

# Foundations of Artificial Intelligence

## B13. State-Space Search: IDA\*

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## State-Space Search: Overview

### Chapter overview: state-space search

- ▶ B1–B3. Foundations
- ▶ B4–B8. Basic Algorithms
- ▶ B9–B15. Heuristic Algorithms
  - ▶ B9. Heuristics
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  - ▶ B11. Best-first Graph Search
  - ▶ B12. Greedy Best-first Search, A\*, Weighted A\*
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## B13.1 IDA\*: Idea

## IDA\*

The main drawback of the presented best-first graph search algorithms is their space complexity.

Idea: use the concepts of iterative-deepening DFS

- ▶ depth-limited search with increasing limits
- ▶ instead of **depth** we limit  **$f$**   
(in this chapter  $f(n) := g(n) + h(n.state)$  as in A\*)
- ↔ **IDA\*** (iterative-deepening A\*)
- ▶ **tree search**, unlike the previous best-first search algorithms

## B13.2 IDA\*: Algorithm

## Reminder: Iterative Deepening Depth-first Search

reminder from Chapter B8: iterative deepening depth-first search

## Iterative Deepening DFS

```
for  $depth\_limit \in \{0, 1, 2, \dots\}$ :
     $solution := depth\_limited\_search(init(), depth\_limit)$ 
    if  $solution \neq none$ :
        return  $solution$ 
```

function  $depth\_limited\_search(s, depth\_limit)$ :

```
if  $is\_goal(s)$ :
    return  $\langle \rangle$ 
if  $depth\_limit > 0$ :
    for each  $\langle a, s' \rangle \in succ(s)$ :
         $solution := depth\_limited\_search(s', depth\_limit - 1)$ 
        if  $solution \neq none$ :
             $solution.push\_front(a)$ 
            return  $solution$ 
return none
```

## First Attempt: IDA\* Main Function

first attempt: iterative deepening A\* (IDA\*)

## IDA\* (First Attempt)

```
for  $f\_limit \in \{0, 1, 2, \dots\}$ :
     $solution := f\_limited\_search(init(), 0, f\_limit)$ 
    if  $solution \neq none$ :
        return  $solution$ 
```

## First Attempt: $f$ -Limited Search

```

function f_limited_search( $s, g, f\_limit$ ):
  if  $g + h(s) > f\_limit$ :
    return none
  if is_goal( $s$ ):
    return  $\langle \rangle$ 
  for each  $\langle a, s' \rangle \in succ(s)$ :
     $solution := f\_limited\_search(s', g + cost(a), f\_limit)$ 
    if  $solution \neq none$ :
       $solution.push\_front(a)$ 
      return  $solution$ 
  return none
  
```

## IDA\* First Attempt: Discussion

- ▶ The pseudo-code can be rewritten to be even more similar to our IDDFS pseudo-code. However, this would make our next modification more complicated.
- ▶ The algorithm follows the same principles as IDDFS, but takes path costs and heuristic information into account.
- ▶ For unit-cost state spaces and the trivial heuristic  $h : s \mapsto 0$  for all states  $s$ , it behaves **identically** to IDDFS.
- ▶ For general state spaces, there is a problem with this first attempt, however.

## Growing the $f$ Limit

- ▶ In IDDFS, we grow the limit from the smallest limit that gives a non-empty search tree (0) by 1 at a time.
- ▶ This usually leads to exponential growth of the tree between rounds, so that re-exploration work can be amortized.
- ▶ In our first attempt at IDA\*, there is no guarantee that increasing the  $f$  limit by 1 will lead to a larger search tree than in the previous round.
- ▶ This problem becomes worse if we also allow non-integer (fractional) costs, where increasing the limit by 1 would be very arbitrary.

## Setting the Next $f$ Limit

- idea:** let the  $f$ -limited search compute the next sensible  $f$  limit
- ▶ Start with  $h(\text{init}())$ , the smallest  $f$  limit that results in a non-empty search tree.
  - ▶ In every round, increase the  $f$  limit to the **smallest** value that ensures that in the next round at least one additional path will be considered by the search.
- ↪ `f_limited_search` now returns two values:
- ▶ the next  $f$  limit that would include at least one new node in the search tree ( $\infty$  if no such limit exists; **none** if a solution was found), and
  - ▶ the solution that was found (or **none**).

## Final Algorithm: IDA\* Main Function

final algorithm: iterative deepening A\* (IDA\*)

```

IDA*
f_limit = h(init())
while f_limit ≠ ∞:
    ⟨f_limit, solution⟩ := f_limited_search(init(), 0, f_limit)
    if solution ≠ none:
        return solution
return unsolvable
  
```

## Final Algorithm: $f$ -Limited Search

```

function f_limited_search(s, g, f_limit):
    if g + h(s) > f_limit:
        return ⟨g + h(s), none⟩
    if is_goal(s):
        return ⟨none, ⟨⟩⟩
    new_limit := ∞
    for each ⟨a, s'⟩ ∈ succ(s):
        ⟨child_limit, solution⟩ := f_limited_search(s', g + cost(a), f_limit)
        if solution ≠ none:
            solution.push_front(a)
            return ⟨none, solution⟩
        new_limit := min(new_limit, child_limit)
    return ⟨new_limit, none⟩
  
```

## Final Algorithm: $f$ -Limited Search

```

function f_limited_search(s, g, f_limit):
    if g + h(s) > f_limit:
        return ⟨g + h(s), none⟩
    if is_goal(s):
        return ⟨none, ⟨⟩⟩
    new_limit := ∞
    for each ⟨a, s'⟩ ∈ succ(s):
        ⟨child_limit, solution⟩ := f_limited_search(s', g + cost(a), f_limit)
        if solution ≠ none:
            solution.push_front(a)
            return ⟨none, solution⟩
        new_limit := min(new_limit, child_limit)
    return ⟨new_limit, none⟩
  
```

## B13.3 IDA\*: Properties

## IDA\*: Properties

Inherits important properties of A\* and depth-first search:

- ▶ **semi-complete** if  $h$  safe and  $cost(a) > 0$  for all actions  $a$
- ▶ **optimal** if  $h$  admissible
- ▶ **space complexity**  $O(\ell b)$ , where
  - ▶  $\ell$ : length of longest generated path  
(for unit cost problems: bounded by optimal solution cost)
  - ▶  $b$ : branching factor

We state these without proof.

## IDA\*: Discussion

- ▶ compared to A\* potentially considerable overhead because no **duplicates** are detected
  - ↪ exponentially slower in many state spaces
  - ↪ often combined with partial duplicate elimination (cycle detection, transposition tables)
- ▶ overhead due to **iterative increases** of  $f$  limit **often negligible**, but **not always**
  - ▶ especially problematic if action costs vary a lot: then it can easily happen that each new  $f$  limit only considers a small number of new paths

## B13.4 Summary

## Summary

- ▶ **IDA\*** is a tree search variant of A\* based on iterative deepening depth-first search
- ▶ main advantage: **low space complexity**
- ▶ disadvantage: **repeated work** can be significant
- ▶ most useful when there are **few duplicates**