Planning and Optimization

A7. Invariants, Mutexes and Finite Domain Representation

Gabriele Röger and Thomas Keller

Universität Basel

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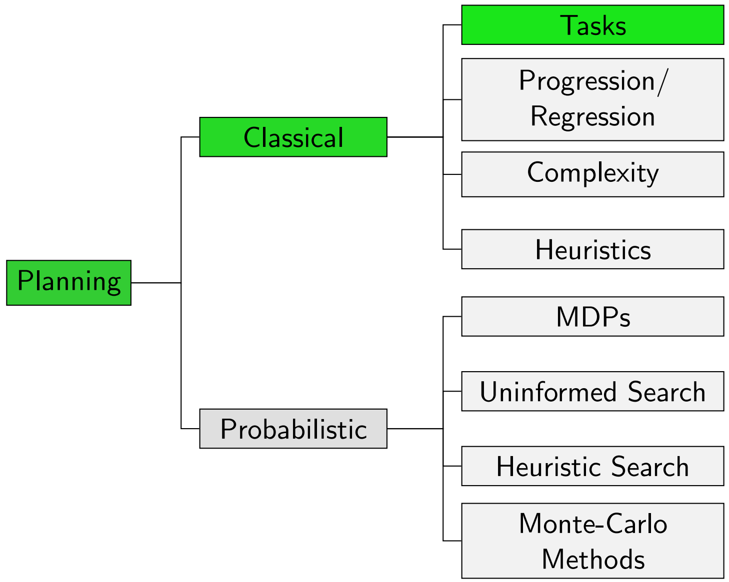
Content of this Course



A7.1 Invariants

Invariants

- When we as humans reason about planning tasks, we implicitly make use of "obvious" properties of these tasks.
  - Example: we are never in two places at the same time
- We can represent such properties as logical formulas φ that are true in all reachable states.
  - Example: $\varphi = \neg(\textit{at-uni} \land \textit{at-home})$
- Such formulas are called invariants of the task.

Invariants: Definition

Definition (Invariant)

An invariant of a planning task Π with state variables V is a logical formula φ over V such that $s \models \varphi$ for all reachable states s of Π.

Computing Invariants

- Theoretically, testing if an arbitrary formula φ is an invariant is as hard as planning itself.
  $\leadsto$ proof idea: a planning task is unsolvable iff the negation of its goal is an invariant
- Still, many practical invariant synthesis algorithms exist.
- To remain efficient (= polynomial-time), these algorithms only compute a subset of all useful invariants.
  $\leadsto$ sound, but not complete
- Empirically, they tend to at least find the "obvious" invariants of a planning task.

Exploiting Invariants

Invariants have many uses in planning:

- Regression search:
  Prune states that violate (are inconsistent with) invariants.
- Planning as satisfiability:
  Add invariants to a SAT encoding of a planning task to get tighter constraints.
- Reformulation:
  Derive a more compact state space representation (i.e., with fewer unreachable states).

We now briefly discuss the last point because it is important for planning tasks in finite-domain representation, introduced in the following chapter.

A7.2 Mutexes

Mutexes

Invariants that take the form of binary clauses are called mutexes because they express that certain variable assignments cannot be simultaneously true and are hence mutually exclusive.

Example (Blocks World)

The invariant $\neg A\text{-}on\text{-}B \lor \neg A\text{-}on\text{-}C$ states that $A\text{-}on\text{-}B$ and $A\text{-}on\text{-}C$ are mutex.

We say that a larger set of literals is mutually exclusive if every subset of two literals is a mutex.

Example (Blocks World)

Every pair in $\{B\text{-}on\text{-}A, C\text{-}on\text{-}A, D\text{-}on\text{-}A, A\text{-}clear\}$ is mutex.

Encoding Mutex Groups as Finite-Domain Variables

Let $L = \{\ell_1, \dots, \ell_n\}$ be mutually exclusive literals over n different variables $V_L = \{v_1, \dots, v_n\}$.

Then the planning task can be rephrased using a single finite-domain (i.e., non-binary) state variable v_L with $n + 1$ possible values in place of the n variables in V_L:

- n of the possible values represent situations in which exactly one of the literals in L is true.
- The remaining value represents situations in which none of the literals in L is true.
  - Note: If we can prove that one of the literals in L must be true in each state (i.e., $\ell_1 \lor \dots \lor \ell_n$ is an invariant), this additional value can be omitted.

