

Algorithms and Data Structures

C7. Graphs: Outlook

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May 21, 2026

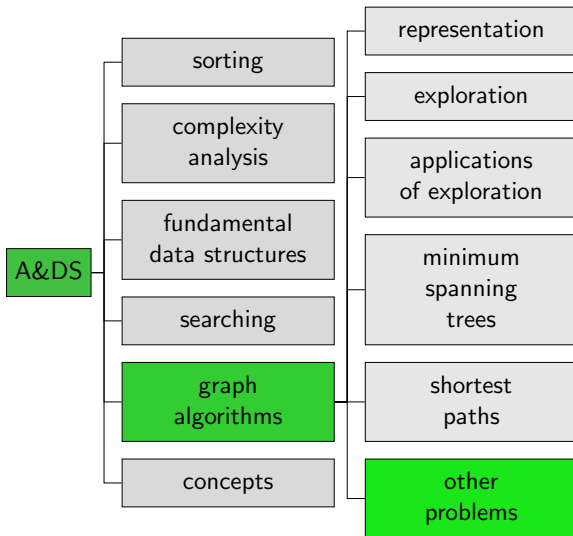
Algorithms and Data Structures

May 21, 2026 — C7. Graphs: Outlook

C7.1 Other Graph Problems

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Content of the Course



Crash Course Complexity Theory

- ▶ **Decision problems:** Seeking a Yes/No answer
Given weighted graph, vertices s , t and number K .
Is there a path from s to t that costs at most K ?
- ▶ **Search problem:** Seeking an actual solution
Given weighted graph and vertices s , t .
Find a shortest path from s to t .

Crash Course Complexity Theory

We distinguish different classes of problems:

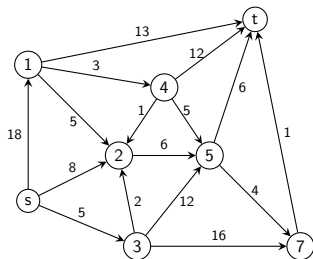
- ▶ **P**: decision problems that can be **solved with a polynomial-time algorithm** (in $O(p)$ for some polynomial p).
- ▶ **NP**: decision problems, where the yes instances have **proofs** that can be **verified in polynomial time**.
Proof: e.g. specific path of cost $\leq K$
- ▶ **$P \neq NP$?** We do not know.
- ▶ **NP-hard problems**: Problems that are at least as hard as the hardest problems in NP.
→ no polynomial-time algorithms known.
- ▶ **NP-complete** decision problems: NP-hard & in NP
- ▶ **NP-equivalent** search problems: corresponding decision problem NP-complete.

Flows in Graphs I

Definition (Flow Network)

A **flow network** $N = (G, s, t, k)$ is given by

- ▶ a **directed graph** $G = (V, E)$,
- ▶ a **source** $s \in V$,
- ▶ a **sink** $t \in V$, and
- ▶ a **capacity function** $k : E \rightarrow \mathbb{R}_+^\infty$.



Flows in Graphs II

Definition (Flow)

An **s-t flow** f assigns every edge a value from $\mathbb{R}_{\geq 0}$, where

- ▶ the **value does not exceed the capacity** of the edge::

$$f(e) \leq k(e) \text{ for all } e \in E$$

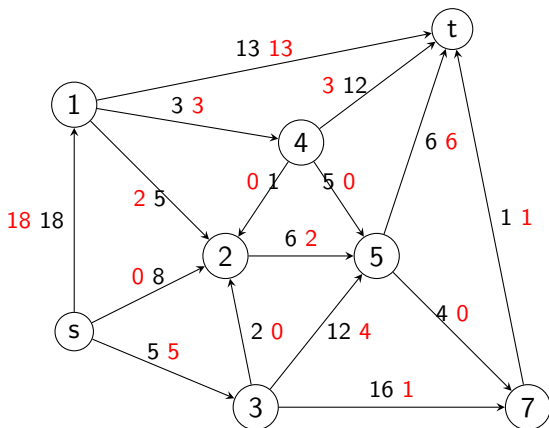
- ▶ for all vertices except for the source and the sink
the **incoming flow matches the outgoing flow**:

$$\sum_{\substack{(u,w) \in E \\ w=v}} f((u,w)) = \sum_{\substack{(u,w) \in E \\ u=v}} f((u,w)) \text{ for all } v \in V \setminus \{s, t\}$$

The **value** of the flow is the net flow into the sink:

$$|f| = \sum_{\substack{(u,w) \in E \\ w=t}} f((u,w)) - \sum_{\substack{(u,w) \in E \\ u=t}} f((u,w))$$

Example



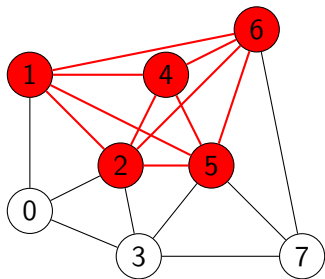
How hard is it to find a maximal flow?

