Foundations of Artificial Intelligence

B10. State-Space Search: Analysis of Heuristics

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State-Space Search: Overview

Chapter overview: state-space search

▶ B1–B3. Foundations
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  ▶ B13. IDA*
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Reminder: Heuristics

Definition (heuristic)

Let $\mathcal{S}$ be a state space with states S.
A heuristic function or heuristic for $\mathcal{S}$ is a function

$$h : S \to \mathbb{R}_0^+ \cup \{\infty\},$$

mapping each state to a nonnegative number (or ∞).

B10.1 Properties of Heuristics

Perfect Heuristic

> **Definition (perfect heuristic)**
>
> Let $\mathcal{S}$ be a state space with states S.
>
> The perfect heuristic for $\mathcal{S}$, written h^*, maps each state $s \in S$
>
> ► to the cost of an optimal solution for s, or
>
> ► to ∞ if no solution for s exists.

German: perfekte Heuristik

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Properties of Heuristics

> **Definition (safe, goal-aware, admissible, consistent)**
>
> Let $\mathcal{S}$ be a state space with states S.
>
> A heuristic h for $\mathcal{S}$ is called
>
> ► safe if $h^*(s) = \infty$ for all $s \in S$ with $h(s) = \infty$
>
> ► goal-aware if $h(s) = 0$ for all goal states s
>
> ► admissible if $h(s) \leq h^*(s)$ for all states $s \in S$
>
> ► consistent if $h(s) \leq cost(a) + h(s')$ for all transitions $s \xrightarrow{a} s'$

German: sicher, zielerkennend, zulässig, konsistent
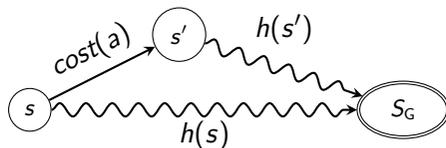
B10.2 Examples

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

> ### Route Planning in Romania
>
> straight-line distance:
> - safe
> - goal-aware
> - admissible
> - consistent
>
> Why?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

> ### Blocks World
>
> misplaced blocks:
> - safe?
> - goal-aware?
> - admissible?
> - consistent?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

> ### Missionaries and Cannibals
>
> people on wrong river bank:
> - safe?
> - goal-aware?
> - admissible?
> - consistent?

B10.3 Connections

Properties of Heuristics: Connections (1)

> **Theorem (admissible ⟹ safe + goal-aware)**
>
> Let h be an admissible heuristic.
>
> Then h is safe and goal-aware.

Why?

Properties of Heuristics: Connections (2)

> **Theorem (goal-aware + consistent ⟹ admissible)**
>
> Let h be a goal-aware and consistent heuristic.
>
> Then h is admissible.

Why?

Showing All Four Properties

How can one show most easily that a heuristic has all four properties?

B10.4 Summary

Summary

- perfect heuristic h^*: true cost to the goal
- important properties: safe, goal-aware, admissible, consistent
- connections between these properties
  - admissible $\implies$ safe and goal-aware
  - goal-aware and consistent $\implies$ admissible