

Foundations of Artificial Intelligence

B5. State-Space Search: Tree Search and Graph Search

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March 9, 2026

State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
 - B4. Data Structures for Search Algorithms
 - B5. Tree Search and Graph Search
 - B6. Breadth-first Search
 - B7. Uniform Cost Search
 - B8. Depth-first Search and Iterative Deepening
- B9–B15. Heuristic Algorithms

Introduction

Search Algorithms

General Search Algorithm

iteratively create a **search tree**:

- starting with the **initial state**,
- repeatedly **expand** a state by **generating** its **successors**
(which state depends on the used search algorithm)
- stop when a **goal state** is expanded (sometimes: generated)
- or **all reachable states** have been considered

Search Algorithms

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In this chapter, we study two essential classes of search algorithms:

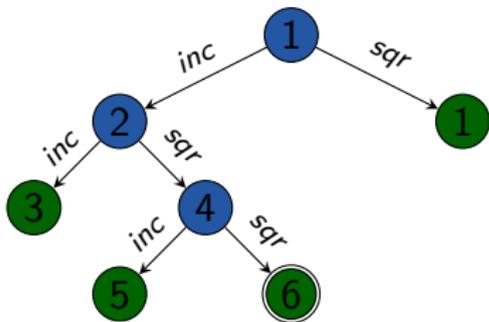
- **tree search**
- **graph search**

Each class consists of a large number of concrete algorithms.

German: expandieren, erzeugen, Baumsuche, Graphensuche

Tree Search

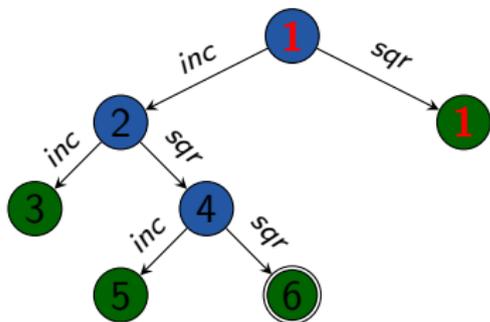
Tree Search: General Idea



- possible paths to be explored organized in a tree (**search tree**)
- **search nodes** correspond 1:1 to **paths** from initial state
- **duplicates** a.k.a. **transpositions** (i.e., multiple nodes with identical state) possible
- search tree can have **unbounded depth**

German: Suchbaum, Duplikate, Transpositionen

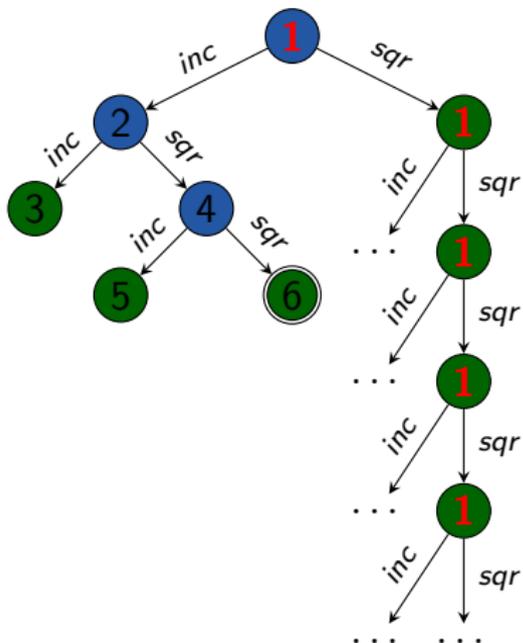
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Generic Tree Search Algorithm

Generic Tree Search Algorithm

```
open := new OpenList
open.insert(make_root_node())
while not open.is_empty():
    n := open.pop()
    if is_goal(n.state):
        return extract_path(n)
    for each  $\langle a, s' \rangle \in \text{succ}(n.\text{state})$ :
        n' := make_node(n, a, s')
        open.insert(n')
return unsolvable
```

Generic Tree Search Algorithm: Discussion

discussion:

