Algorithms and Data Structures

A14. Sorting: Counting Sort & Radix Sort

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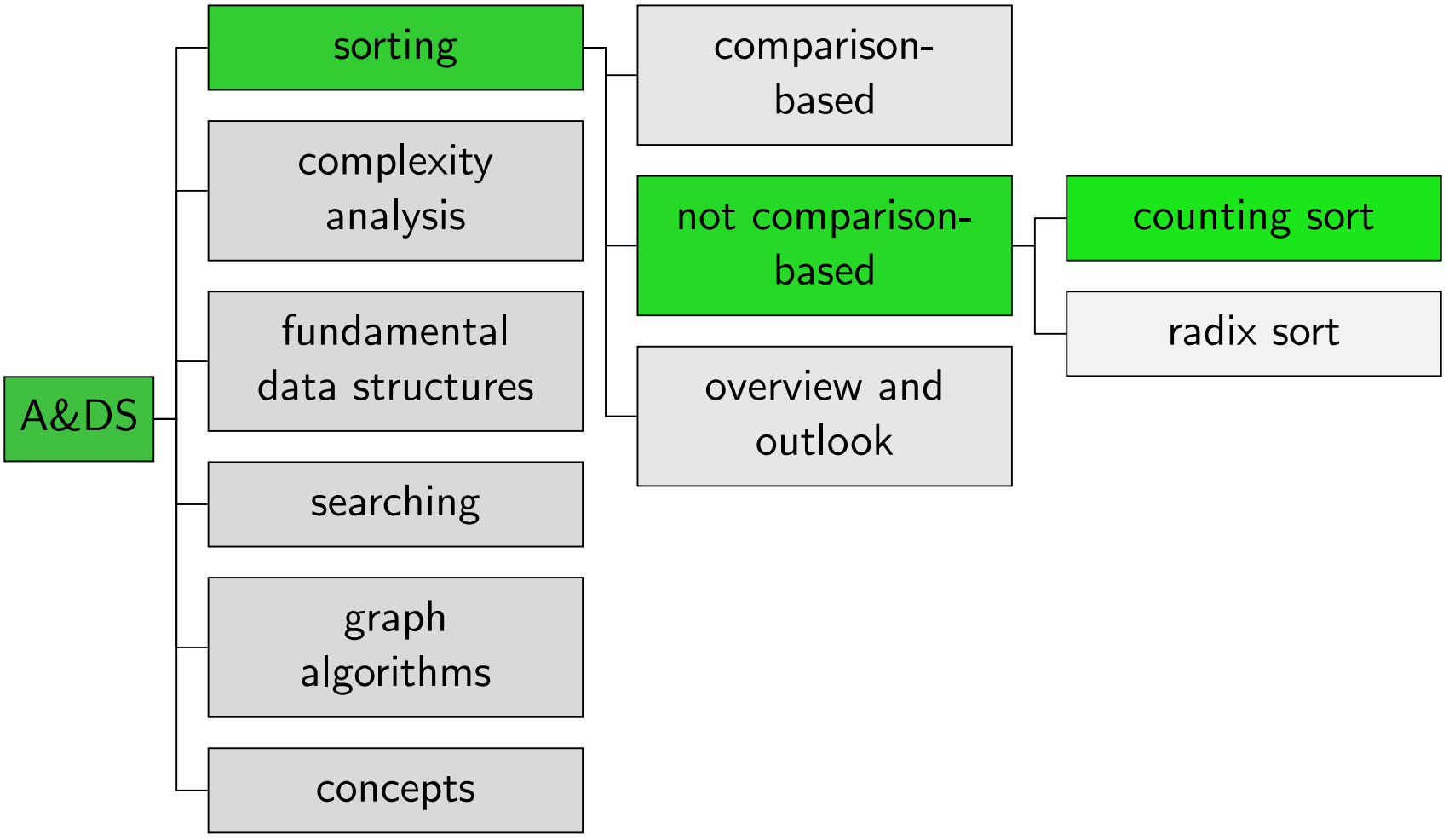
Counting Sort

Content of the Course



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  - ...
  - elements with key i fill positions $\sum_{j=0}^{i-1} \#j$ to $\left(\sum_{j=0}^{i-1} \#j\right) + \#i - 1$.
- (Backwards) iterate over the input array and copy the entries to the corresponding positions in the output array.

