Definitions for Finite-Domain Planning Tasks

Definition 1 (Finite-Domain State Variable)**.** *A* finite-domain state variable *is a symbol v with an associated* finite domain*, i.e., a non-empty finite set. We write* $\mathrm{dom}(v)$ *for the domain of v.*

Definition 2 (Logical Formula over Finite-Domain State Variables)**.** *Let V be a set of finite-domain state variables. The* logical formulas *over V are constructed by finite application of the following rules:*

- *$\top$ and $\bot$ are logical formulas.*
- *For all $v \in V$ and $d \in \mathrm{dom}(v)$, $v = d$ is a logical formula.*
- *If φ is a logical formula, then so is $\neg\varphi$.*
- *If φ and ψ are logical formulas, then so are $(\varphi \lor \psi)$ and $(\varphi \land \psi)$.*

Definition 3 (Finite-Domain State)**.** *Let V be a finite set of finite-domain state variables. A* state *over V is an assignment $s : V \to \bigcup_{v \in V} \mathrm{dom}(v)$ such that $s(v) \in \mathrm{dom}(v)$ for all $v \in V$.*

Definition 4 (Semantics of Formulas)**.** *Let V be a finite set of finite-domain variables, s be a state over V and φ be a formula over V. We say that s* satisfies *φ (written $s \models \varphi$) according to the following definition:*

- $s \models \top$
- $s \not\models \bot$
- $s \models v = d$ *iff* $s(v) = d$ *(for all $v \in V, d \in \mathrm{dom}(v)$)*
- $v \models \neg\varphi$ *iff* $v \not\models \varphi$
- $v \models (\varphi \lor \psi)$ *iff* $v \models \varphi$ *or* $v \models \psi$
- $v \models (\varphi \land \psi)$ *iff* $v \models \varphi$ *and* $v \models \psi$

Definition 5 (Finite-Domain Effect)**.** Effects *over finite-domain state variables V are inductively defined as follows:*

- *If $v \in V$ is a state variable and $d \in \mathrm{dom}(v)$, then $v := d$ is an (atomic) effect.*
- *If $e_1, \dots, e_n$ are effects, then $(e_1 \land \dots \land e_n)$ is an effect.*
  The special case with $n = 0$ is the empty effect $\top$.
- *If χ is a logical formula over V and e is an effect, then $(\chi \rhd e)$ is an effect.*

Definition 6 (Finite-Domain Operator)**.** *An operator o over finite-domain state variables V consists of a* precondition *$pre(o)$ (a logical formula over V), an* effect *$eff(o)$ over V, and a* cost *$cost(o) \in \mathbb{R}^+_0$.*

Definition 7 (Effect Condition for a Finite-Domain Effect)**.** *Let v be a finite-domain variable and $d \in \mathrm{dom}(v)$. The* effect condition *$effcond(v := d, e)$ under which $v := d$ triggers given the effect e is a formula defined as follows:*

- $\mathit{effcond}(v := d, v := d) = \top$
- $\mathit{effcond}(v := d, v' := d') = \bot$ *for atomic effects with* $v' \neq v$ *or* $d' \neq d$
- $\mathit{effcond}(v := d, (e_1 \wedge \cdots \wedge e_n)) = \mathit{effcond}(v := d, e_1) \vee \cdots \vee \mathit{effcond}(v := d, e_n)$
- $\mathit{effcond}(v := d, (\chi \rhd e)) = \chi \wedge \mathit{effcond}(v := d, e)$

Definition 8 (Applicable, Resulting State)**.** *Let* V *be a finite set of finite-domain state variables. Let* s *be a state over* V*, and let* o *be an operator over* V*. Operator* o *is* applicable *in* s *if*

$$s \models \mathit{pre}(o) \wedge \bigwedge_{v \in V} \bigwedge_{d, d' \in \mathrm{dom}(v), d \neq d'} \neg(\mathit{effcond}(v := d, \mathit{eff}(o)) \wedge \mathit{effcond}(v := d', \mathit{eff}(o))).$$

If o *is applicable in* s*, the* resulting state *of applying* o *in* s*, written* $s[\![o]\!]$*, is the state* s' *defined as follows for all* $v \in V$*:*

$$s'(v) = \begin{cases} d & \text{if } s \models \mathit{effcond}(v := d, \mathit{eff}(o)) \\ s(v) & \text{if } s \not\models \mathit{effcond}(v := d, \mathit{eff}(o)) \text{ for all } d \in \mathrm{dom}(v) \end{cases}$$

Definition 9 (Planning Task in Finite-Domain Representation)**.** *A* planning task in finite-domain representation *or* FDR planning task *is a 4-tuple* $\Pi = \langle V, I, O, \gamma \rangle$ *where*

- V *is a finite set of* finite-domain state variables*,*
- I *is a state over* V *called the* initial state*,*
- O *is a finite set of* finite-domain operators *over* V*, and*
- γ *is a formula over* V *called the* goal*.*

Definition 10 (Transition System Induced by a Planning Task)**.** *The planning task* $\Pi = \langle V, I, O, \gamma \rangle$ induces *the transition system* $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$*, where*

- S *is the set of states over* V*,*
- L *is the set of operators* O*,*
- $c(o) = \mathit{cost}(o)$ *for all operators* $o \in O$*,*
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$*,*
- $s_0 = I$*, and*
- $S_\star = \{s \in S \mid s \models \gamma\}$*.*