For example with the Edmonds-Karp algorithm in $O(|E|^2|V|)$

Cliques

Definition (Clique)

A **clique** in an undirected graph (V, E) is a subset $C \subseteq V$ of the vertices such that each pair of distinct vertices in C is connected by an edge: for $u, v \in C$ with $u \neq v$ it holds that $\{u, v\} \in E$.



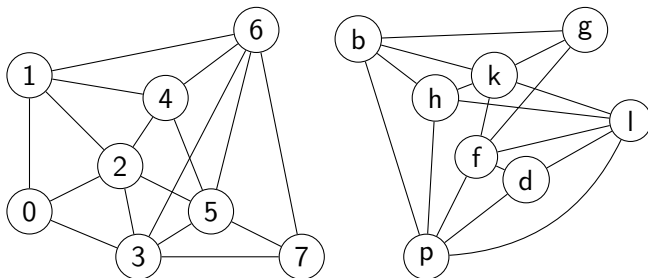
How hard is it to determine a largest clique in a graph?

NP-equivalent

Graph Isomorphism

Definition (Graph Isomorphism)

Two graphs are **isomorphic**, if they are identical up to renaming vertices.



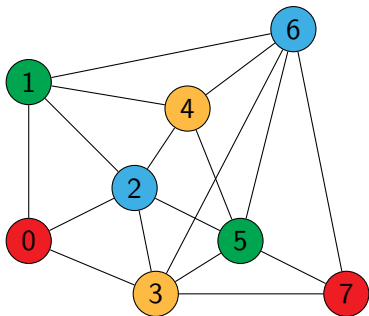
How hard is it to decide whether two given graphs are isomorphic?

In NP, but unknown whether in P and/or NP-complete

Graph Coloring

Definition (k -Colorability)

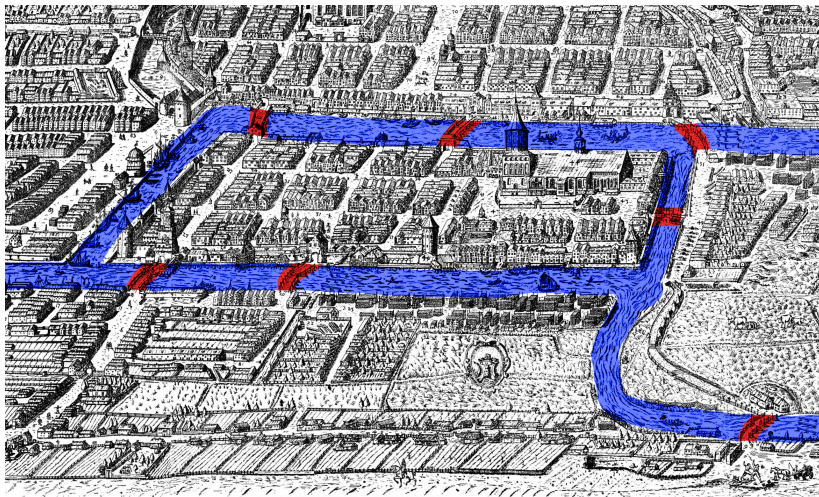
An undirected graph $G = (V, E)$ is **k -colorable** ($k \in \mathbb{N}$), if there is a coloring $f : V \rightarrow \{1, \dots, k\}$ with $f(v) \neq f(w)$ for all $\{v, w\} \in E$.



How hard is it to decide whether a given graph is k -colorable?

NP-complete

Seven Bridges of Königsberg

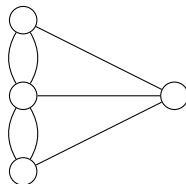
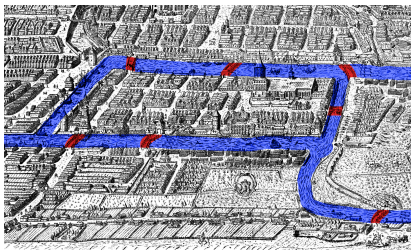


Is there a walk through the city crossing each bridge exactly once?

Eulerian Trail

Definition (Eulerian Trail)

An **Eulerian trail** is a path that uses every edge exactly once.



How hard is it to decide whether a graph has an Eulerian trail?

Has Eulerian trail iff exactly zero or two vertices have odd degree, and all of its vertices with nonzero degree belong to a single connected component.