Foundations of Artificial Intelligence

B12. State-Space Search: Greedy Best-first Search, A*, Weighted A*

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms
  - B9. Heuristics
  - B10. Analysis of Heuristics
  - B11. Best-first Graph Search
  - B12. Greedy Best-first Search, A*, Weighted A*
  - B13. IDA*
  - B14. Properties of A*, Part I
  - B15. Properties of A*, Part II

Introduction

What Is It About?

In this chapter we study last chapter's algorithms in more detail:

- greedy best-first search
- A*
- weighted A*

Greedy Best-first Search

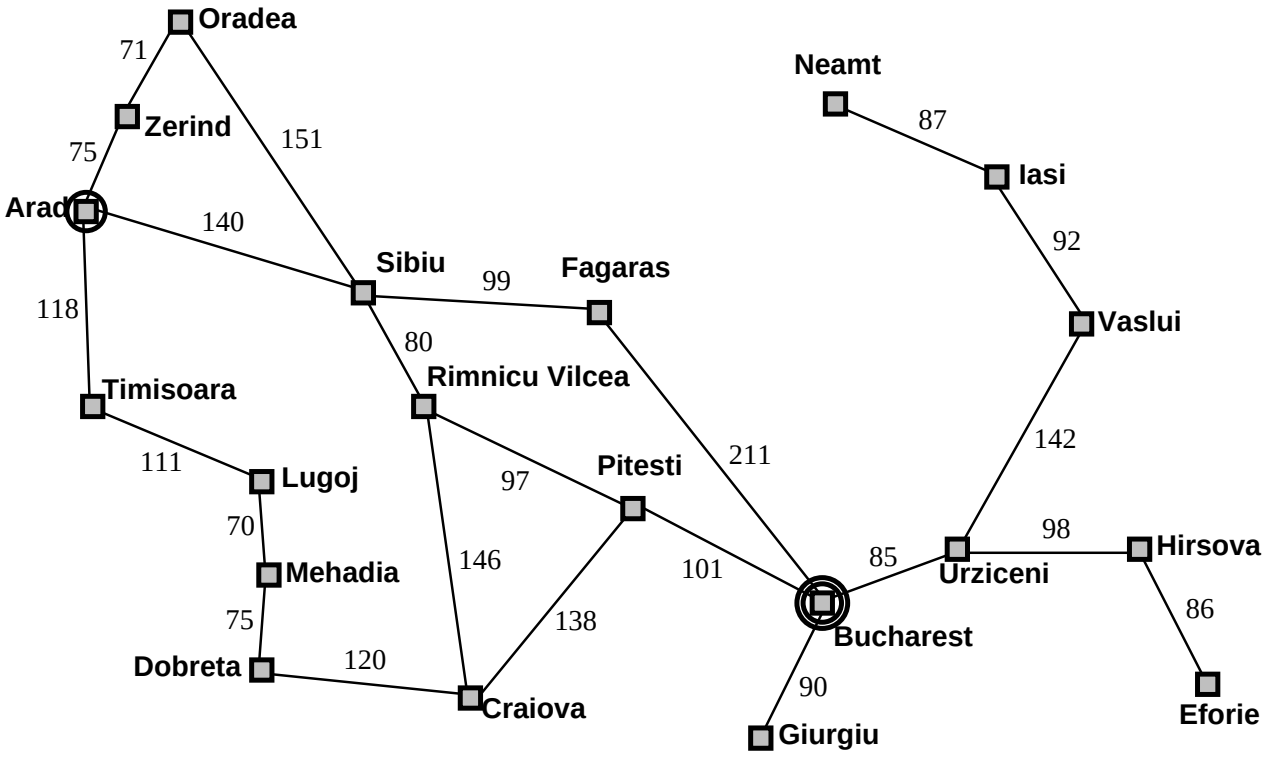
Greedy Best-first Search

Greedy Best-first Search

only consider the heuristic: $f(n) = h(n.\text{state})$

Note: usually without reopening (for reasons of efficiency)

Example: Greedy Best-first Search for Route Planning



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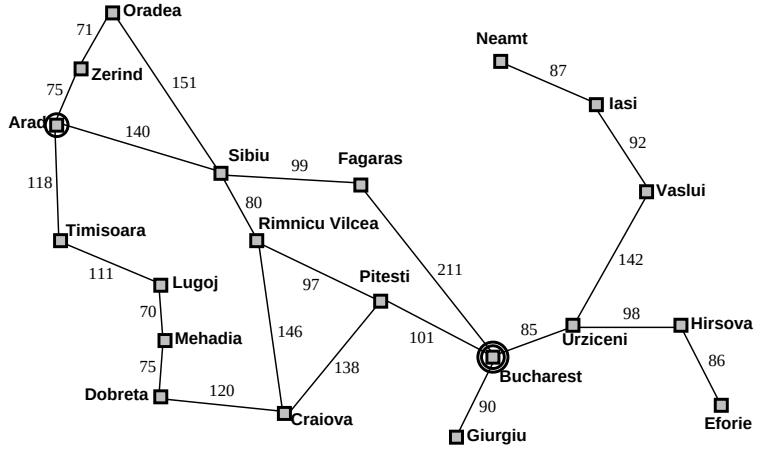

