Foundations of Artificial Intelligence

F4. Automated Planning: Delete Relaxation Heuristics

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Automated Planning: Overview

Chapter overview: automated planning

- F1. Introduction
- F2. Planning Formalisms
- F3. Delete Relaxation
- F4. Delete Relaxation Heuristics
- F5. Abstraction
- F6. Abstraction Heuristics

Relaxed Planning Graphs

Relaxed Planning Graphs

- relaxed planning graphs: represent which variables in Π^+ can be reached and how
- graphs with variable layers V^i and action layers A^i
  - variable layer V^0 contains the variable vertex v^0 for all $v \in I$
  - action layer A^{i+1} contains the action vertex a^{i+1} for action a if V^i contains the vertex v^i for all $v \in pre(a)$
  - variable layer V^{i+1} contains the variable vertex v^{i+1} if previous variable layer contains v^i, or previous action layer contains a^{i+1} with $v \in add(a)$

German: relaxierter Planungsgraph, Variablenknoten, Aktionsknoten

Relaxed Planning Graphs (Continued)

- a goal vertex g if $v^n \in V^n$ for all $v \in G$, where n is last layer
- graph can be constructed for arbitrary many layers but stabilizes after a bounded number of layers $\rightsquigarrow V^{i+1} = V^i$ and $A^{i+1} = A^i$ (Why?)
- directed edges:
  - from v^i to a^{i+1} if $v \in pre(a)$ (precondition edges)
  - from a^i to v^i if $v \in add(a)$ (effect edges)
  - from v^i to v^{i+1} (no-op edges)
  - from v^n to g if $v \in G$ (goal edges)

German: Zielknoten, Vorbedingungskanten, Effektkanten, Zielkanten, No-Op-Kanten

Illustrative Example

We write actions a with $pre(a) = \{p_1, \dots, p_k\}$, $add(a) = \{q_1, \dots, q_l\}$, $del(a) = \emptyset$ and $cost(a) = c$ as $p_1, \dots, p_k \xrightarrow{c} q_1, \dots, q_l$

$$V = \{m, n, o, p, q, r, s, t\}$$
$$I = \{m\}$$
$$G = \{o, p, q, r, s\}$$
$$A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$$
$$a_1 = m \xrightarrow{3} n, o$$
$$a_2 = m, o \xrightarrow{1} p$$
$$a_3 = n, o \xrightarrow{1} q$$
$$a_4 = n \xrightarrow{1} r$$
$$a_5 = p \xrightarrow{1} q, r$$
$$a_6 = p \xrightarrow{1} s$$

Illustrative Example: Relaxed Planning Graph

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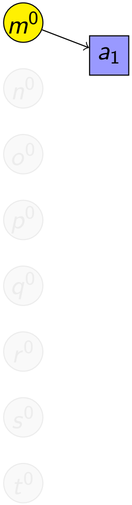

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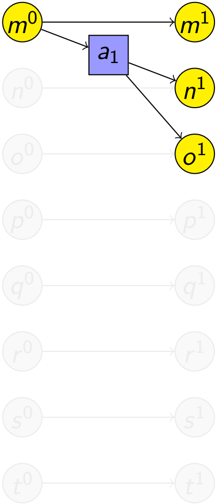

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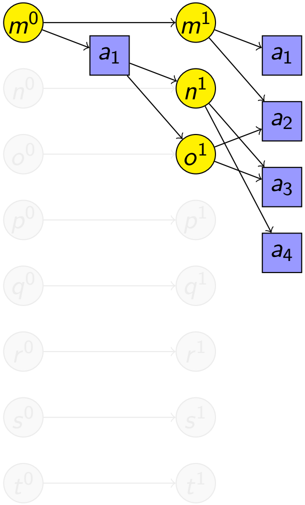

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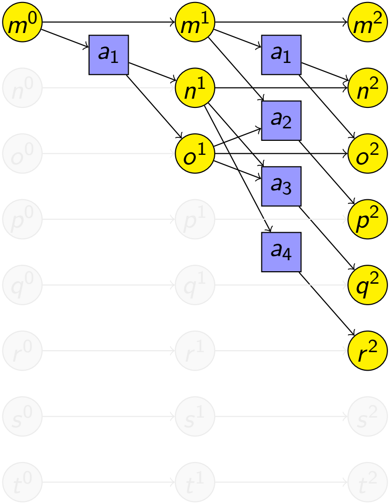

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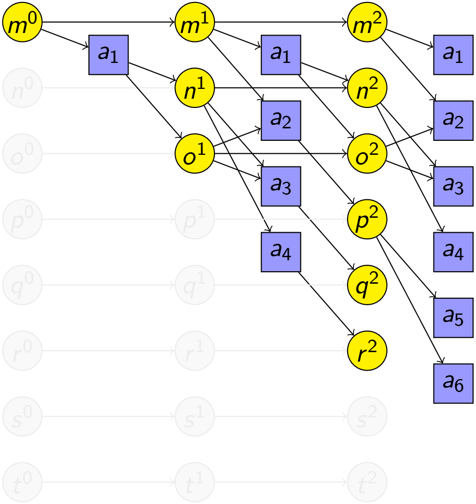

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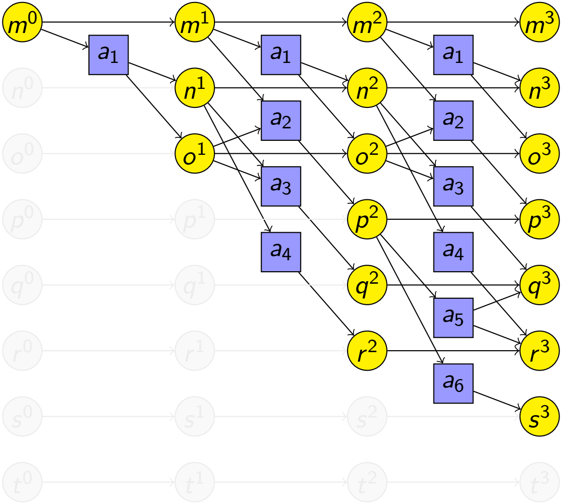

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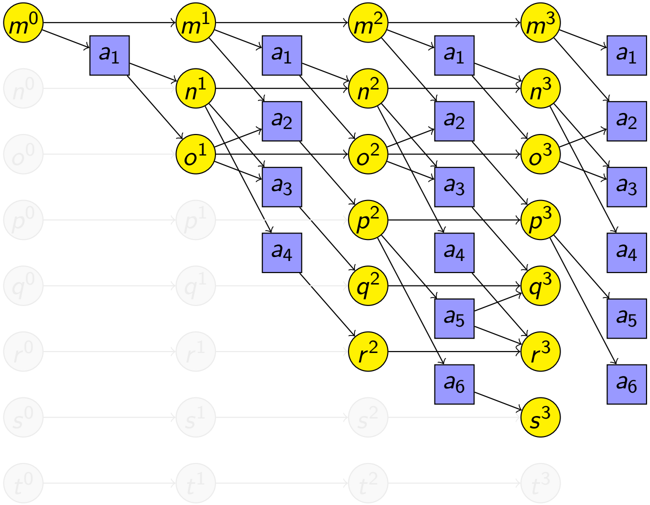

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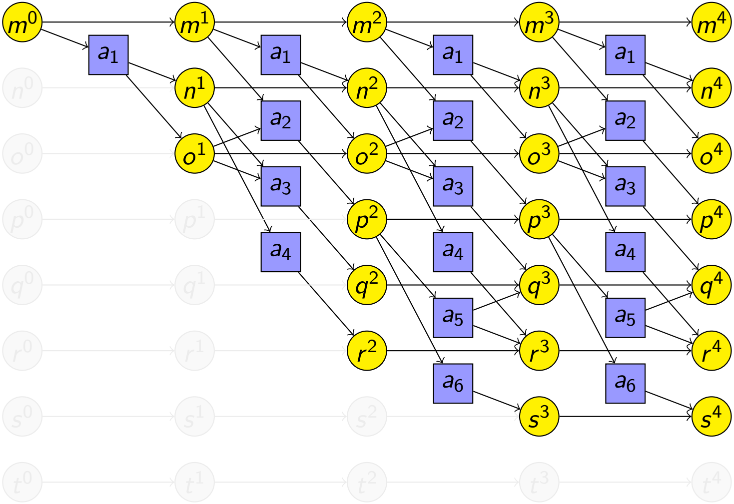

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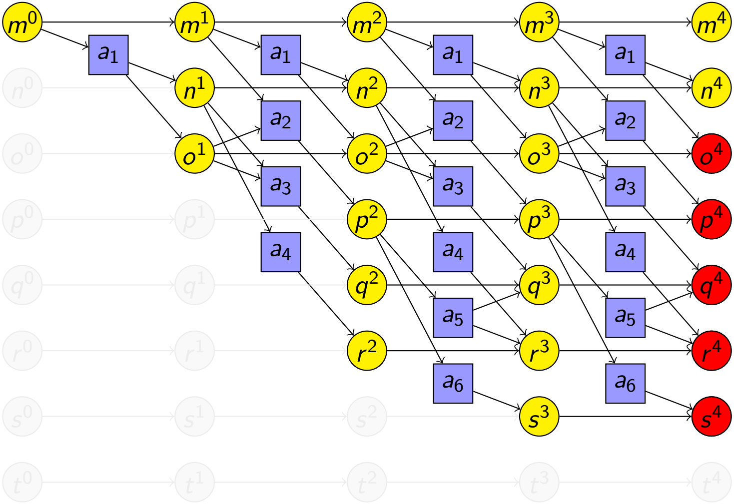