Counting Sort: Algorithm

```
1  def sort(array, k):
2      counts = [0] * (k + 1)  # list of k + 1 zeros
3      result = [0] * len(array) # list of same size as array
4
5      for elem in array:
6          counts[elem] += 1
7      # counts[j] contains number of occurrences of j
8
9      for i in range(1, k+1): # i = 1,2, ... , k
10         counts[i] += counts[i-1]
11     # counts[j] now contains number of occurrences of elements <= j
12
13     # copy elements from array to result, starting from the end
14     for elem in reversed(array):
15         result[counts[elem]-1] = elem
16         counts[elem] -= 1
17
18     return result
```

Jupyter Notebook

Jupyter notebook: `counting_sort.ipynb`

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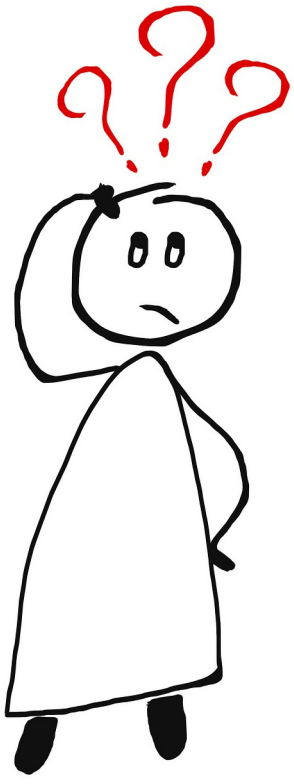
- Running time: $\Theta(n + k)$ (n size of input sequence)
  $\rightarrow$ For fixed k or $k \in O(n)$ linear.
- Memory: $\Theta(n + k)$ (not in-place)
- Counting sort is stable. Why?

Questions



Questions?

Radix Sort

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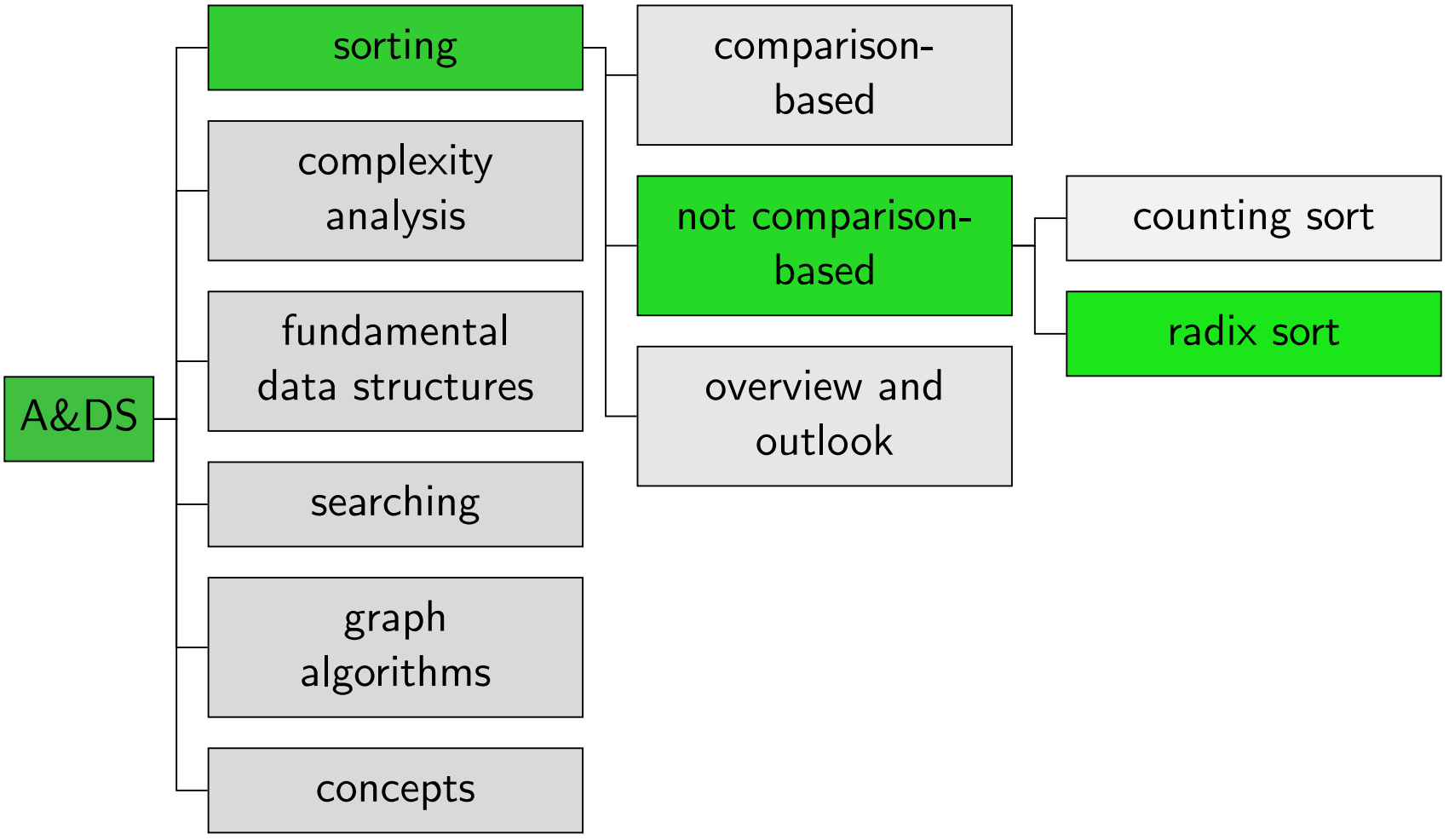