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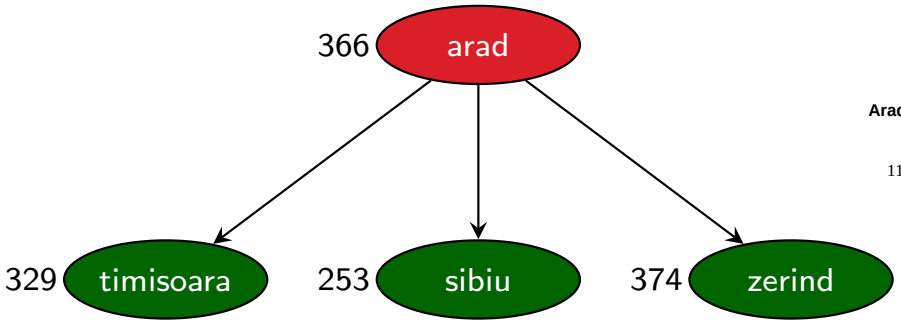

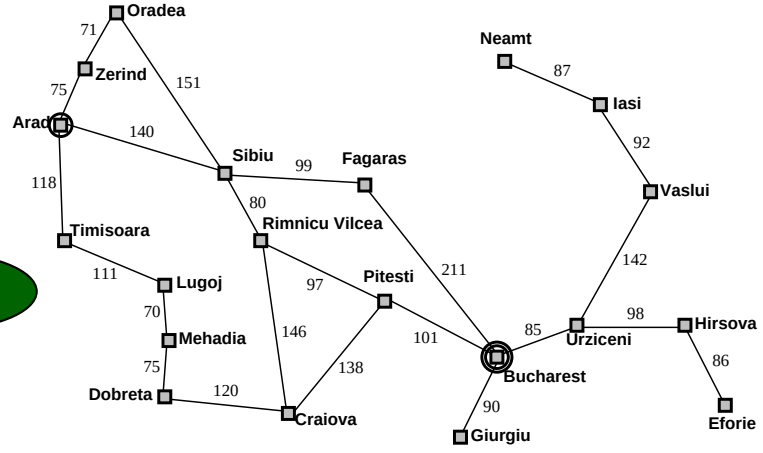

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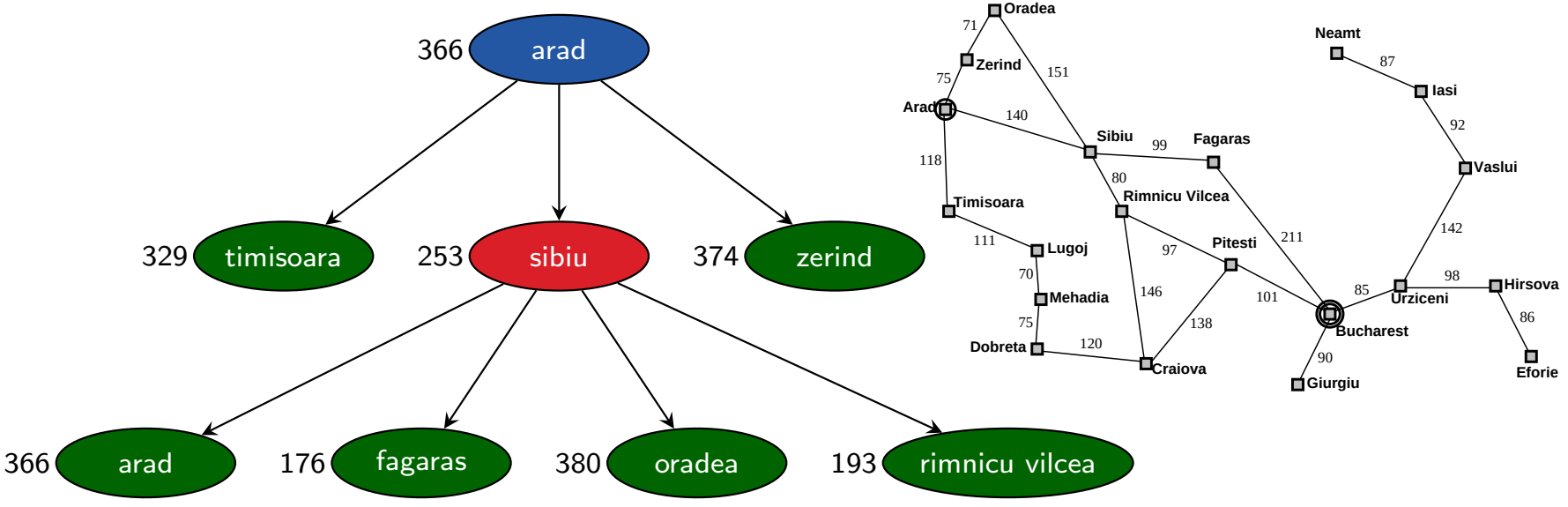