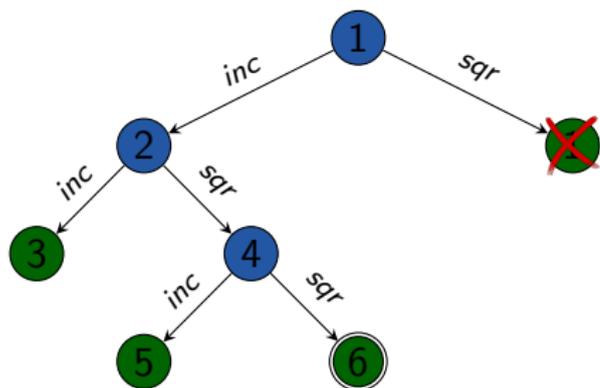
- **generic template** for tree search algorithms
- ↪ for concrete algorithm, we must (at least) decide how to implement the open list
- concrete algorithms often **conceptually** follow template, (= generate the same search tree), but deviate from details for efficiency reasons

Graph Search

Graph Search

differences to tree search:

- recognize **duplicates**: when a state is reached on multiple paths, only keep one search node
- **search nodes** correspond **1:1** to **reachable states**
- depth of search tree **bounded**



remarks:

- some graph search algorithms do not immediately eliminate all duplicates (\rightsquigarrow later)
- one possible reason: find optimal solutions when a path to state s found later is cheaper than one found earlier

Generic Graph Search Algorithm

Generic Graph Search Algorithm

```
open := new OpenList
open.insert(make_root_node())
closed := new ClosedList
while not open.is_empty():
    n := open.pop()
    if closed.lookup(n.state) = none:
        closed.insert(n)
        if is_goal(n.state):
            return extract_path(n)
        for each  $\langle a, s' \rangle \in$  succ(n.state):
            n' := make_node(n, a, s')
            open.insert(n')
return unsolvable
```

Generic Graph Search Algorithm: Discussion

discussion:

- same comments as for generic tree search apply
- in “pure” algorithm, closed list does not actually need to store the search nodes
 - sufficient to implement *closed* as set of states
 - advanced algorithms often need access to the nodes, hence we show this more general version here
- some variants perform goal and duplicate tests elsewhere (earlier) \rightsquigarrow following chapters

Evaluating Search Algorithms

Criteria: Completeness

four criteria for evaluating search algorithms:

Completeness

Is the algorithm guaranteed to find a solution if one exists?

Does it terminate if no solution exists?

first property: **semi-complete**

both properties: **complete**

German: Vollständigkeit, semi-vollständig, vollständig

Criteria: Optimality

four criteria for evaluating search algorithms:

Optimality

Are the solutions returned by the algorithm always optimal?

German: Optimalität

Criteria: Time Complexity

four criteria for evaluating search algorithms:

Time Complexity

How much **time** does the algorithm need until termination?

- usually **worst case** analysis
- usually measured in **generated nodes**

often a function of the following quantities:

- **b** : (**branching factor**) of state space
(max. number of successors of a state)
- **d** : **search depth**
(length of longest path in generated search tree)

German: Zeitaufwand, Verzweigungsgrad, Suchtiefe

Criteria: Space Complexity

four criteria for evaluating search algorithms:

Space Complexity

How much **memory** does the algorithm use?

- usually **worst case** analysis
- usually measured in (concurrently) **stored nodes**

often a function of the following quantities:

- **b** : (**branching factor**) of state space
(max. number of successors of a state)
- **d** : **search depth**
(length of longest path in generated search tree)

German: Speicheraufwand

Analyzing the Generic Search Algorithms

Generic Tree Search Algorithm

- Is it complete? Is it semi-complete?
- Is it optimal?
- What is its worst-case time complexity?
- What is its worst-case space complexity?

Generic Graph Search Algorithm

- Is it complete? Is it semi-complete?
- Is it optimal?
- What is its worst-case time complexity?
- What is its worst-case space complexity?

Summary

Summary (1)

tree search:

- search nodes correspond 1:1 to paths from initial state

graph search:

- search nodes correspond 1:1 to reachable states

↪ duplicate elimination

generic methods with many possible variants

Summary (2)

evaluating search algorithms:

- **completeness** and **semi-completeness**
- **optimality**
- **time complexity** and **space complexity**