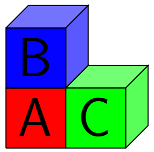
In many cases, the reduction in the number of variables dramatically improves performance of a planning algorithm.

A7.3 FDR Planning Tasks

Reminder: Blocks World with Boolean State Variables

Example

$$s(\textit{A-on-B}) = \mathbf{F}$$
$$s(\textit{A-on-C}) = \mathbf{F}$$
$$s(\textit{A-on-table}) = \mathbf{T}$$
$$s(\textit{B-on-A}) = \mathbf{T}$$
$$s(\textit{B-on-C}) = \mathbf{F}$$
$$s(\textit{B-on-table}) = \mathbf{F}$$
$$s(\textit{C-on-A}) = \mathbf{F}$$
$$s(\textit{C-on-B}) = \mathbf{F}$$
$$s(\textit{C-on-table}) = \mathbf{T}$$



$\rightsquigarrow 2^9 = 512$ states

Note: it may be useful to add auxiliary state variables like *A-clear*.

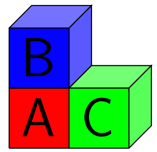
Blocks World with Finite-Domain State Variables

Use three finite-domain state variables:

- *below-a*: $\{\text{b}, \text{c}, \text{table}\}$
- *below-b*: $\{\text{a}, \text{c}, \text{table}\}$
- *below-c*: $\{\text{a}, \text{b}, \text{table}\}$

Example

$$s(\textit{below-a}) = \text{table}$$
$$s(\textit{below-b}) = \text{a}$$
$$s(\textit{below-c}) = \text{table}$$



$\rightsquigarrow 3^3 = 27$ states

Note: it may be useful to add auxiliary state variables like *above-a*.

Finite-Domain State Variables

Definition (Finite-Domain State Variable)

A finite-domain state variable is a symbol v with an associated finite domain, i.e., a non-empty finite set.

We write $\text{dom}(v)$ for the domain of v.

Example (Blocks World)

$v = \textit{above-a}$, $\text{dom}(\textit{above-a}) = \{\text{b}, \text{c}, \text{nothing}\}$

This state variable encodes the same information as the propositional variables *B-on-A*, *C-on-A* and *A-clear*.

Finite-Domain States

Definition (Finite-Domain State)

Let V be a finite set of finite-domain state variables.

A state over V is an assignment $s : V \to \bigcup_{v \in V} \text{dom}(v)$ such that $s(v) \in \text{dom}(v)$ for all $v \in V$.

Example (Blocks World)

$$s = \{\textit{above-a} \mapsto \text{nothing}, \textit{above-b} \mapsto \text{a}, \textit{above-c} \mapsto \text{b}, \textit{below-a} \mapsto \text{b}, \textit{below-b} \mapsto \text{c}, \textit{below-c} \mapsto \text{table}\}$$

Finite-Domain Formulas

Definition (Finite-Domain Formula)

Logical formulas over finite-domain state variables V are defined identically to the propositional case, except that instead of atomic formulas of the form $v' \in V'$ with propositional state variables V', there are atomic formulas of the form $v = d$, where $v \in V$ and $d \in \text{dom}(v)$.

Example (Blocks World)

The formula $(\textit{above-a} = \text{nothing}) \lor \neg(\textit{below-b} = \text{c})$ corresponds to the formula $\textit{A-clear} \lor \neg \textit{B-on-C}$.

Finite-Domain Effects

Definition (Finite-Domain Effect)

Effects over finite-domain state variables V are defined identically to the propositional case, except that instead of atomic effects of the form v' and $\neg v'$ with propositional state variables $v' \in V'$, there are atomic effects of the form $v := d$, where $v \in V$ and $d \in \text{dom}(v)$.

Example (Blocks World)

The effect
$(\textit{below-a} := \text{table}) \land ((\textit{above-b} = \text{a}) \rhd (\textit{above-b} := \text{nothing}))$
corresponds to the effect
$\textit{A-on-table} \land \neg \textit{A-on-B} \land \neg \textit{A-on-C} \land (\textit{A-on-B} \rhd (\textit{B-clear} \land \neg \textit{A-on-B}))$.

$\rightsquigarrow$ finite-domain operators, effect conditions etc. follow

Planning Tasks in Finite-Domain Representation

Definition (Planning Task in Finite-Domain Representation)

A planning task in finite-domain representation or FDR planning task is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- V is a finite set of finite-domain state variables,
- I is a state over V called the initial state,
- O is a finite set of finite-domain operators over V, and
- γ is a formula over V called the goal.