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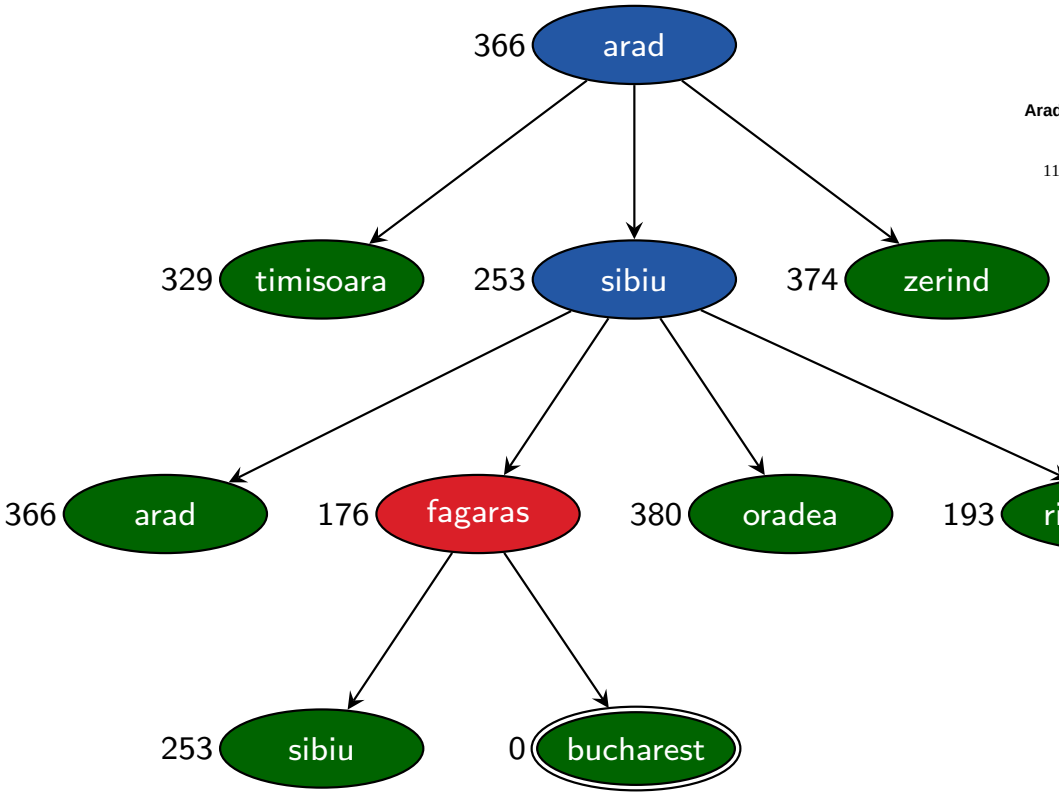

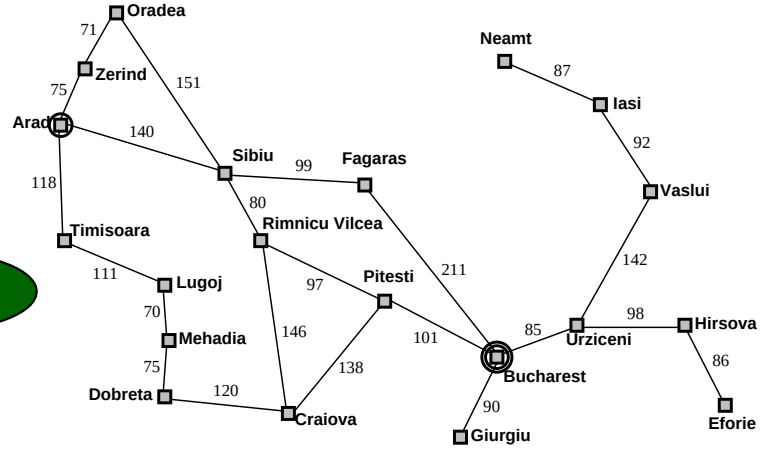

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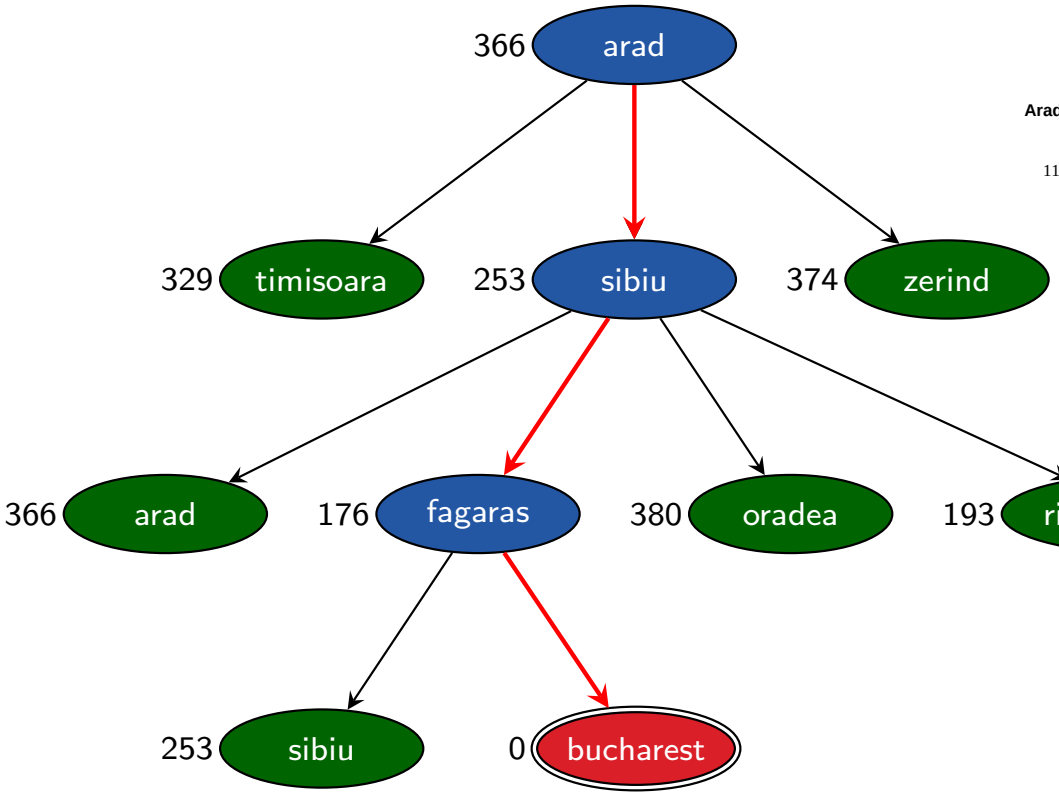

