hard to compute.
- $\rightsquigarrow$ Use approximations that can be efficiently computed.
- Now: uniform cost partitioning
- Later in the course: optimal cost partitioning

Uniform Cost Partitioning

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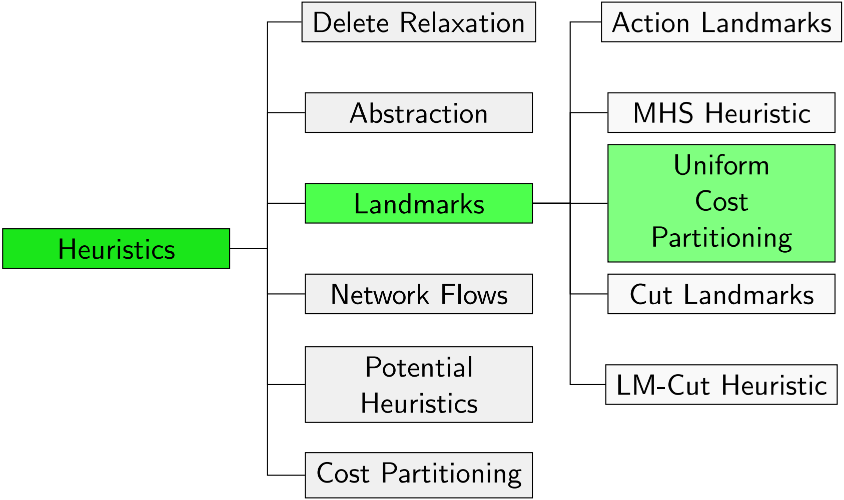

Uniform Cost Partitioning (1)

Idea: Distribute cost of operators uniformly among the landmarks.

Definition (Uniform Cost Partitioning Heuristic for Landmarks)

Let $\mathcal{L}$ be a set of disjunctive action landmarks.

The uniform cost partitioning heuristic $h^{\mathrm{UCP}}(\mathcal{L})$ is defined as

$$h^{\mathrm{UCP}}(\mathcal{L}) = \sum_{L \in \mathcal{L}} \min_{o \in L} c'(o) \text{ with}$$

$$c'(o) = cost(o) / |\{L \in \mathcal{L} \mid o \in L\}|.$$

Uniform Cost Partitioning (2)

Theorem (Uniform Cost Partitioning Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s of Π. Then $h^{UCP}(\mathcal{L})$ is an admissible heuristic estimate for s.

Uniform Cost Partitioning (2)

Theorem (Uniform Cost Partitioning Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s of Π. Then $h^{UCP}(\mathcal{L})$ is an admissible heuristic estimate for s.

Proof.

Let $\pi = \langle o_1, \dots, o_n \rangle$ be an optimal plan for s. For $L \in \mathcal{L}$ define a new cost function $cost_L$ as $cost_L(o) = c'(o)$ if $o \in L$ and $cost_L(o) = 0$ otherwise. Let Π_L be a modified version of Π, where for all operators o the cost is replaced with $cost_L(o)$. We make three independent observations:

1. For $L \in \mathcal{L}$ the value $cost'(L) := \min_{o \in L} c'(o)$ is an admissible estimate for s in Π_L.
2. π is also a plan for s in Π_L, so $h^*_{\Pi_L}(s) \leq \sum_{i=1}^{n} cost_L(o_i)$.
3. $\sum_{L \in \mathcal{L}} cost_L(o) = cost(o)$ for each operator o.

...

Uniform Cost Partitioning (3)

Proof (continued).

Together, this leads to the following inequality (subscripts indicate for which task the heuristic is computed):

$$
\begin{aligned}
h_{\Pi}^{\mathrm{UCP}}(\mathcal{L}) = \sum_{L\in\mathcal{L}} cost'(L) &\overset{(1)}{\leq} \sum_{L\in\mathcal{L}} h^*_{\Pi_L}(s) \\
&\overset{(2)}{\leq} \sum_{L\in\mathcal{L}} \sum_{i=1}^{n} cost_L(o_i) = \sum_{i=1}^{n} \sum_{L\in\mathcal{L}} cost_L(o_i) \\
&\overset{(3)}{=} \sum_{i=1}^{n} cost(o) = h^*_{\Pi}(s)
\end{aligned}
$$

□

Relationship

Theorem

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s.

Then $h^{UCP}(\mathcal{L}) \leq h^{MHS}(\mathcal{L}) \leq h^*(s)$.

(Proof omitted.)

Summary

Summary

- Disjunctive action landmark: set L of operators such that every plan uses some operator from L
- The cost of the landmark is the cost of its cheapest operator.
- Hitting sets yield the most accurate heuristic for a given set of disjunctive action landmarks, but the computation is NP-hard.
- Uniform cost partitioning is a polynomial approach for the computation of informative heuristics from disjunctive action landmarks.