Planning and Optimization

D2. Abstractions: Additive Abstractions

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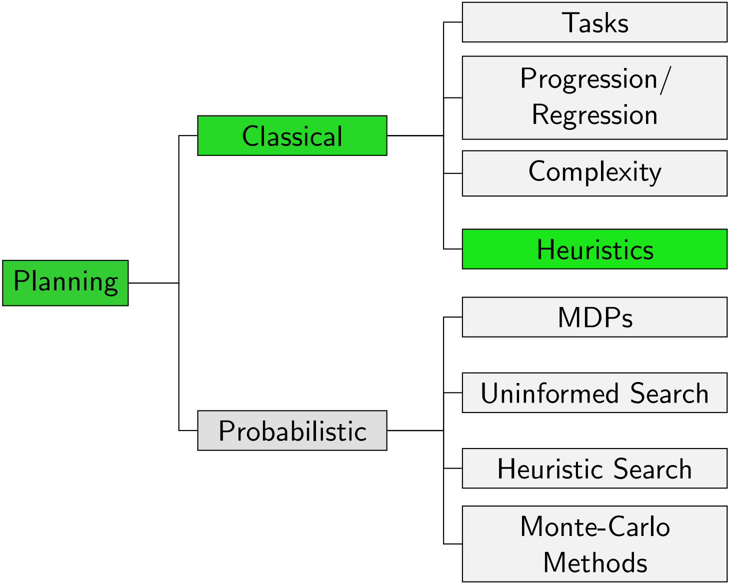
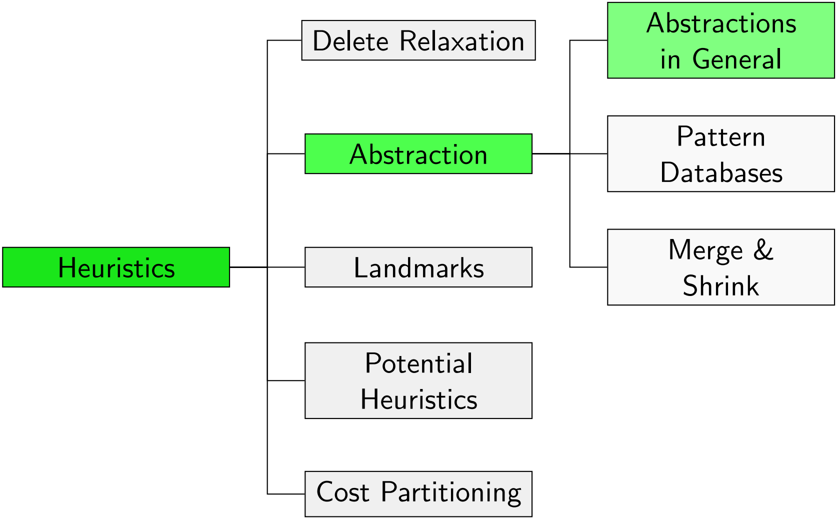
Content of this Course



Content of this Course: Heuristics



Multiple Abstractions

Multiple Abstractions

- One important practical question is how to come up with a suitable abstraction mapping α.
- Indeed, there is usually a huge number of possibilities, and it is important to pick good abstractions (i.e., ones that lead to informative heuristics).
- However, it is generally not necessary to commit to a single abstraction.

Combining Multiple Abstractions

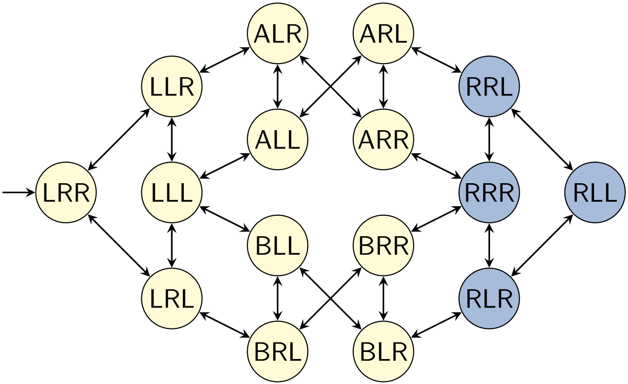
Maximizing several abstractions:

- Each abstraction mapping gives rise to an admissible heuristic.
- By computing the maximum of several admissible heuristics, we obtain another admissible heuristic which dominates the component heuristics.
- Thus, we can always compute several abstractions and maximize over the individual abstract goal distances.