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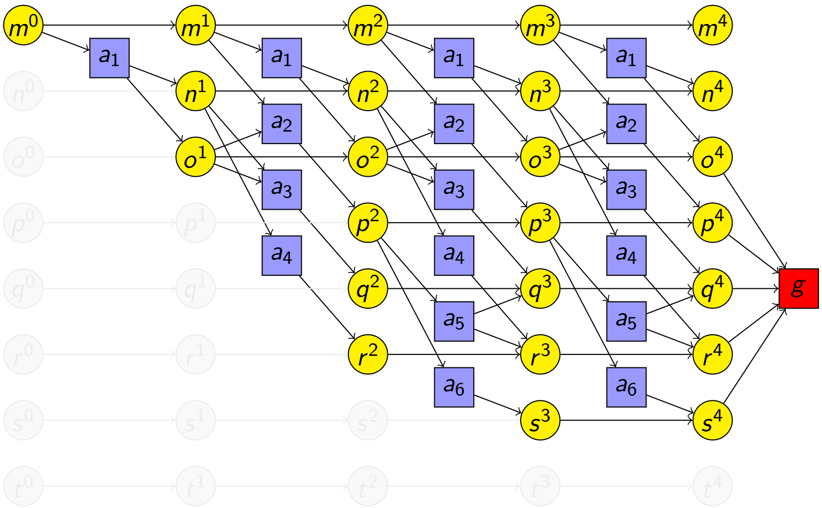


Generic Relaxed Planning Graph Heuristic

Heuristic Values from Relaxed Planning Graph

function *generic-rpg-heuristic*($\langle V, I, G, A\rangle, s$):
    $\Pi^+ := \langle V, s, G, A^+\rangle$
    for $k \in \{0, 1, 2, \dots\}$:
        $rpg := RPG_k(\Pi^+)$ [relaxed planning graph to layer k]
        if *rpg* contains a goal node:
            Annotate nodes of *rpg*.
            if termination criterion is true:
                return heuristic value from annotations
        else if graph has stabilized:
            return ∞

- ⇝ general template for RPG heuristics
- ⇝ to obtain concrete heuristic: instantiate highlighted elements

Concrete Examples for Generic RPG Heuristic

Many planning heuristics fit this general template.

In this course:

- maximum heuristic $h^{\max}$ (Bonet & Geffner, 1999)
- additive heuristic h^{add} (Bonet, Loerincs & Geffner, 1997)
- Keyder & Geffner's (2008) variant of the FF heuristic h^{FF} (Hoffmann & Nebel, 2001)

German: Maximum-Heuristik, additive Heuristik, FF-Heuristik

remark:

- The most efficient implementations of these heuristics do not use explicit planning graphs, but rather alternative (equivalent) definitions.

Maximum and Additive Heuristics

Maximum and Additive Heuristics

- $h^{\max}$ and h^{add} are the simplest RPG heuristics.
- Vertex annotations are numerical values.
- The vertex values estimate the costs
  - to make a given variable true
  - to reach and apply a given action
  - to reach the goal

Maximum and Additive Heuristics: Filled-in Template

$h^{\max}$ and h^{add}

computation of annotations:

- costs of variable vertices:
  0 in layer 0;
  otherwise minimum of the costs of predecessor vertices
- costs of action and goal vertices:
  maximum ($h^{\max}$) or sum (h^{add}) of predecessor vertex costs;
  for action vertices a^i, also add $cost(a)$

termination criterion:

- stability: terminate if $V^i = V^{i-1}$ and costs of all vertices in V^i equal corresponding vertex costs in V^{i-1}

heuristic value:

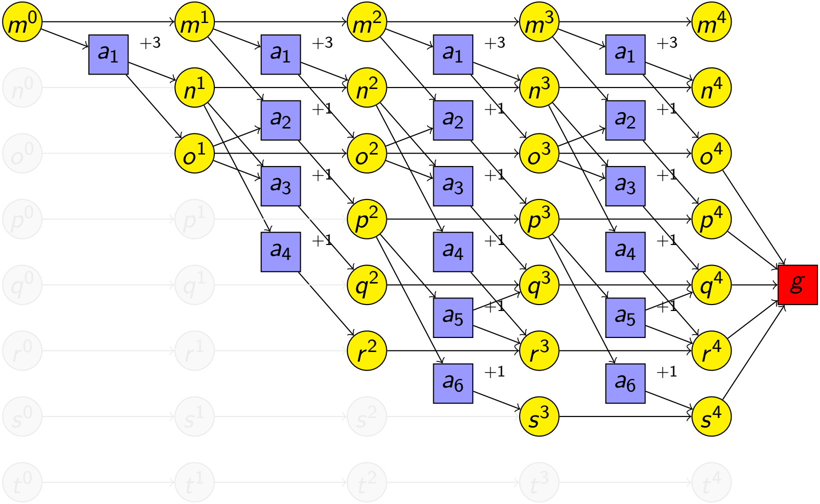
- value of goal vertex

Maximum and Additive Heuristics: Intuition

intuition:

- variable vertices:
  - choose cheapest way of reaching the variable
- action/goal vertices:
  - $h^{\max}$ is optimistic: assumption:
    when reaching the most expensive precondition variable, we can reach the other precondition variables in parallel (hence maximization of costs)
  - h^{add} is pessimistic: assumption:
    all precondition variables must be reached completely independently of each other (hence summation of costs)

Illustrative Example: $h^{\max}$



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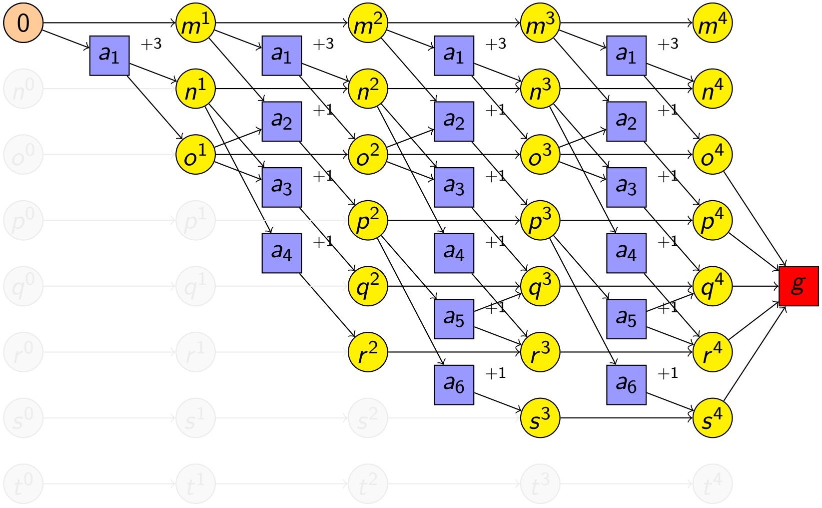

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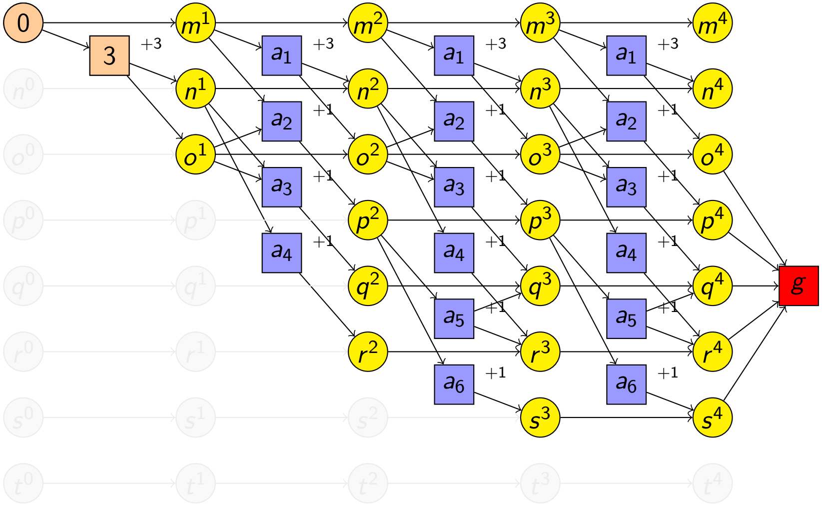