A7.4 FDR Task Semantics

FDR Task Semantics: Introduction

- We have now defined what FDR tasks look like.
- We still have to define their semantics.
- Because they are similar to propositional planning tasks, we can define their semantics in a very similar way.

Direct vs. Compilation Semantics

We describe two ways of defining semantics for FDR tasks:

- directly, mirroring our definitions for propositional tasks
- by compilation to propositional tasks

Comparison of the semantics:

- The two semantics are equivalent in terms of the reachable state space and hence in terms of the set of solutions. (We will not prove this.)
- They are not equivalent w.r.t. the set of all states.

Where the distinction matters, we use the direct semantics in this course unless stated otherwise.

Conflicting Effects

- As with propositional planning tasks, there is a subtlety: what should an effect of the form $v := \mathsf{a} \wedge v := \mathsf{b}$ mean?
- For FDR tasks, the common convention is to make this illegal, i.e., to make an operator inapplicable if it would lead to conflicting effects.

Consistency Condition and Applicability

Definition (Consistency Condition)

Let e be an effect over finite-domain state variables V.

The consistency condition for e, $consist(e)$ is defined as

$$\bigwedge_{v \in V} \bigwedge_{d, d' \in \mathrm{dom}(v), d \neq d'} \neg(effcond(v := d, e) \wedge effcond(v := d', e)).$$

Definition (Applicable FDR Operator)

An FDR operator o is applicable in a state s if $s \models pre(o) \wedge consist(eff(o))$.

The definitions of $s[\![o]\!]$ etc. then follow in the natural way.

Reminder: Semantics of Propositional Planning Tasks

Reminder from Chapter A4:

Definition (Transition System Induced by a Prop. Planning Task)

The propositional planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- S is the set of all valuations of V,
- L is the set of operators O,
- $c(o) = cost(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

Semantics of Planning Tasks

A definition that works for both types of planning tasks:

Definition (Transition System Induced by a Planning Task)

The planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- S is the set of states over V,
- L is the set of operators O,
- $c(o) = cost(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

Planning task here refers to either a propositional or FDR task.

Compilation Semantics

Definition (Induced Propositional Planning Task)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an FDR planning task. The induced propositional planning task Π' is the (regular) planning task $\Pi' = \langle V', I', O', \gamma' \rangle$, where

- $V' = \{\langle v, d \rangle \mid v \in V, d \in \text{dom}(v)\}$
- $I'(\langle v, d \rangle) = \mathbf{T}$ iff $I(v) = d$
- O' and γ' are obtained from O and γ by
  - replacing each operator precondition $pre(o)$ by $pre(o) \land consist(eff(o))$, and then
  - replacing each atomic formula $v = d$ by the proposition $\langle v, d \rangle$,
  - replacing each atomic effect $v := d$ by the effect $\langle v, d \rangle \land \bigwedge_{d' \in \text{dom}(v) \setminus \{d\}} \neg \langle v, d' \rangle$.

A7.5 SAS$^+$ Planning Tasks

SAS$^+$ Planning Tasks

Definition (SAS$^+$ Planning Task)

An FDR planning task $\Pi = \langle V, I, O, \gamma \rangle$ is called a SAS$^+$ planning task if

- there are no conditional effects in O, and
- all operator preconditions in O and the goal formula γ are conjunctions of atoms.

SAS$^+$ vs. STRIPS

- SAS$^+$ is the analogue of STRIPS planning tasks for FDR
- induced propositional planning task of a SAS$^+$ task is a STRIPS planning task after simplification (consistency conditions simplify to $\bot$ or $\top$)
- FDR tasks obtained by mutex-based reformulation of STRIPS planning tasks are SAS$^+$ tasks

A7.6 Summary

Summary

- Invariants are common properties of all reachable states, expressed as logical formulas.
- Mutexes are invariants that express that certain pairs of literals are mutually exclusive.
- Planning tasks in finite-domain representation (FDR) are an alternative to propositional planning tasks.
- FDR tasks are often more compact (have fewer states).
- This makes many planning algorithms more efficient when working with a finite-domain representation.
- SAS$^+$ tasks are a restricted form of FDR tasks where only conjunctions of atoms are allowed in the preconditions, effects and goal. No conditional effects are allowed.