Adding several abstractions:

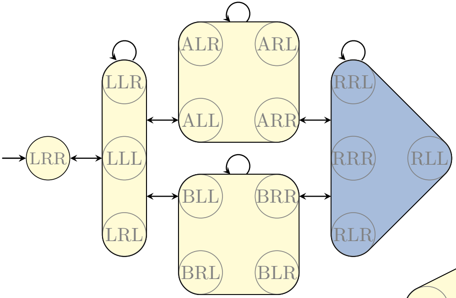
- In some cases, we can even compute the sum of individual estimates and still stay admissible.
- Summation often leads to much higher estimates than maximization, so it is important to understand under which conditions summation of heuristics is admissible.

Adding Several Abstractions: Example (1)

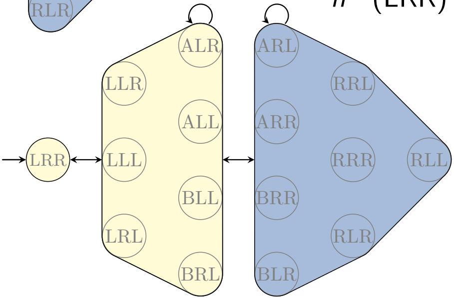


$h^*(\text{LRR}) = 4$

Adding Several Abstractions: Example (2)

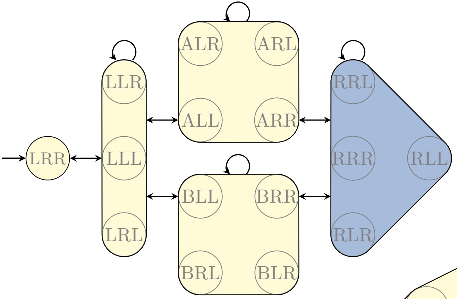


$h^{\alpha_1}(\mathrm{LRR}) = 3$



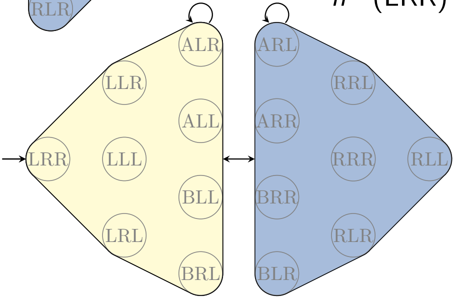
$h^{\alpha_2}(\mathrm{LRR}) = 2$

Adding Several Abstractions: Example (3)



$h^{\alpha_2}(\mathsf{LRR}) = 1$

$h^{\alpha_1}(\mathsf{LRR}) = 3$



Additivity

Orthogonality of Abstractions

Definition (Orthogonal)

Let α_1 and α_2 be abstractions of transition system $\mathcal{T}$.

We say that α_1 and α_2 are orthogonal if for all transitions $s \xrightarrow{\ell} t$ of $\mathcal{T}$, we have $\alpha_i(s) = \alpha_i(t)$ for at least one $i \in \{1, 2\}$.

Affecting Transition Labels

Definition (Affecting Transition Labels)

Let $\mathcal{T}$ be a transition system, and let ℓ be one of its labels.

We say that ℓ affects $\mathcal{T}$ if $\mathcal{T}$ has a transition $s \xrightarrow{\ell} t$ with $s \neq t$.

Theorem (Affecting Labels vs. Orthogonality)

Let α_1 and α_2 be abstractions of transition system $\mathcal{T}$.

If no label of $\mathcal{T}$ affects both $\mathcal{T}^{\alpha_1}$ and $\mathcal{T}^{\alpha_2}$,
then α_1 and α_2 are orthogonal.

(Easy proof omitted.)

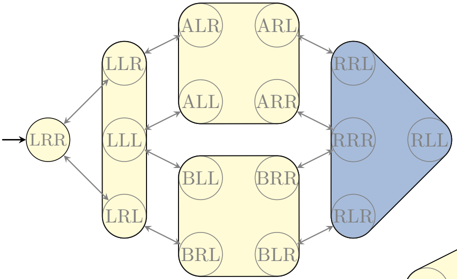
Orthogonality and Additivity

Theorem (Additivity for Orthogonal Abstractions)

Let $h^{\alpha_1}, \dots, h^{\alpha_n}$ be abstraction heuristics of the same transition system such that α_i and α_j are orthogonal for all $i \neq j$.

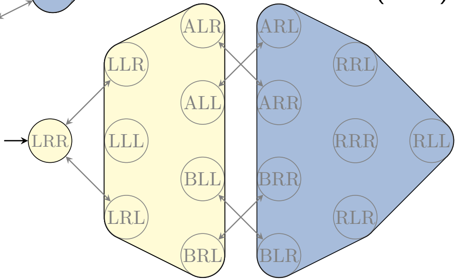
Then $\sum_{i=1}^{n} h^{\alpha_i}$ is a safe, goal-aware, admissible and consistent heuristic for Π.