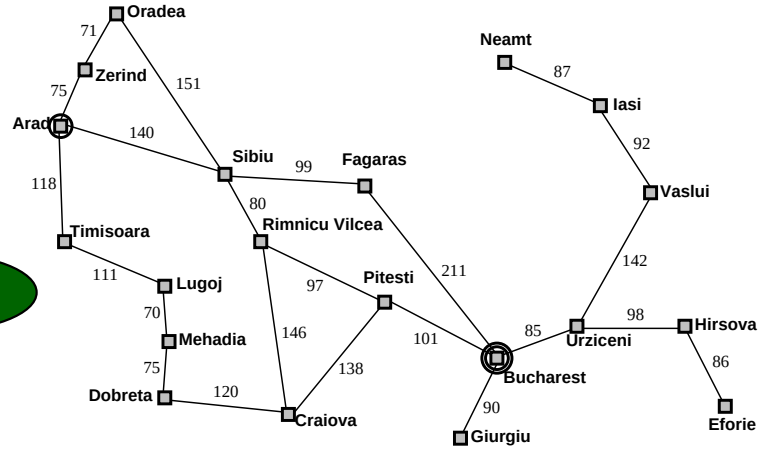

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Greedy Best-first Search: Properties

- complete with safe heuristics
  (like all variants of best-first graph search)
- suboptimal: solutions can be arbitrarily bad
- often very fast: one of the fastest search algorithms in practice
- monotonic transformations of h (e.g. scaling, additive constants) do not affect behaviour (Why is this interesting?)

A*

A*

A*

combine greedy best-first search with uniform cost search:
$f(n) = g(n) + h(n.\text{state})$

- trade-off between path cost and proximity to goal
- $f(n)$ estimates overall cost of cheapest solution
  from initial state via n to the goal

A*: Citations

hart nilsson raphael

About 17.800 results (**0,07** sec)

A formal basis for the heuristic determination of minimum cost paths

PE **Hart**, NJ **Nilsson**, B **Raphael** - IEEE transactions on Systems …, 1968 - ieeexplore.ieee.org

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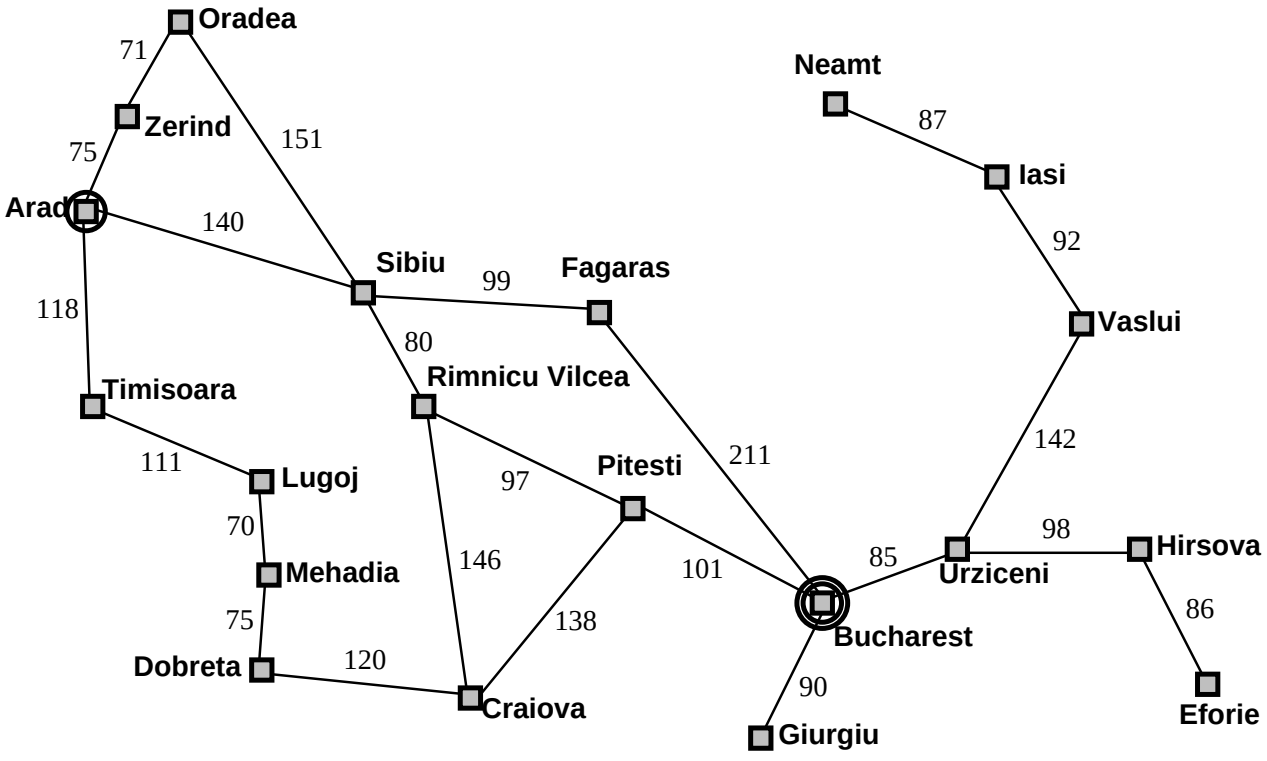

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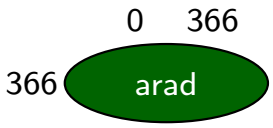