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		462	763			96			
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- Collect items from left to right/top to bottom:
  462, 763, 983, 96, 286

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- Separate items by the least significant (= last) digit:

0	1	2	3	4	5	6	7	8	9
		462	763			96			
			983			286			

- Collect items from left to right/top to bottom:
  462, 763, 983, 96, 286
- Separate items by the second last digit and collect them.
- Separate items by the third last digit and collect them.
- ... until you considered all positions of digits.

Radix Sort: Example

- Input: 263, 983, 96, 462, 286

Radix Sort: Example

- Input: 263, 983, 96, 462, 286
- Separation by last digit:

0	1	2	3	4	5	6	7	8	9
		462	263			96			
			983			286			

After collection: 462, 263, 983, 96, 286

Radix Sort: Example

- Input: 263, 983, 96, 462, 286
- Separation by last digit:

0	1	2	3	4	5	6	7	8	9
		462	263			96			
			983			286			

After collection: 462, 263, 983, 96, 286

- Separation by second last digit:

0	1	2	3	4	5	6	7	8	9
						462		983	96
						263		286	

After collection: 462, 263, 983, 286, 96

Radix Sort: Example

- Input: 263, 983, 96, 462, 286
- Separation by last digit:

0	1	2	3	4	5	6	7	8	9
		462	263			96			
			983			286			

After collection: 462, 263, 983, 96, 286

- Separation by second last digit:

0	1	2	3	4	5	6	7	8	9
						462		983	96
						263		286	

After collection: 462, 263, 983, 286, 96

- Separation by third last digit:

0	1	2	3	4	5	6	7	8	9
096		263		462					983
		286							

After collection: 96, 263, 286, 462, 983

Jupyter Notebook

Jupyter notebook: `radix_sort.ipynb`

Radix Sort: Algorithm (for arbitrary base)

```
1  def sort(array, base=10):
2      if not array:  # array is empty
3          return
4      iteration = 0
5      max_val = max(array) # identify largest element
6      while base ** iteration <= max_val:
7          buckets = [[] for num in range(base)]
8          for elem in array:
9              digit = (elem // (base ** iteration)) % base
10             buckets[digit].append(elem)
11         pos = 0
12         for bucket in buckets:
13             for elem in bucket:
14                 array[pos] = elem
15                 pos += 1
16         iteration += 1
```

Radix Sort: Running Time

- m: Maximal number of digits in representation with given base b.
- n: length of input sequence
- Running time $O(m \cdot (n + b))$

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For fixed m and b, radix sort has linear running time.

Radix Sort: High-level Perspective

All entries in the array have d digits, where the lowest-order digit is at position 0 and the highest-order digit at position d-1.

```
1  def radix_sort(array, d)
2    for i in range(d):
3      # use a stable sort to sort array on the digit at position i
```

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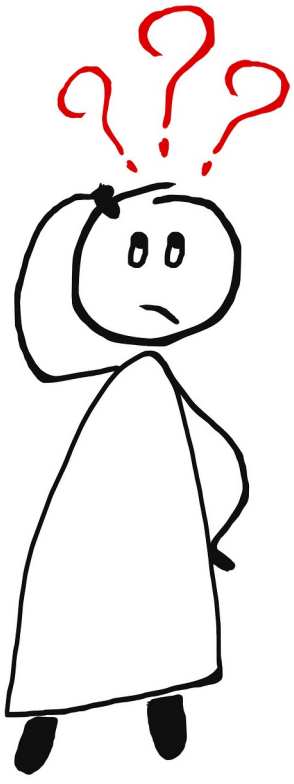

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Summary

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- Counting sort and radix sort are not comparison-based and allow us (under certain restrictions) to sort in linear time.
- However, they place additional restrictions on the keys used.