Orthogonality and Additivity: Example (1)

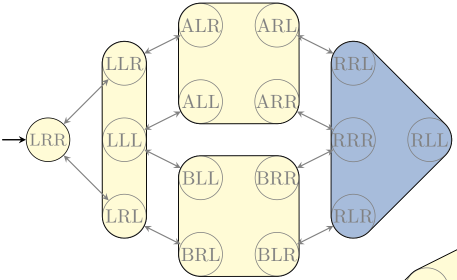


$$h^{\alpha_1}(\mathsf{LRR}) = 3$$

$$h^{\alpha_2}(\mathsf{LRR}) = 2$$

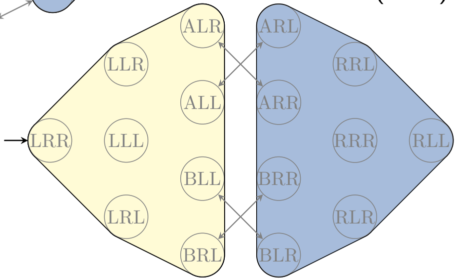


Orthogonality and Additivity: Example (2)



$h^{\alpha_1}(\text{LRR}) = 3$

$h^{\alpha_2}(\text{LRR}) = 1$



Orthogonality and Additivity: Proof (1)

Proof.

We prove goal-awareness and consistency;
the other properties follow from these two.

Let $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$ be the concrete transition system.

Let $h = \sum_{i=1}^{n} h^{\alpha_i}$.

Orthogonality and Additivity: Proof (1)

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Let $h = \sum_{i=1}^{n} h^{\alpha_i}$.

Goal-awareness: For goal states $s \in S_\star$,
$h(s) = \sum_{i=1}^{n} h^{\alpha_i}(s) = \sum_{i=1}^{n} 0 = 0$ because all individual abstraction heuristics are goal-aware. ...

Orthogonality and Additivity: Proof (2)

Proof (continued).

Consistency: Let $s \xrightarrow{o} t \in T$. We must prove $h(s) \leq c(o) + h(t)$.

Orthogonality and Additivity: Proof (2)

Proof (continued).

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Because the abstractions are orthogonal, $\alpha_i(s) \neq \alpha_i(t)$ for at most one $i \in \{1, \ldots, n\}$.

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Case 1: $\alpha_i(s) = \alpha_i(t)$ for all $i \in \{1, \dots, n\}$.

Orthogonality and Additivity: Proof (2)

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Case 1: $\alpha_i(s) = \alpha_i(t)$ for all $i \in \{1, \dots, n\}$.

$$
\begin{aligned}
\text{Then } h(s) &= \textstyle\sum_{i=1}^{n} h^{\alpha_i}(s) \\
&= \textstyle\sum_{i=1}^{n} h^*_{\mathcal{T}^{\alpha_i}}(\alpha_i(s)) \\
&= \textstyle\sum_{i=1}^{n} h^*_{\mathcal{T}^{\alpha_i}}(\alpha_i(t)) \\
&= \textstyle\sum_{i=1}^{n} h^{\alpha_i}(t) \\
&= h(t) \leq c(o) + h(t).
\end{aligned}
$$

...

Orthogonality and Additivity: Proof (3)

Proof (continued).

Case 2: $\alpha_i(s) \neq \alpha_i(t)$ for exactly one $i \in \{1, \dots, n\}$.

Let $k \in \{1, \dots, n\}$ such that $\alpha_k(s) \neq \alpha_k(t)$.

Orthogonality and Additivity: Proof (3)

Proof (continued).

Case 2: $\alpha_i(s) \neq \alpha_i(t)$ for exactly one $i \in \{1, \dots, n\}$.

Let $k \in \{1, \dots, n\}$ such that $\alpha_k(s) \neq \alpha_k(t)$.