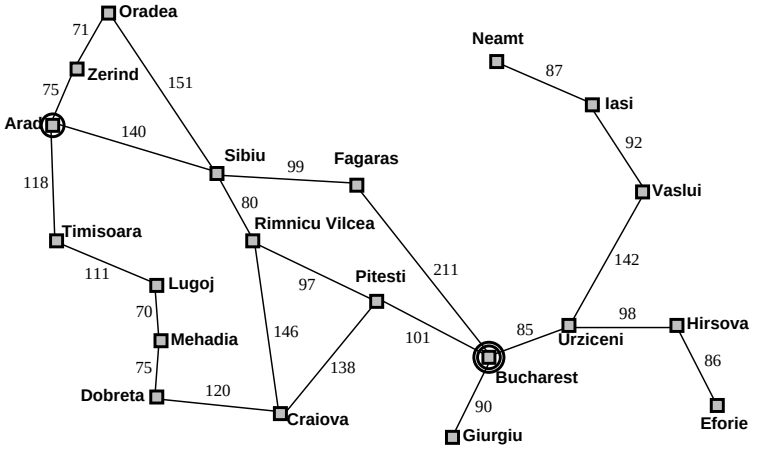

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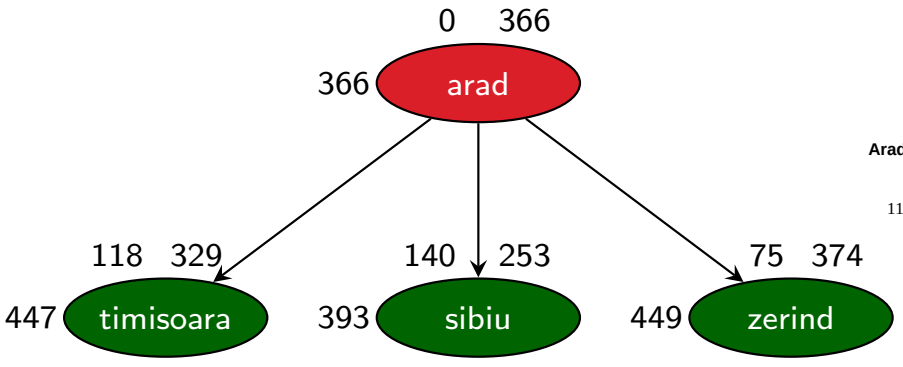

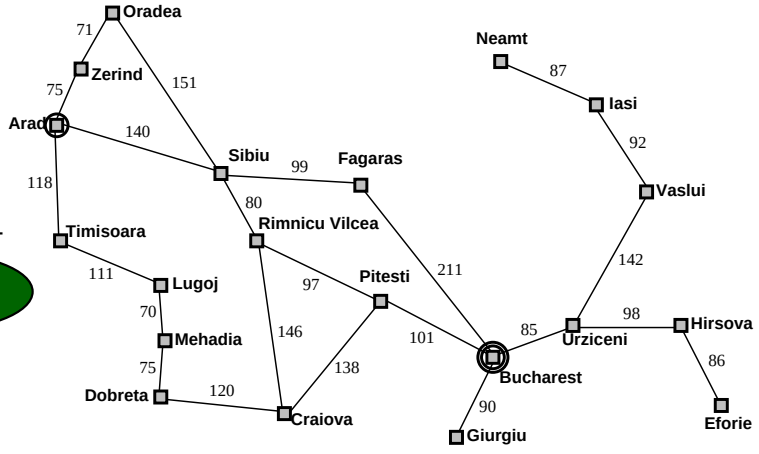

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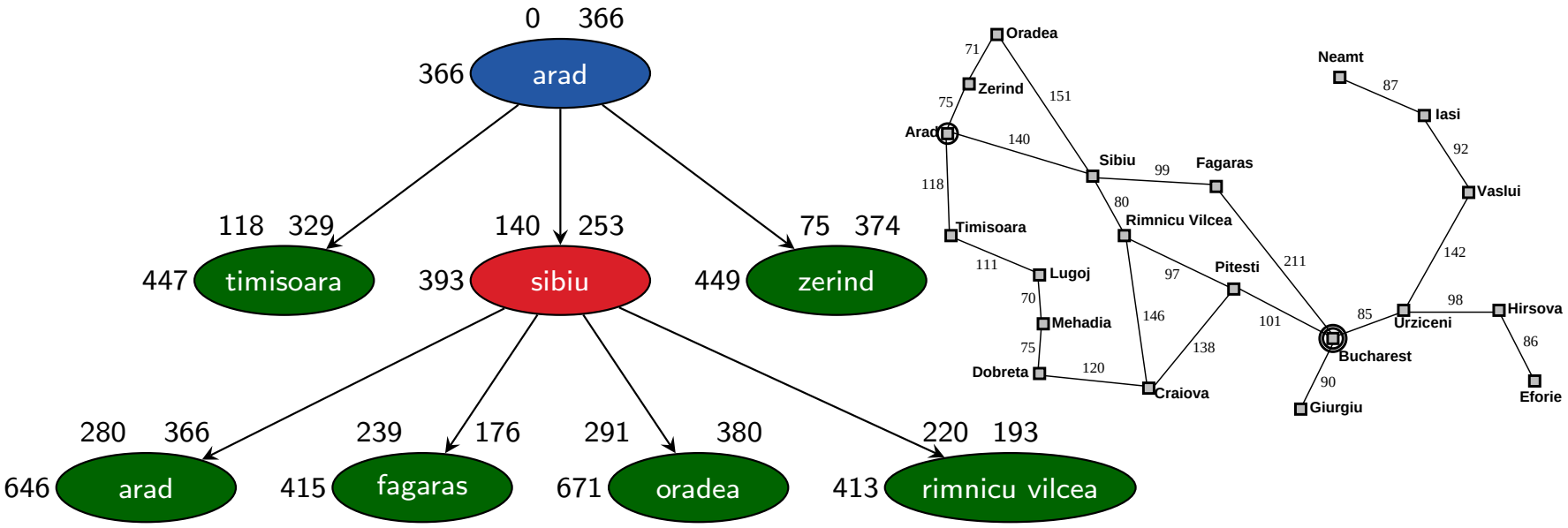