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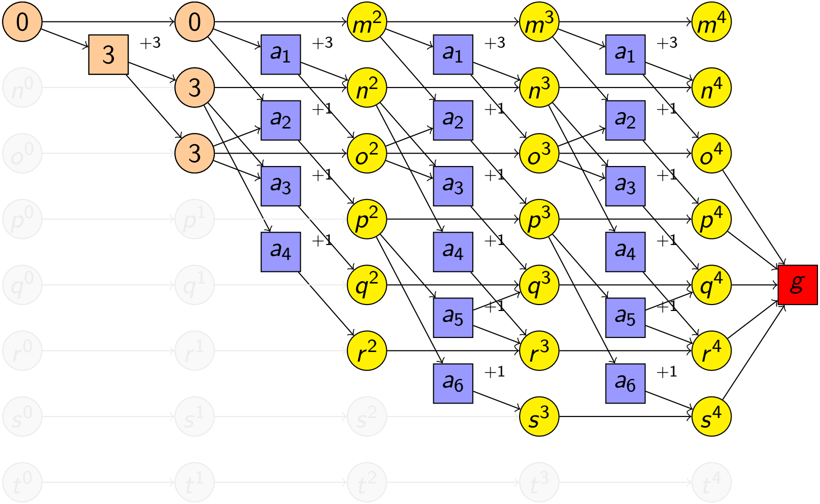

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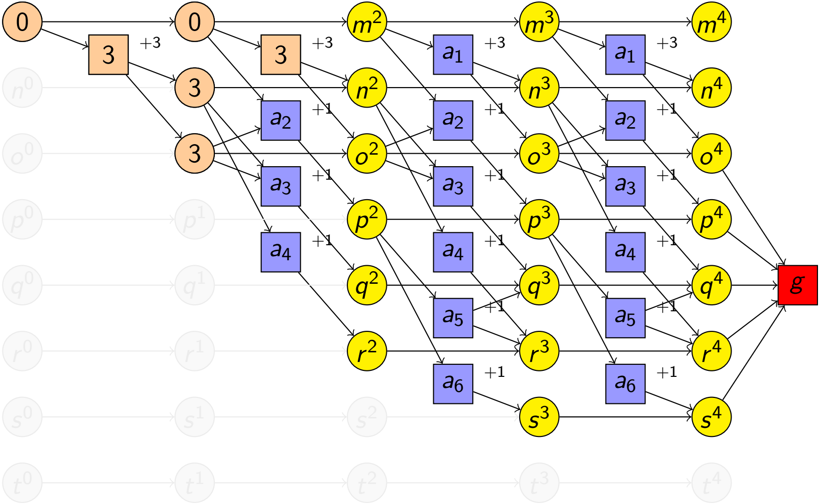

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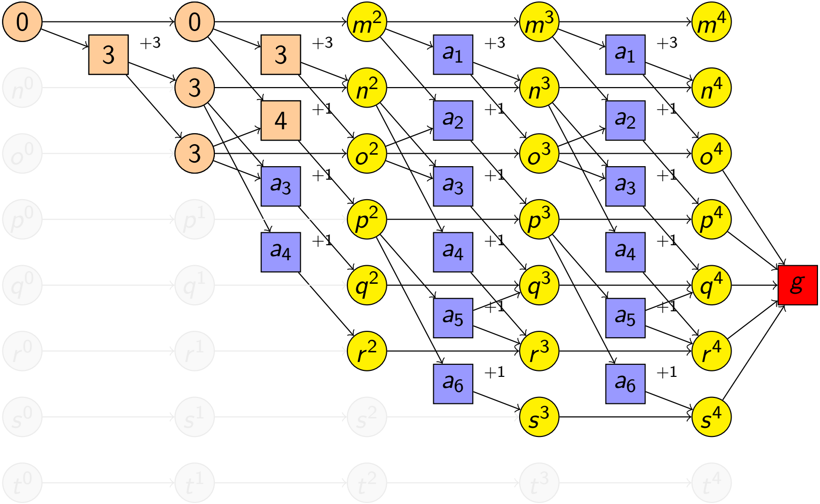

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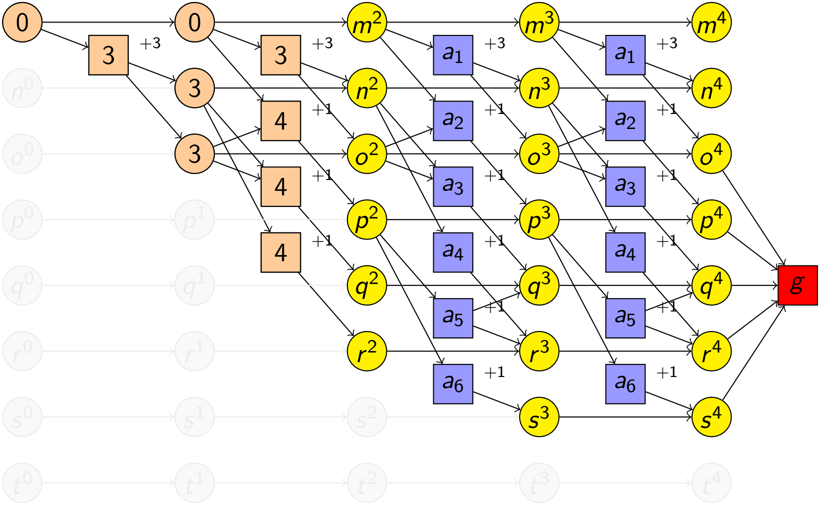

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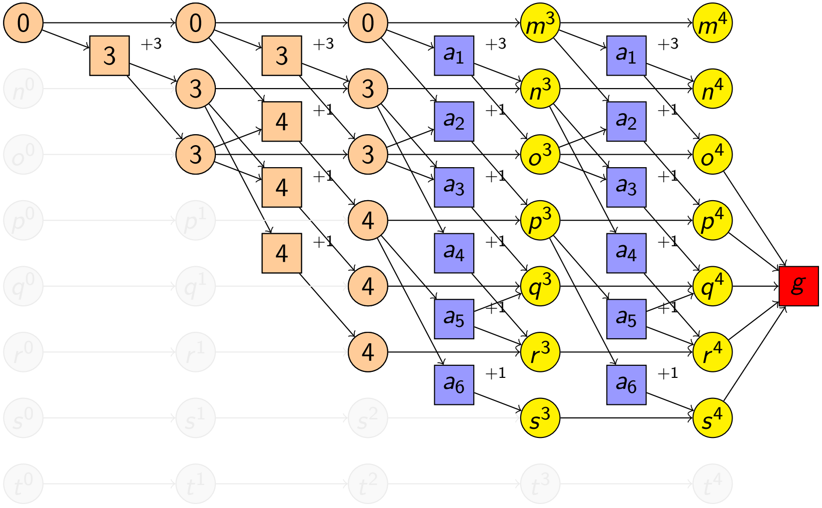

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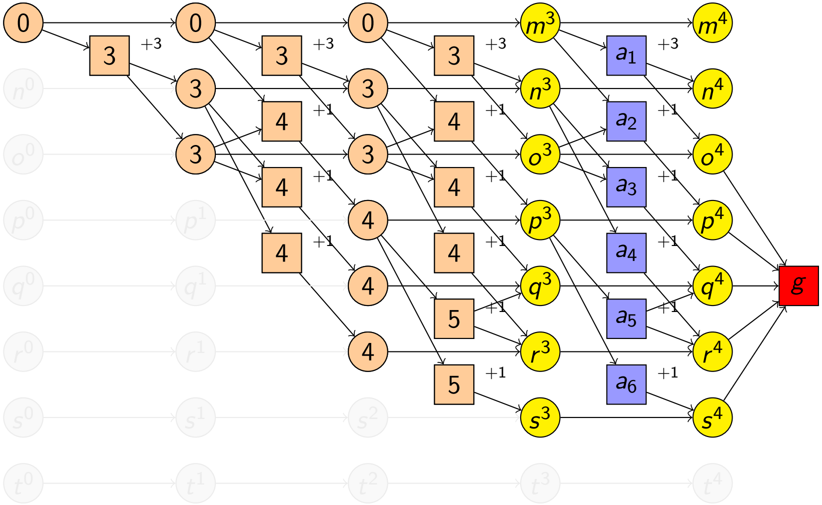

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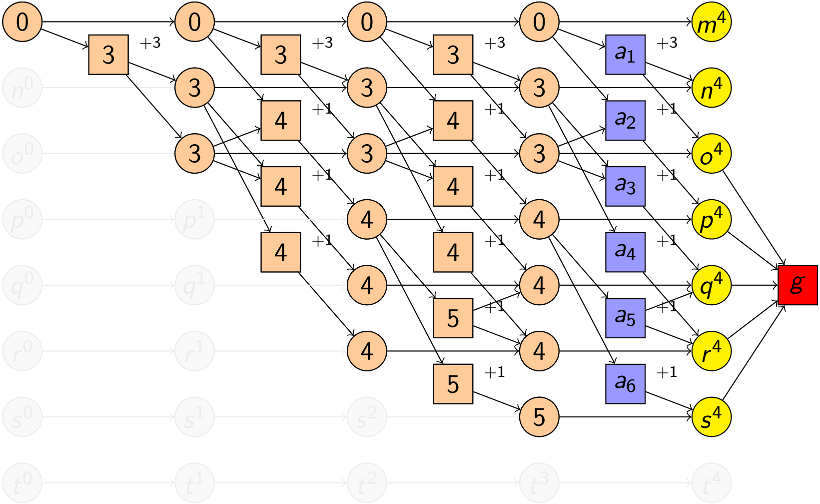

