

Algorithms and Data Structures

A6. Runtime Analysis: Logarithm

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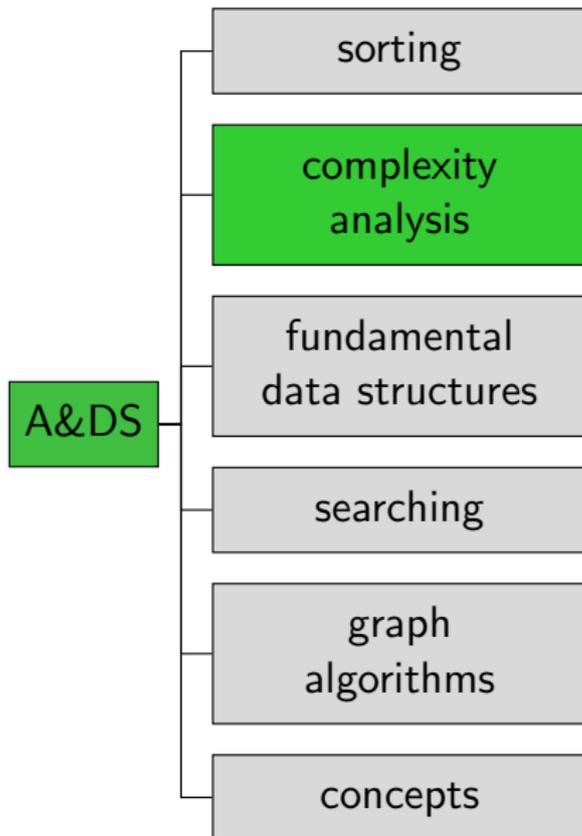
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A6.1 Logarithm

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Content of the Course



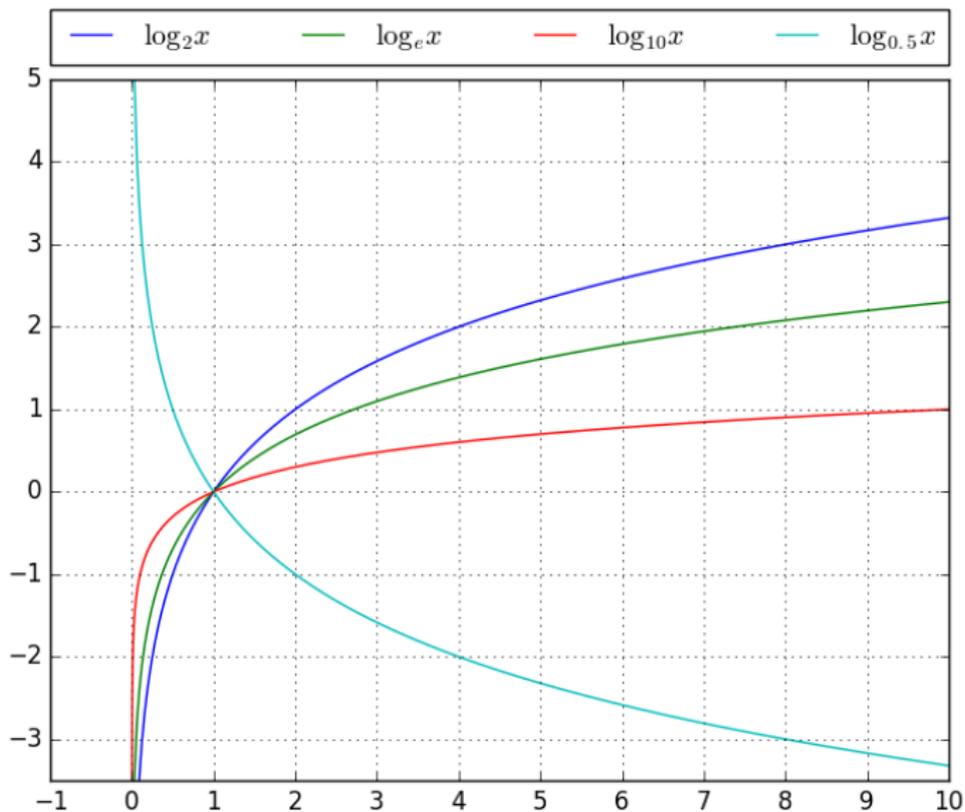
Logarithm

- ▶ For the analysis of merge sort, we will need the **logarithm function**.
- ▶ This is often the case in runtime analysis, in particular for divide-and-conquer algorithms.
- ▶ The logarithm to the base b is the inverse function to exponentiation with base b , i.e.

$$\log_b x = y \text{ iff. } b^y = x.$$

- ▶ **Example:** $\log_2 8 = 3$, because $2^3 = 8$
Example: $\log_3 81 = 4$, because $3^4 = 81$
- ▶ $\log_b a$ intuitively (if this works without remainder):
“How often must we divide a by b to reach 1?”

Logarithm: Illustration



Calculation with Logarithms

The following rules are immediate results of the rules $(b^c)^d = b^{cd} = (b^c)^d$ and $b^c b^d = b^{c+d}$:

product $\log_b(xy) = \log_b x + \log_b y$

power $\log_b(x^r) = r \log_b x$

change of base $\log_b x = \log_a x / \log_a b$

Logarithm: Example Calculation

In the analysis of algorithms, we sometimes see expressions of the form $a^{\log_b x}$. How do we get the logarithm out of the exponent?

Example: $5^{\log_2 x}$

We use $5 = 2^{\log_2 5}$.

$$\begin{aligned} 5^{\log_2 x} &= (2^{\log_2 5})^{\log_2 x} \\ &= 2^{\log_2 5 \log_2 x} \\ &= 2^{\log_2 x \log_2 5} \\ &= (2^{\log_2 x})^{\log_2 5} \\ &= x^{\log_2 5} \\ &\approx x^{2.32} \end{aligned}$$