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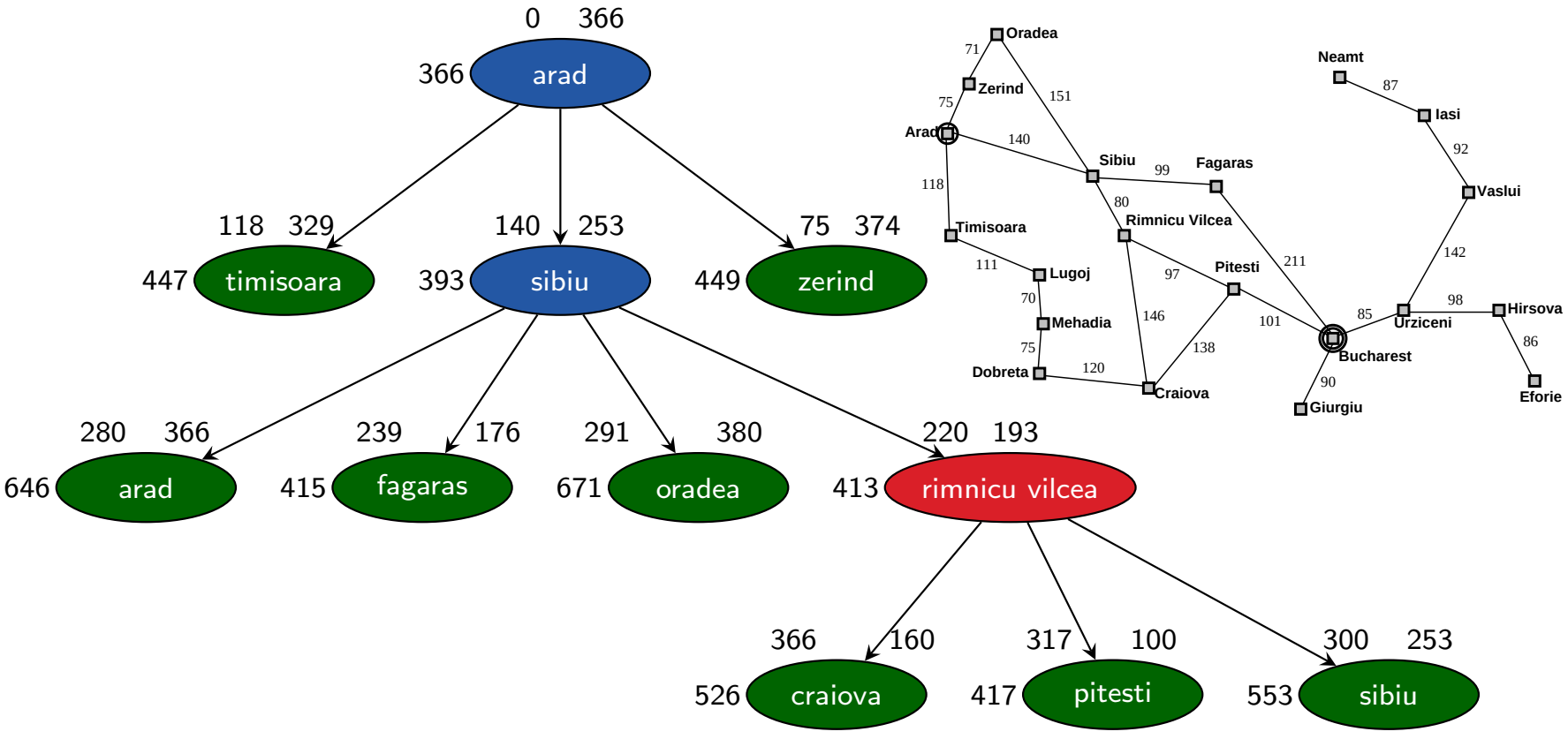

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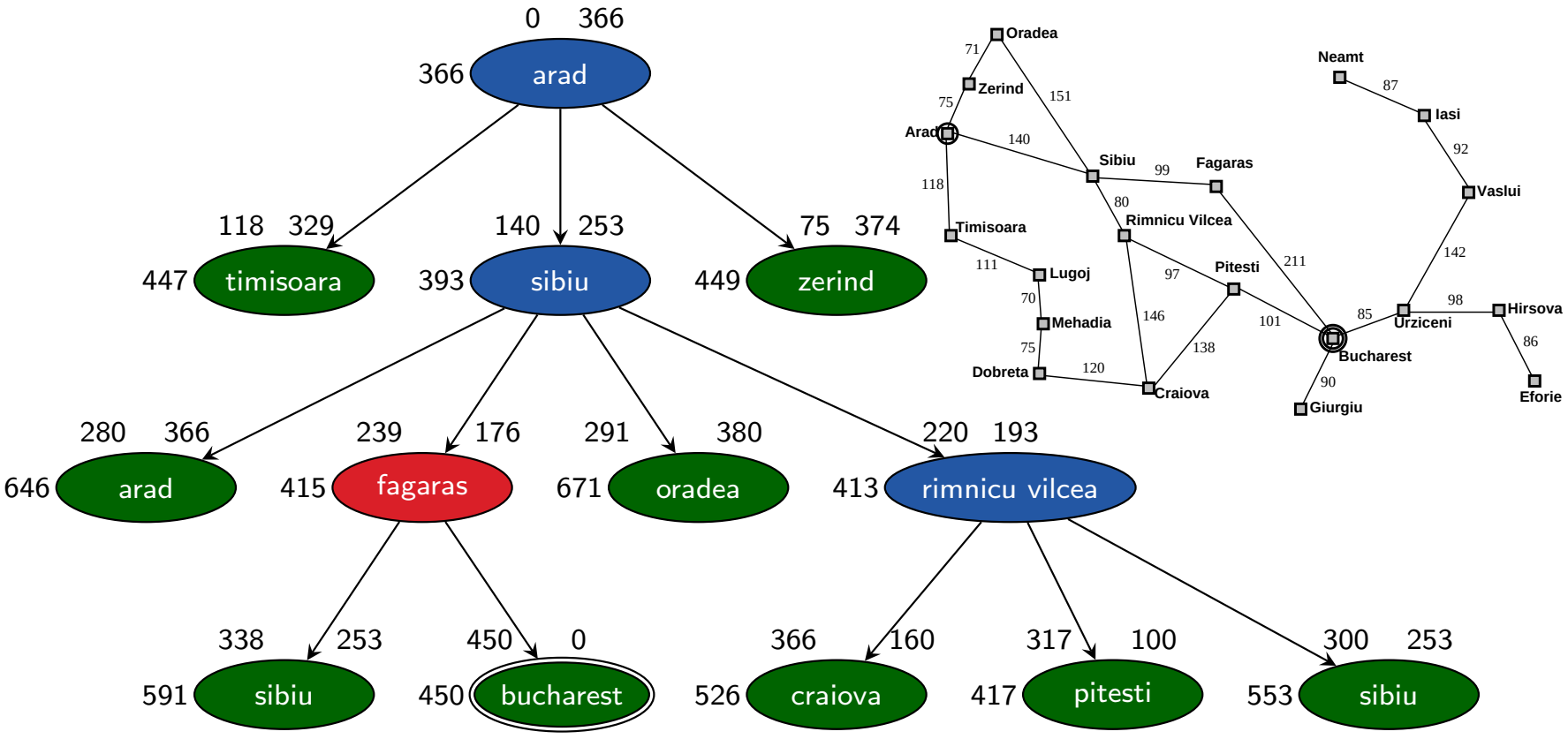