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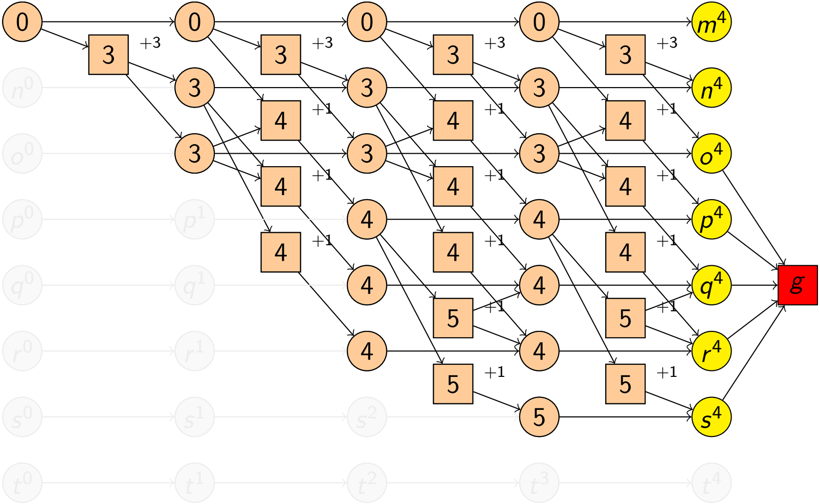

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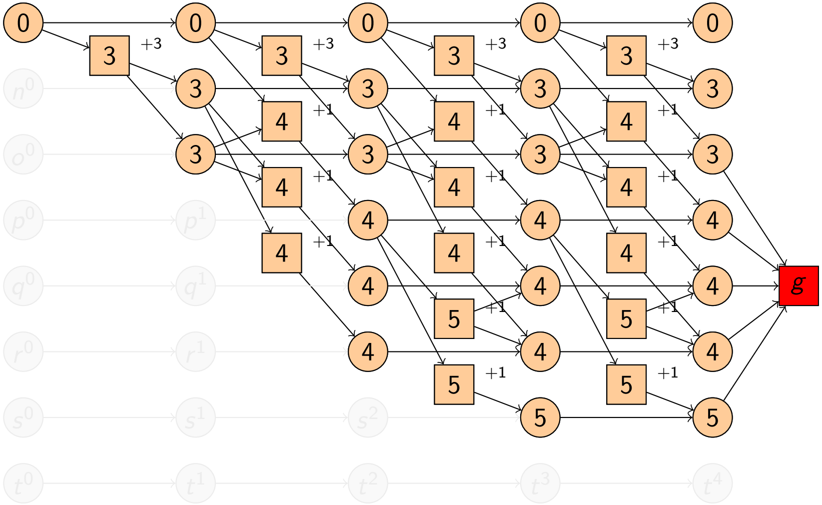

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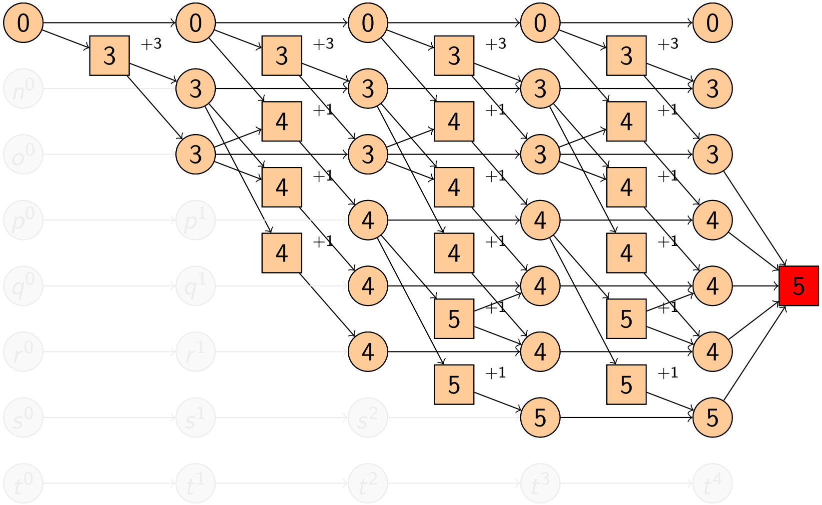
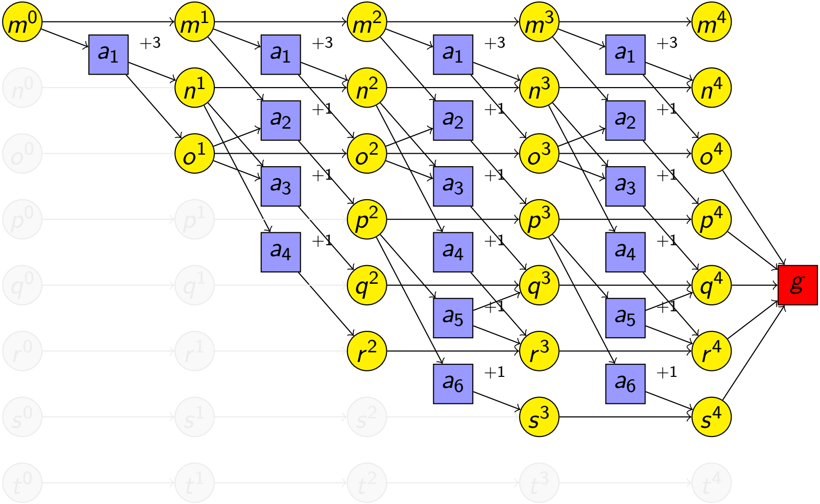


$h^{\max}(\{m\}) = 5$

Illustrative Example: h^{add}



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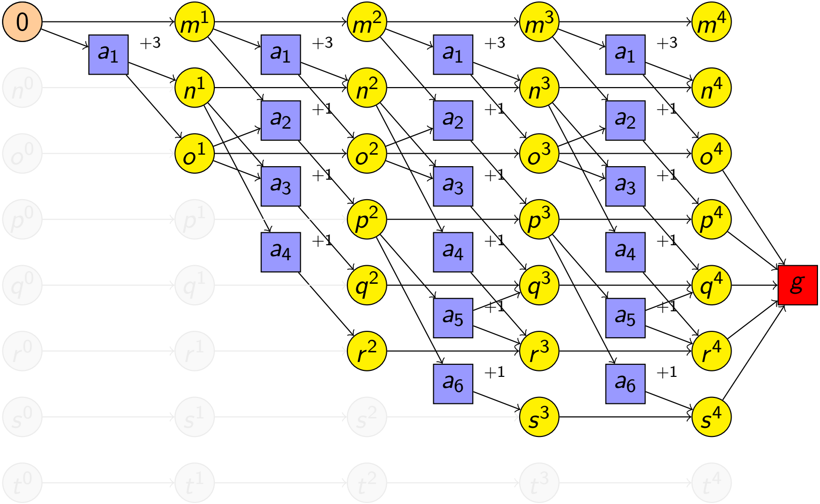

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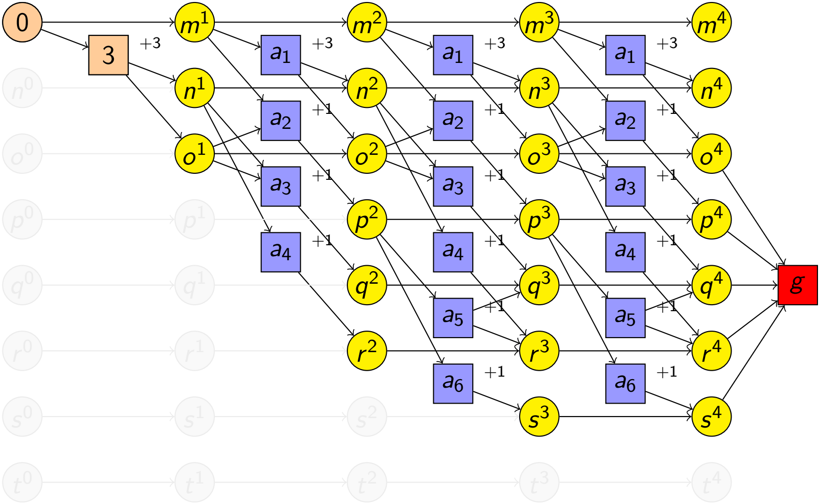

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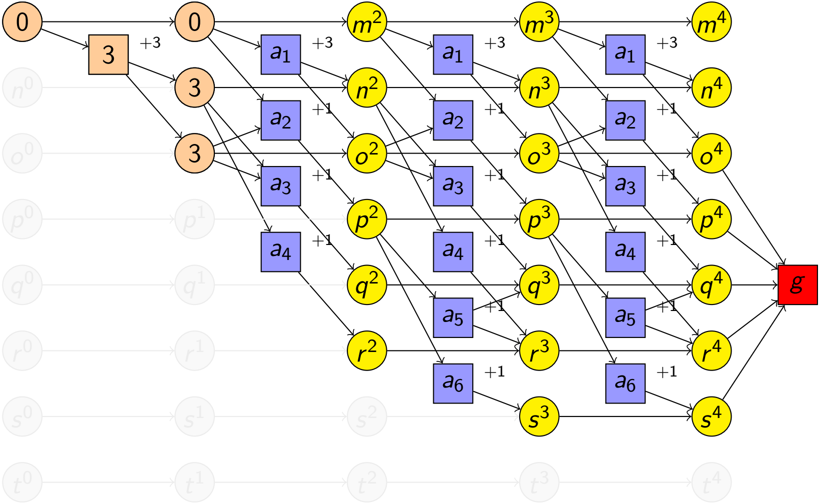

