Foundations of Artificial Intelligence

B4. State-Space Search: Data Structures for Search Algorithms

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State-Space Search: Overview

Chapter overview: state-space search

B4.1 Introduction

Finding Solutions in State Spaces

How can we systematically find a solution?

Search Algorithms

- We now move to search algorithms.
- As everywhere in computer science, suitable data structures are a key to good performance.
  - $\rightsquigarrow$ common operations must be fast
- Well-implemented search algorithms process up to ∼30,000,000 states/second on a single CPU core.
  - $\rightsquigarrow$ bonus materials (Burns et al. paper)

this chapter: some fundamental data structures for search

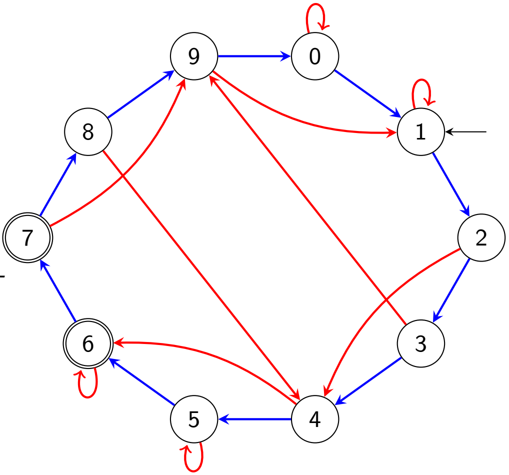
Preview: Search Algorithms

- next chapter: we introduce search algorithms
- now: short preview to motivate data structures for search

Running Example: Reminder

bounded inc-and-square:

- $S = \{0, 1, \ldots, 9\}$
- $A = \{inc, sqr\}$
- $cost(inc) = cost(sqr) = 1$
- T s.t. for $i = 0, \ldots, 9$:
  - $\langle i, inc, (i + 1) \bmod 10\rangle \in T$
  - $\langle i, sqr, i^2 \bmod 10\rangle \in T$
- $s_I = 1$
- $S_G = \{6, 7\}$



Search Algorithms: Idea

iterative create a search tree:

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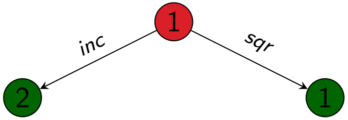
German: expandieren, erzeugen



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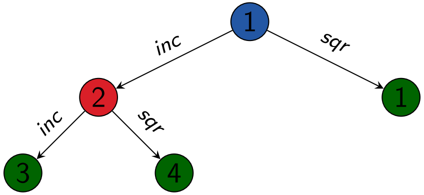

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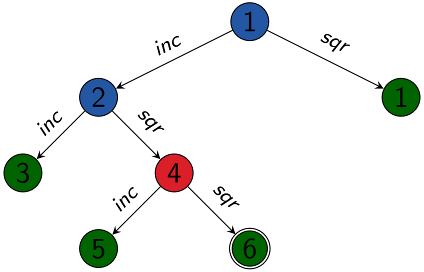
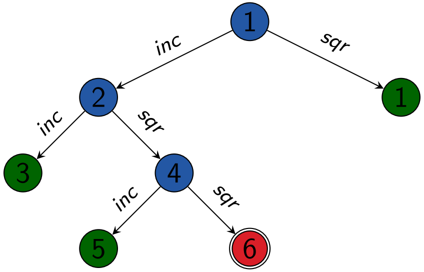


Search Algorithms: Idea

iteratively create a search tree:

- starting with the initial state,
- repeatedly expand a state by generating its successors (which state depends on the used search algorithm)
- stop when a goal state is expanded (sometimes: generated)
- or all reachable states have been considered

German: expandieren, erzeugen



Fundamental Data Structures for Search

We consider three abstract data structures for search:

- search node: stores a state that has been reached, how it was reached, and at which cost
  - ⇝ nodes of the example search tree
- open list: efficiently organizes leaves of search tree
  - ⇝ set of leaves of example search tree
- closed list: remembers expanded states to avoid duplicated expansions of the same state
  - ⇝ inner nodes of a search tree

German: Suchknoten, Open-Liste, Closed-Liste

Not all algorithms use all three data structures, and they are sometimes implicit (e.g., on the CPU stack)

B4.2 Search Nodes

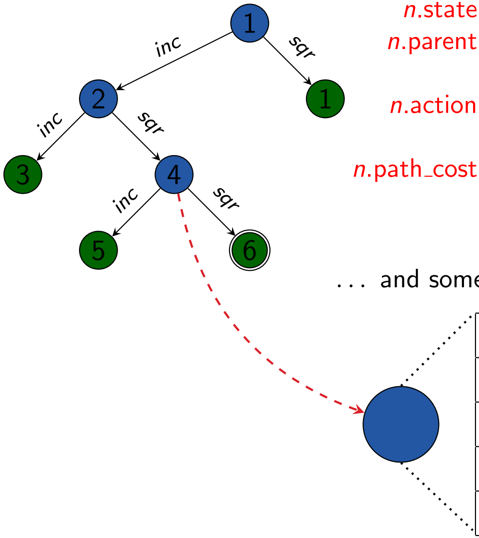
Search Nodes

Search Node

A search node (node for short) stores a state
that has been reached, how it was reached, and at which cost.