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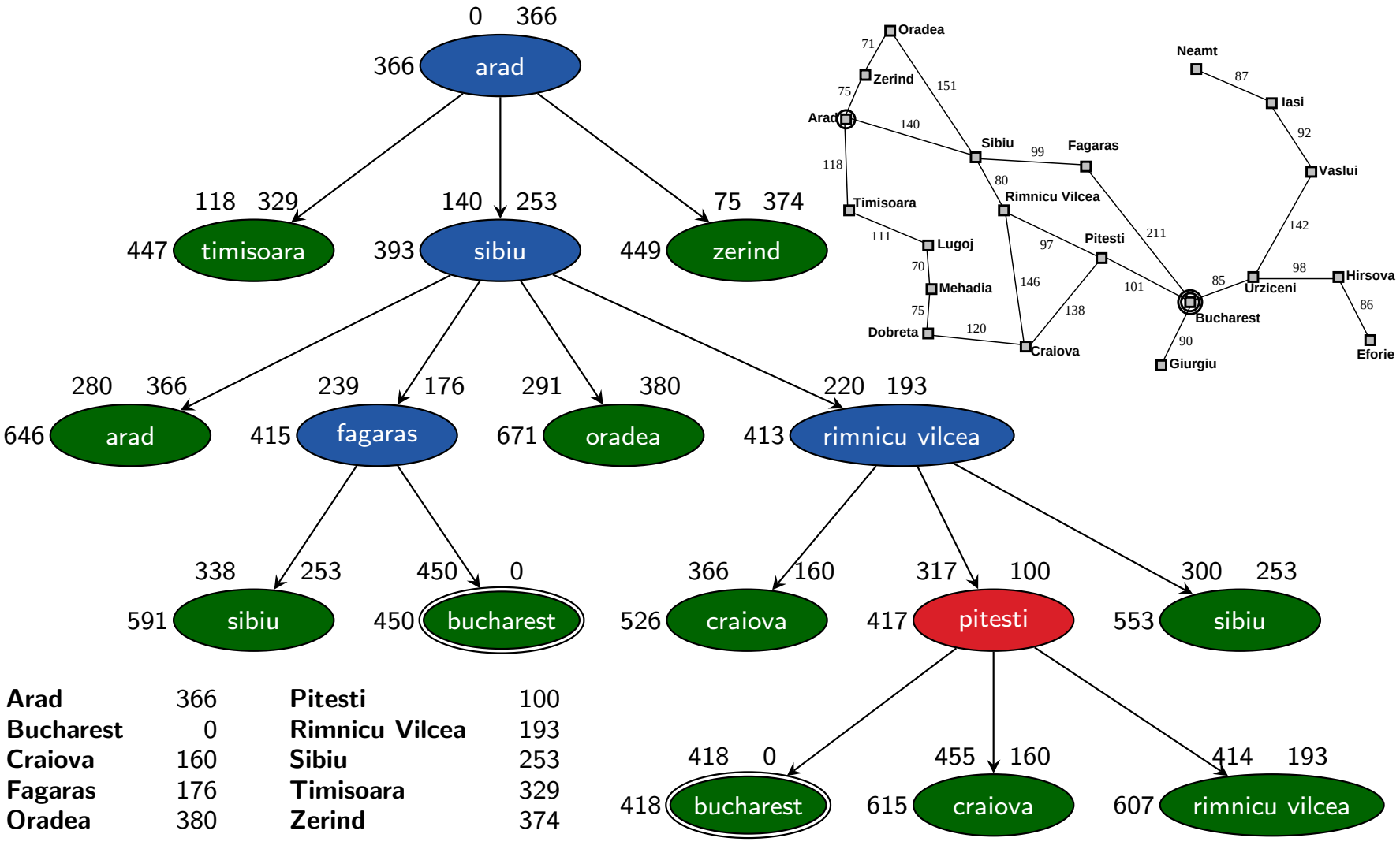