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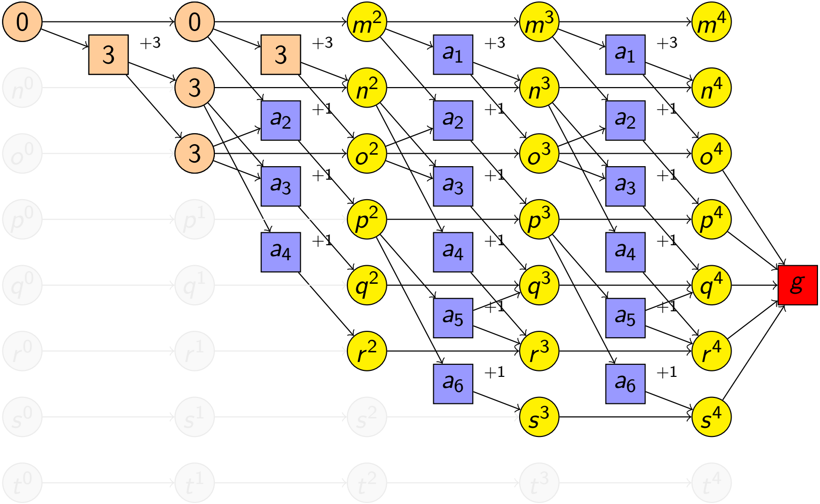

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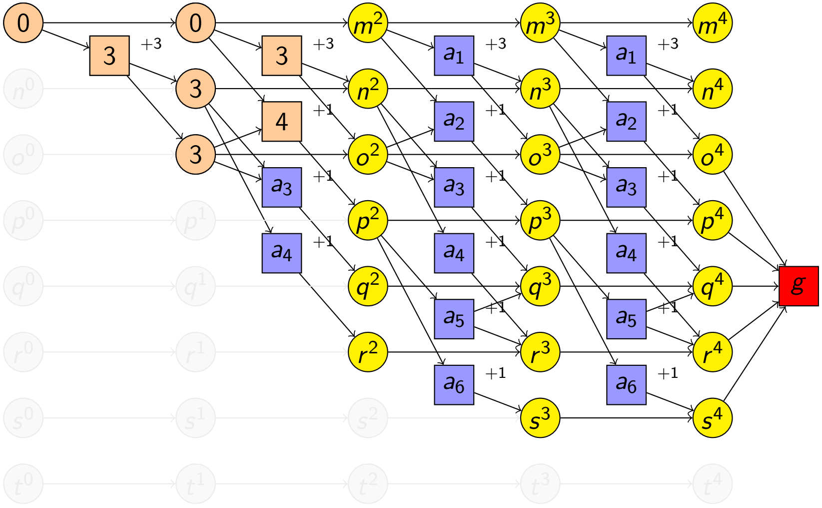

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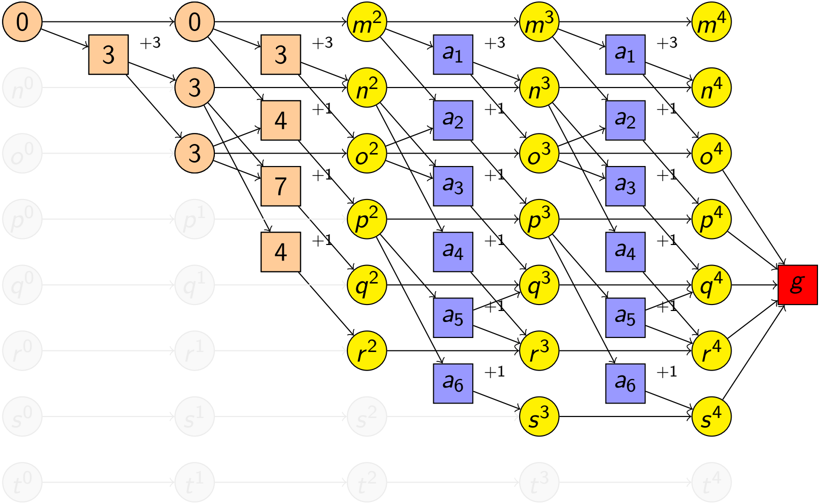

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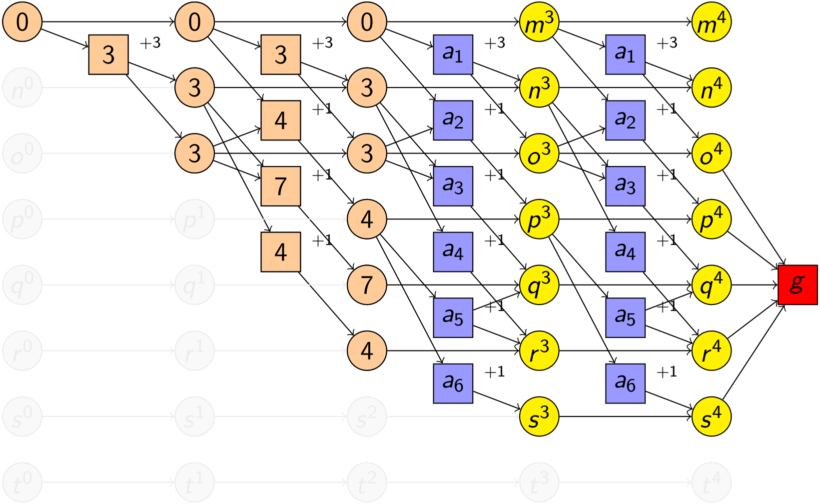

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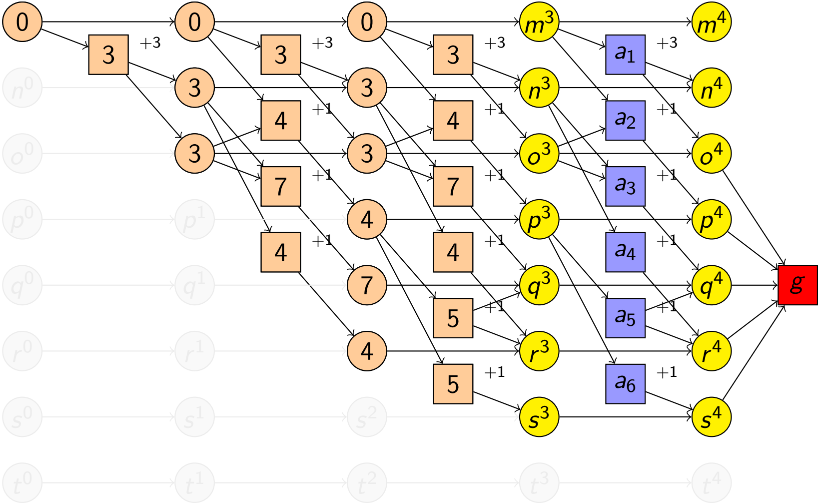

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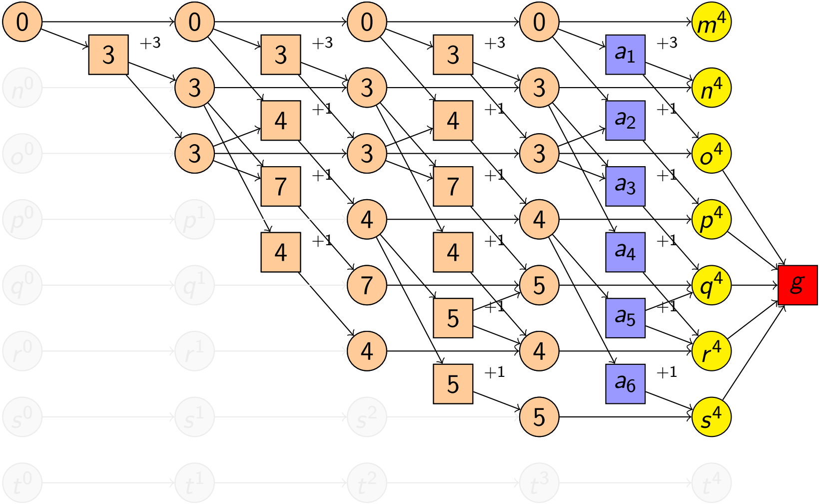

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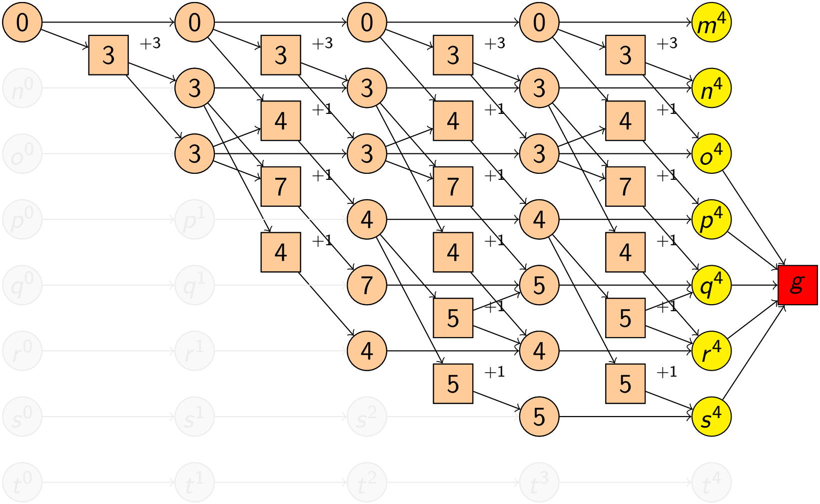