Collectively they form the so-called search tree (Suchbaum).

Data Structure: Search Nodes



attributes of search node *n*:

n.state	state associated with *n*
n.parent	search node that generated *n* (**none** for the root node)
n.action	action leading from *n*.parent to *n* (**none** for the root node)
n.path_cost	cost of path from s_I to *n*.state that results from following parent references (traditionally denoted by $g(n)$)

… and sometimes additional attributes

n.state:	**4**
n.parent:	2
n.action:	*sqr*
n.path_cost:	2
…:	…

Search Nodes: Java

Search Nodes (Java Syntax)

```
public interface State {
}

public interface Action {
}

public class SearchNode {
    State state;
    SearchNode parent;
    Action action;
    int pathCost;
}
```

Implementing Search Nodes

- reasonable implementation of search nodes is easy
- advanced aspects:
  - Do we need explicit nodes at all?
  - Can we use lazy evaluation?
  - Should we manually manage memory?
  - Can we compress information?

Operations on Search Nodes: make_root_node

Generate root node of a search tree:

```
function make_root_node()
node := new SearchNode
node.state := init()
node.parent := none
node.action := none
node.path_cost := 0
return node
```

Operations on Search Nodes: make_node

Generate child node of a search node:

```
function make_node($parent$, $action$, $state$)
$node$ := new SearchNode
$node$.state := $state$
$node$.parent := $parent$
$node$.action := $action$
$node$.path_cost := $parent$.path_cost + cost($action$)
return $node$
```

Operations on Search Nodes: extract_path

Extract the path to a search node:

```
function extract_path($node$)
path := $\langle\rangle$
while $node$.parent $\neq$ none:
    $path$.append($node$.action)
    $node$ := $node$.parent
$path$.reverse()
return $path$
```

B4.3 Open Lists

Open Lists

Open List

The open list (also: frontier) organizes the leaves of a search tree.

It must support two operations efficiently:

- determine and remove the next node to expand
- insert a new node that is a candidate node for expansion

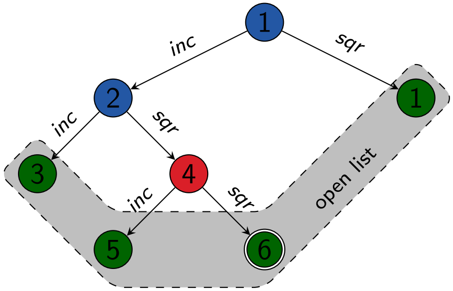
Remark: despite the name, it is usually a very bad idea to implement open lists as simple lists.

Open Lists: Modify Entries

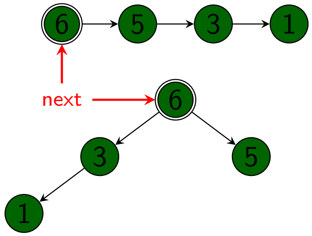
- Some implementations support modifying an open list entry when a shorter path to the corresponding state is found.
- This complicates the implementation.

$\rightsquigarrow$ We do not consider such modifications and instead use delayed duplicate elimination ($\rightsquigarrow$ later).

Interface of Open Lists



examples: deque, min-heap



- open list *open* organizes leaves of search tree with the methods:

open.is_empty()	test if the open list is empty
open.pop()	remove and return the next node to expand
open.insert(*n*)	insert node *n* into the open list

- *open* determines strategy which node to expand next (depends on algorithm)
- underlying data structure choice depends on this strategy

B4.4 Closed Lists

Closed Lists

Closed List

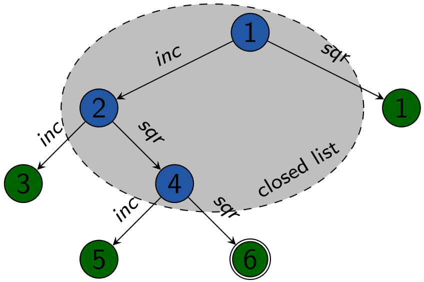
The closed list remembers expanded states
to avoid duplicated expansions of the same state.

It must support two operations efficiently:

- insert a node whose state is not yet in the closed list
- test if a node with a given state is in the closed list;
  if yes, return it

Remark: despite the name, it is usually a very bad idea
to implement closed lists as simple lists. (Why?)

Interface and Implementation of Closed Lists



- closed list *closed* keeps track of expanded states with the methods:

closed.insert(n)	insert node n into *closed*; if a node with this state already exists in *closed*, replace it
closed.lookup(s)	test if a node with state s exists in the closed list; if yes, return it; otherwise, return **none**

- efficient implementation often as hash table with states as keys

B4.5 Summary

Summary

- search node:
  represents states reached during search
  and associated information
- node expansion:
  generate successor nodes of a node by applying all actions
  applicable in the state belonging to the node
- open list or frontier:
  set of nodes that are currently candidates for expansion
- closed list:
  set of already expanded nodes (and their states)