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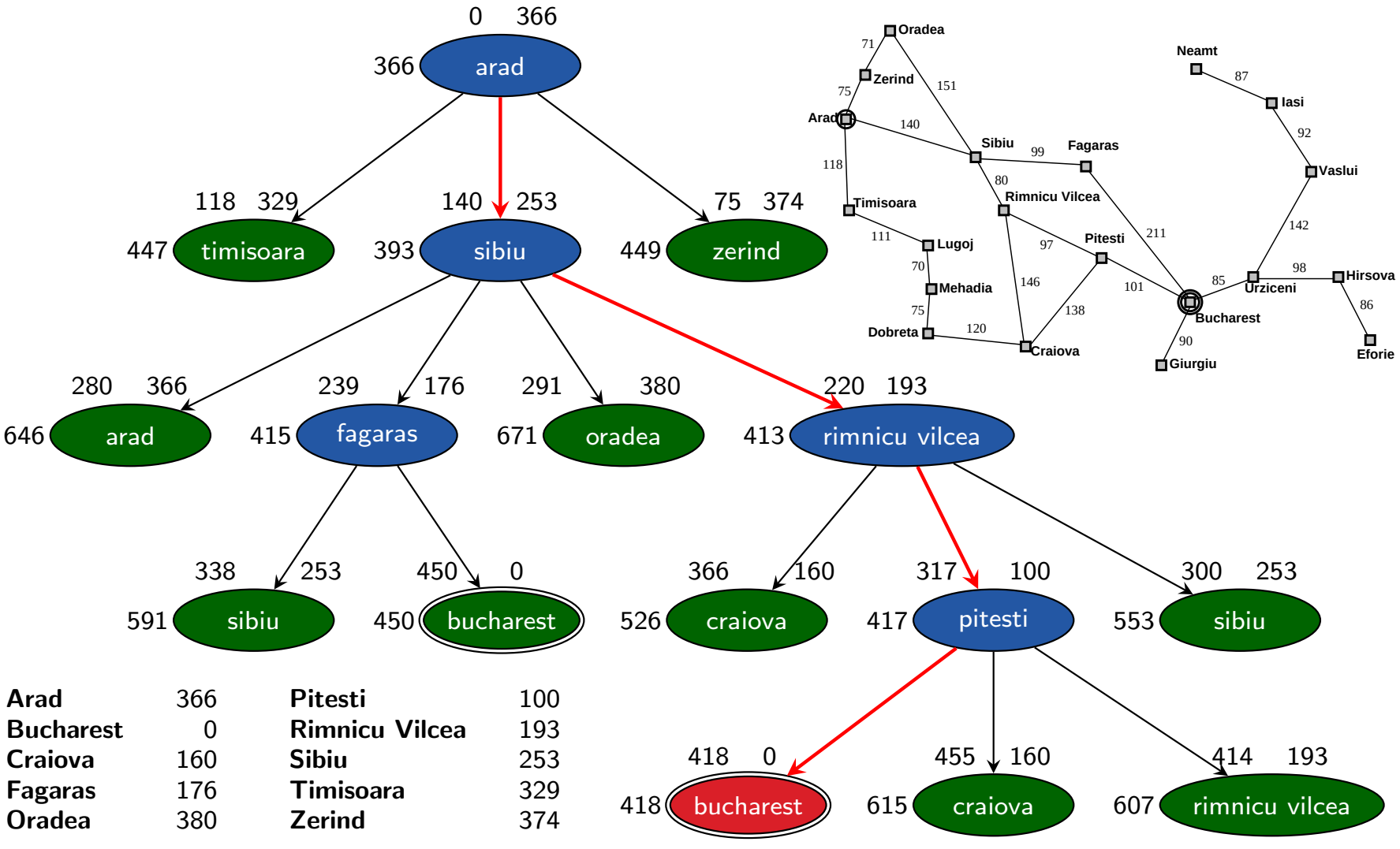

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A*: Properties

- complete with safe heuristics
  (like all variants of best-first graph search)
- with reopening: optimal with admissible heuristics
- without reopening: optimal with heuristics
  that are admissible and consistent

$\rightsquigarrow$ proofs: Chapters B14 and B15

A*: Implementation Aspects

some practical remarks on implementing A*:

- common bug: reopening not implemented
  although heuristic is not consistent
- common bug: duplicate test "too early"
  (upon generation of search nodes)
- common bug: goal test "too early"
  (upon generation of search nodes)
- all these bugs lead to loss of optimality
  and can remain undetected for a long time

Weighted A*

Weighted A*

Weighted A*

A* with more heavily weighted heuristic:
$f(n) = g(n) + w \cdot h(n.\text{state})$,
where weight $w \in \mathbb{R}_0^+$ with $w \geq 1$ is a freely choosable parameter

Note: $w < 1$ is conceivable, but usually not a good idea
(Why not?)

Weighted A*: Properties

weight parameter controls "greediness" of search:

- $w = 0$: like uniform cost search
- $w = 1$: like A*
- $w \to \infty$: like greedy best-first search

with $w \geq 1$ properties analogous to A*:

- *h* admissible:
  found solution guaranteed to be at most w times as expensive as optimum when reopening is used
- *h* admissible and consistent:
  found solution guaranteed to be at most w times as expensive as optimum; no reopening needed

(without proof)

Summary

Summary

best-first graph search with evaluation function f:

- $f = h$: greedy best-first search
  suboptimal, often very fast
- $f = g + h$: A*
  optimal if h admissible and consistent
  or if h admissible and reopening is used
- $f = g + w \cdot h$: weighted A*
  for $w \geq 1$ suboptimality factor at most w
  under same conditions as for optimality of A*