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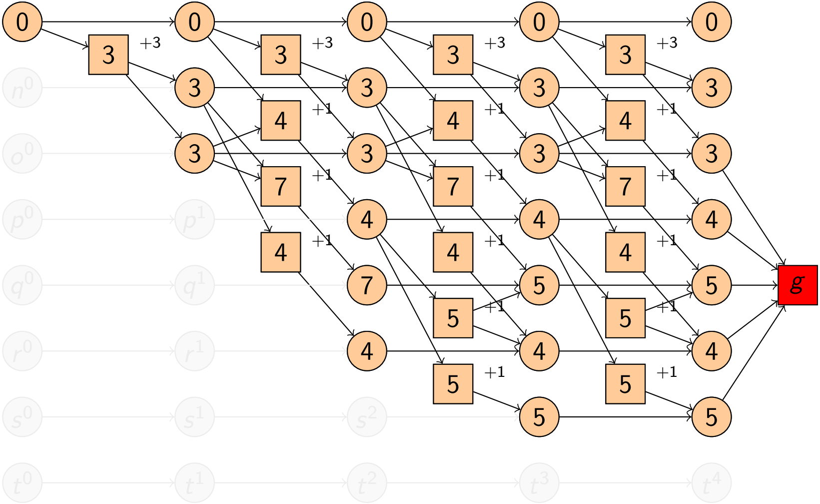

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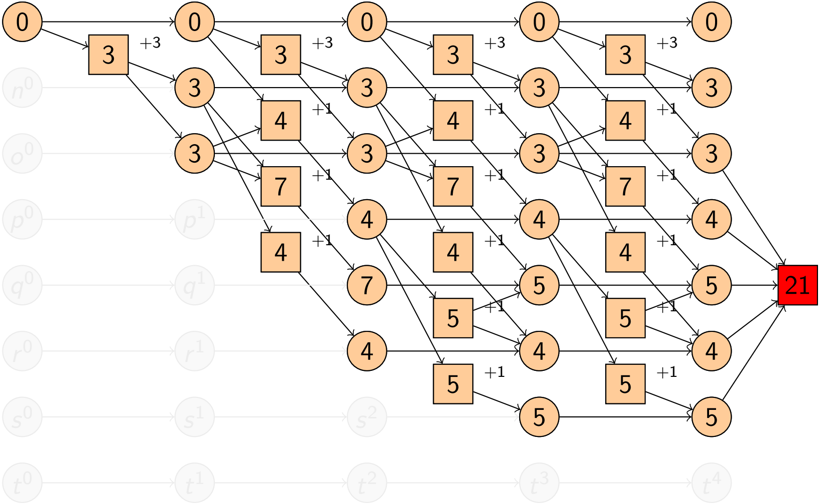

$h^{\text{add}}(\{m\}) = 21$

$h^{\max}$ and h^{add}: Remarks

comparison of $h^{\max}$ and h^{add}:

- both are safe and goal-aware
- $h^{\max}$ is admissible and consistent; h^{add} is neither.

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$\rightsquigarrow$ FF heuristic

FF Heuristic

FF Heuristic

The FF Heuristic

identical to h^{add}, but additional steps at the end:

- Mark goal vertex.
- Apply the following marking rules until nothing more to do:
  - marked action or goal vertex?
    $\rightsquigarrow$ mark all predecessors
  - marked variable vertex v^i in layer $i \geq 1$?
    $\rightsquigarrow$ mark one predecessor with minimal h^{add} value
    (tie-breaking: prefer variable vertices; otherwise arbitrary)

heuristic value:

- The actions corresponding to the marked action vertices build a relaxed plan.
- The cost of this plan is the heuristic value.

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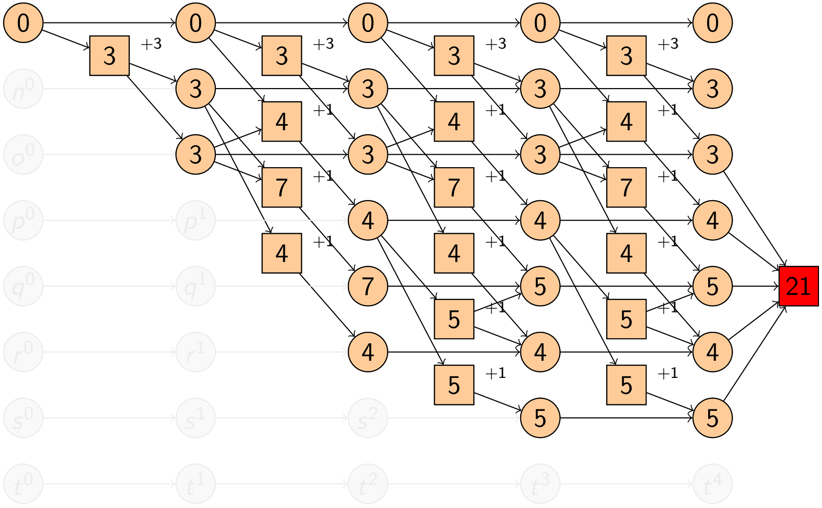

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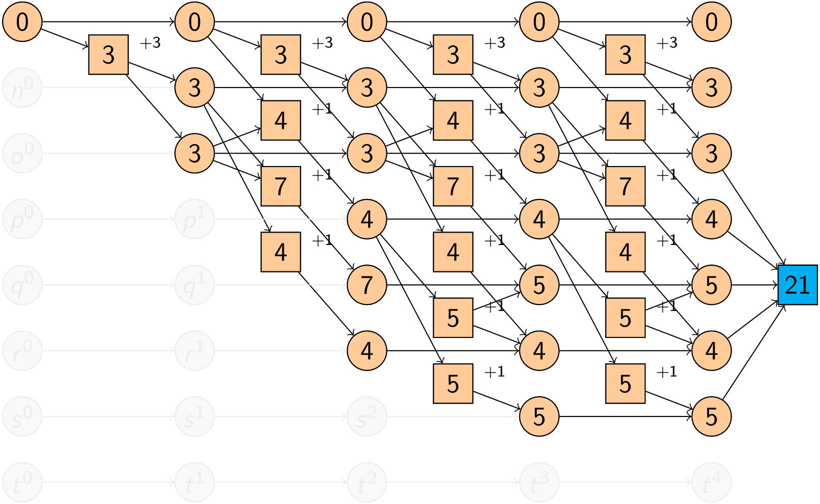

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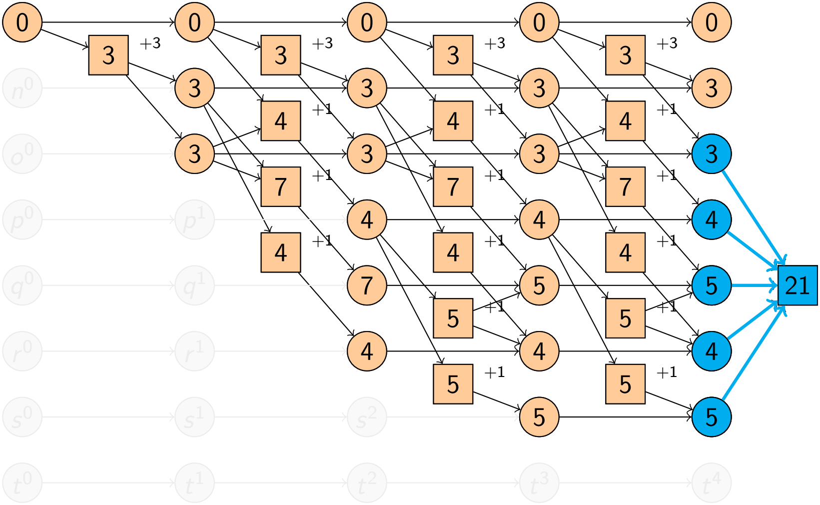

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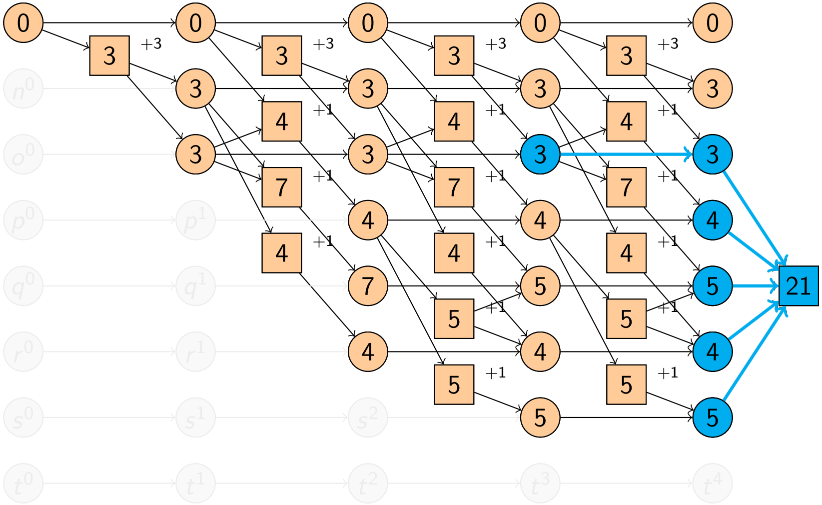