Then
$$\begin{aligned} h(s) &= \textstyle\sum_{i=1}^{n} h^{\alpha_i}(s) \\ &= \textstyle\sum_{i\in\{1,\dots,n\}\setminus\{k\}} h^*_{\mathcal{T}^{\alpha_i}}(\alpha_i(s)) + h^{\alpha_k}(s) \\ &\leq \textstyle\sum_{i\in\{1,\dots,n\}\setminus\{k\}} h^*_{\mathcal{T}^{\alpha_i}}(\alpha_i(t)) + c(o) + h^{\alpha_k}(t) \\ &= c(o) + \textstyle\sum_{i=1}^{n} h^{\alpha_i}(t) \\ &= c(o) + h(t), \end{aligned}$$
where the inequality holds because $\alpha_i(s) = \alpha_i(t)$ for all $i \neq k$ and h^{α_k} is consistent. □

Outlook

Using Abstraction Heuristics in Practice

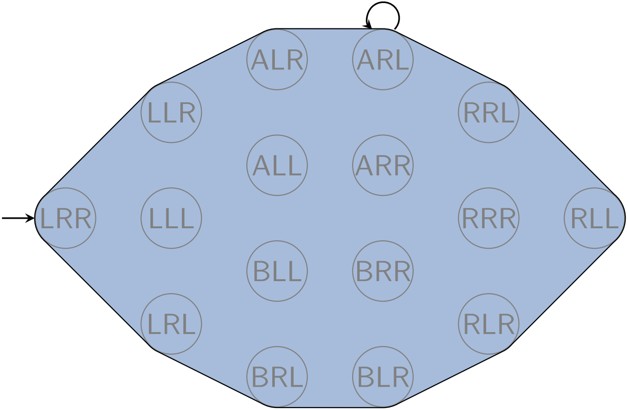
In practice, there are conflicting goals for abstractions:

- we want to obtain an informative heuristic, but
- want to keep its representation small.

Abstractions have small representations if

- there are few abstract states and
- there is a succinct encoding for α.

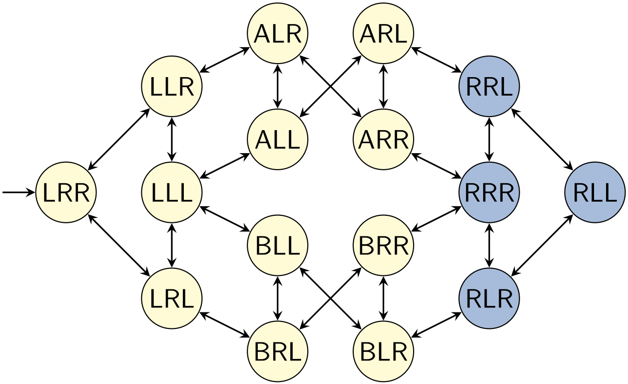
Counterexample: One-State Abstraction



One-state abstraction: $\alpha(s) :=$ const.

+ very few abstract states and succinct encoding for α
− completely uninformative heuristic

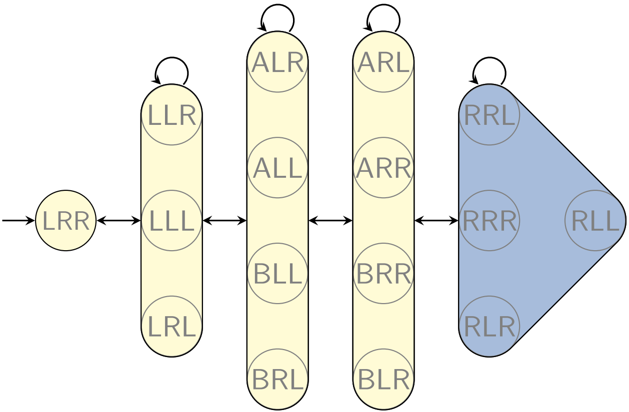
Counterexample: Identity Abstraction



Identity abstraction: $\alpha(s) := s$.

+ perfect heuristic and succinct encoding for α
− too many abstract states

Counterexample: Perfect Abstraction



Perfect abstraction: $\alpha(s) := h^*(s)$.

- + perfect heuristic and usually few abstract states
- − usually no succinct encoding for α

Automatically Deriving Good Abstraction Heuristics

Abstraction Heuristics for Planning: Main Research Problem

Automatically derive effective abstraction heuristics for planning tasks.

⇝ we will study two state-of-the-art approaches in Chapters D3–D8

Summary

Summary

- Often, multiple abstractions are used.
  They can always be maximized admissibly.
- Adding abstraction heuristics is not always admissible.
  When it is, it leads to a stronger heuristic than maximizing.
- Abstraction heuristics from orthogonal abstractions
  can be added without losing admissibility or consistency.
- One sufficient condition for orthogonality is that all
  abstractions are affected by disjoint sets of labels.
- Practically useful abstractions are those which give
  informative heuristics, yet have a small representation.
- Coming up with good abstractions automatically
  is the main research challenge when applying
  abstraction heuristics in planning.