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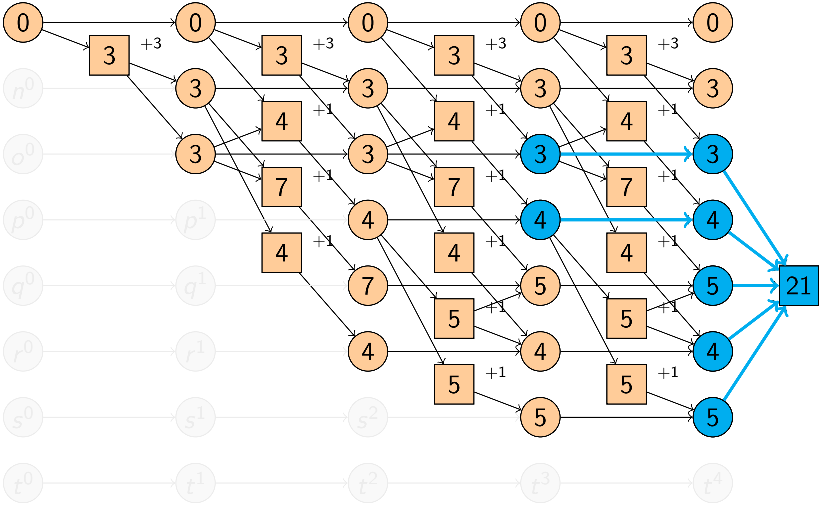

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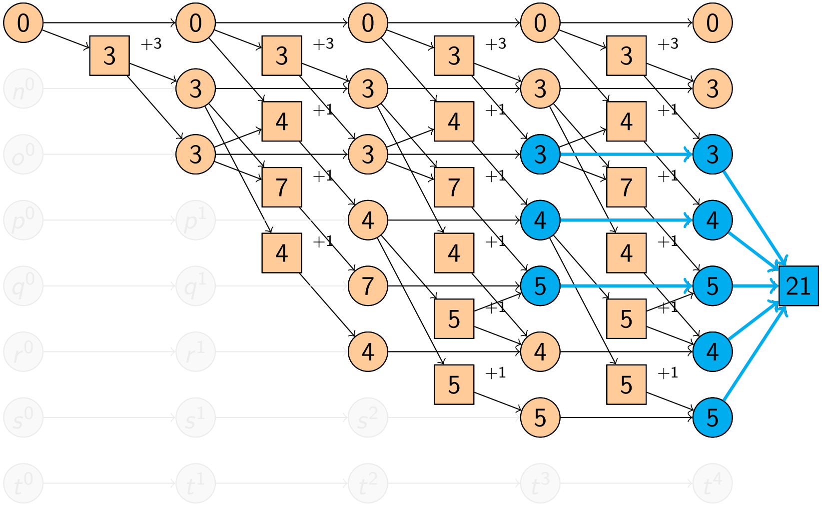

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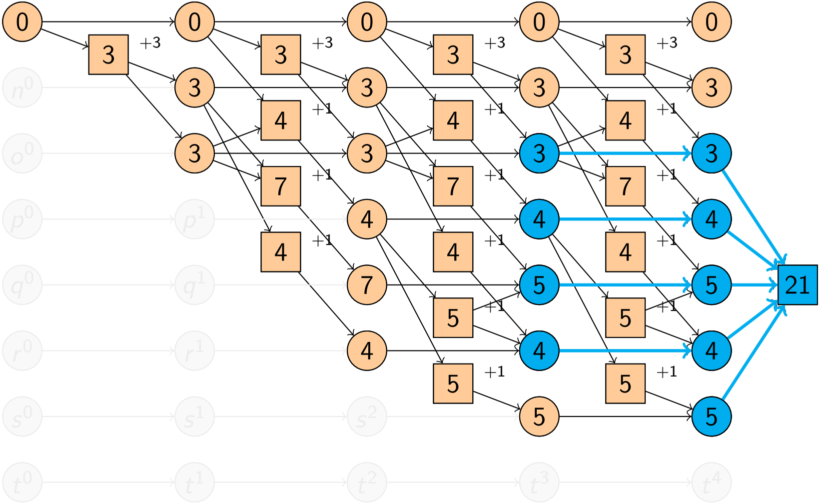

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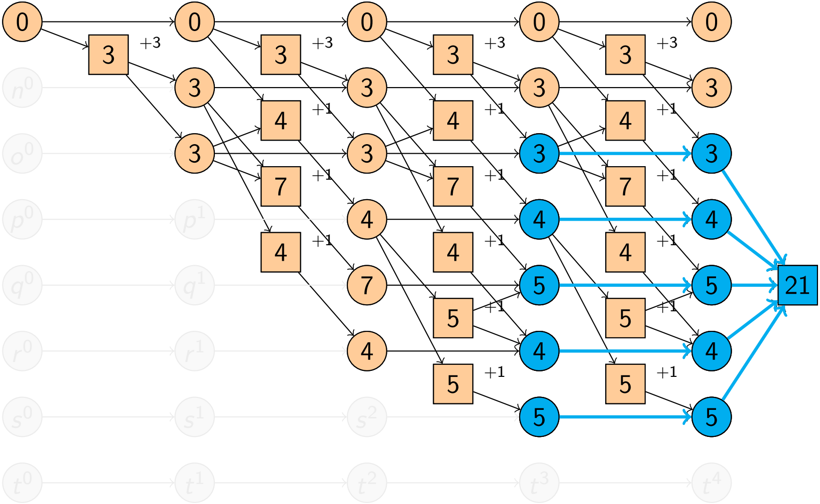

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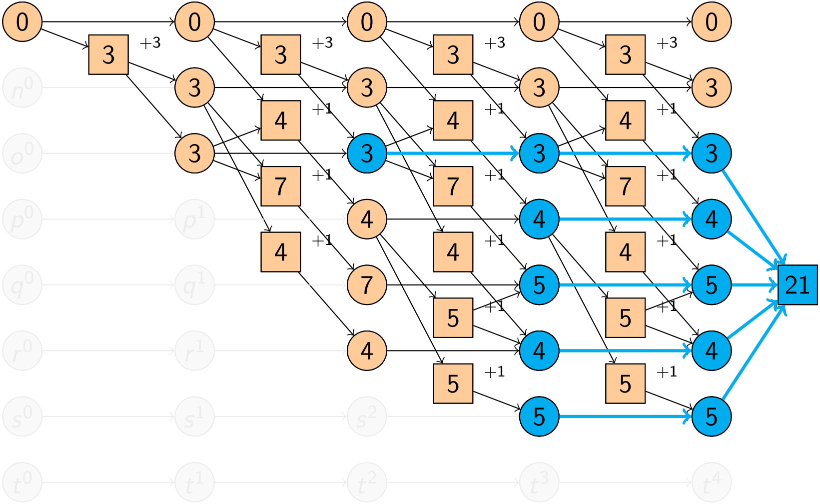

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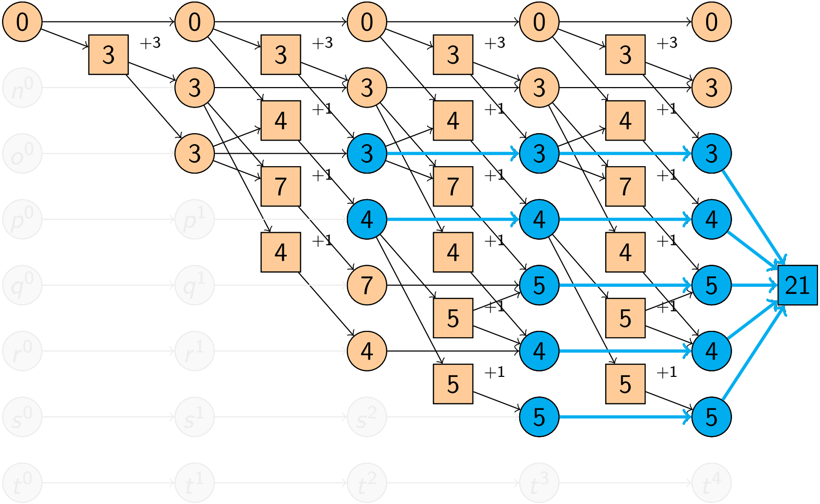

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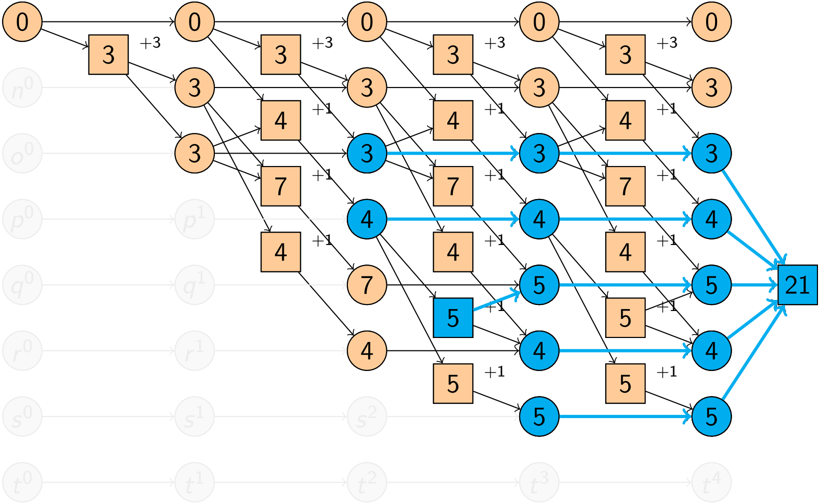

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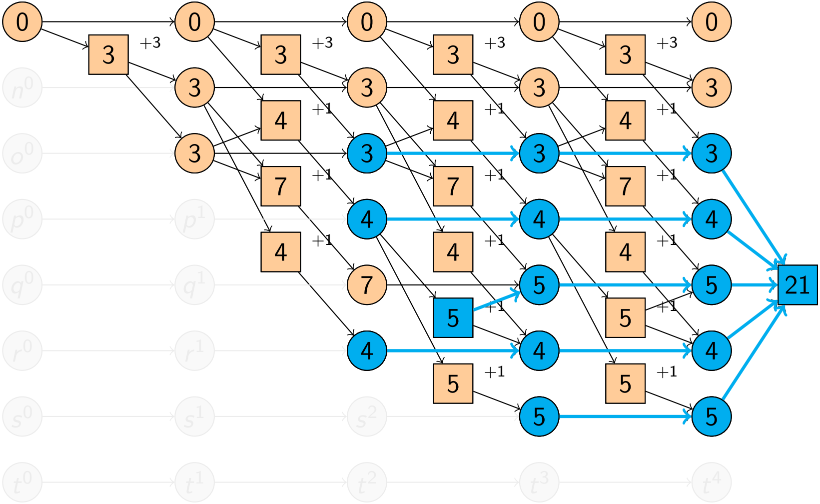

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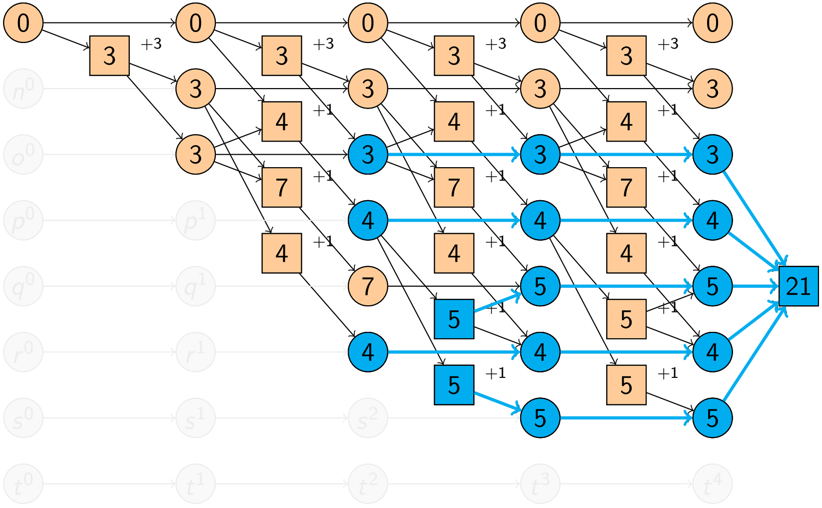

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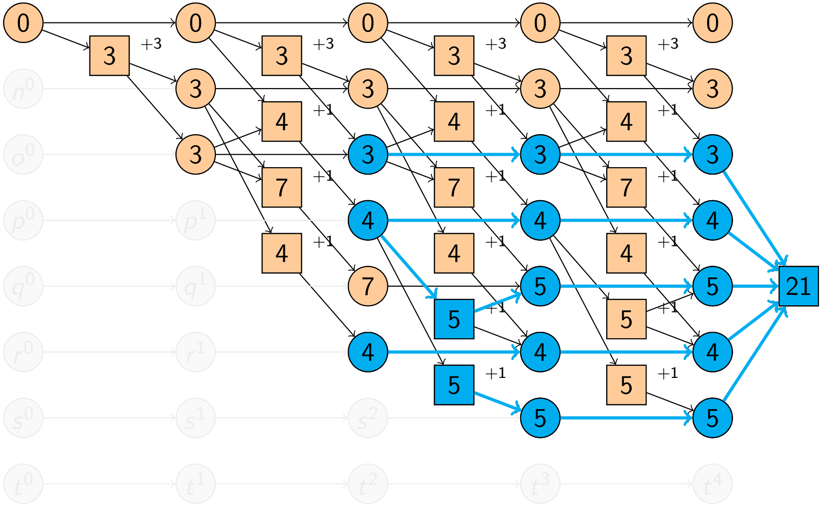

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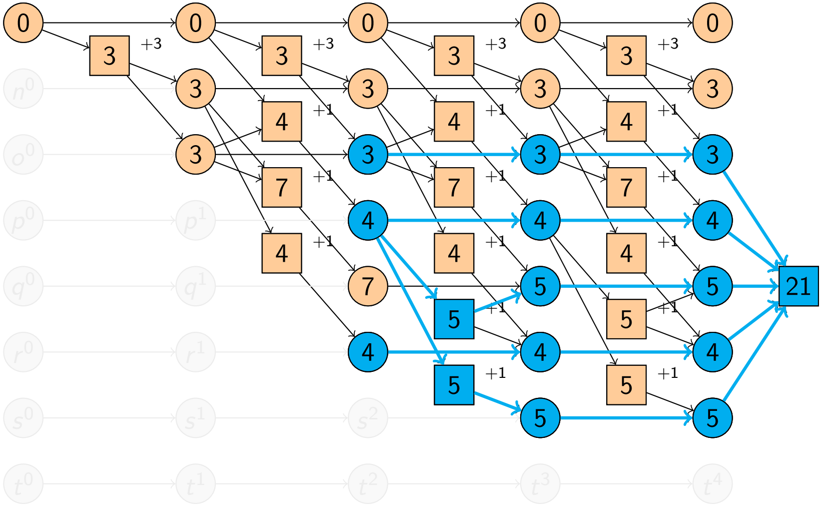
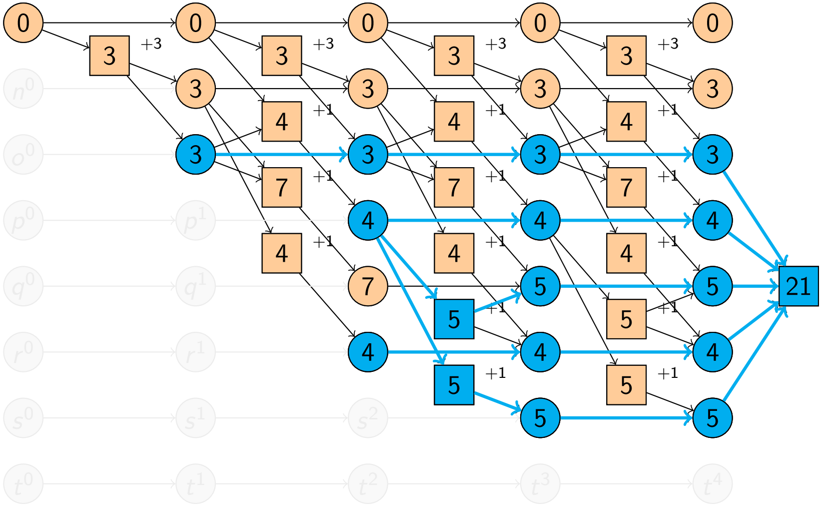


Illustrative Example: h^{FF}



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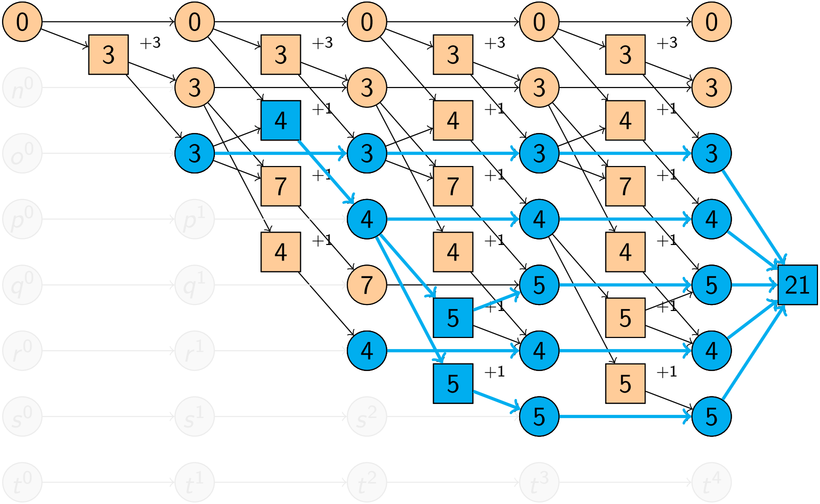

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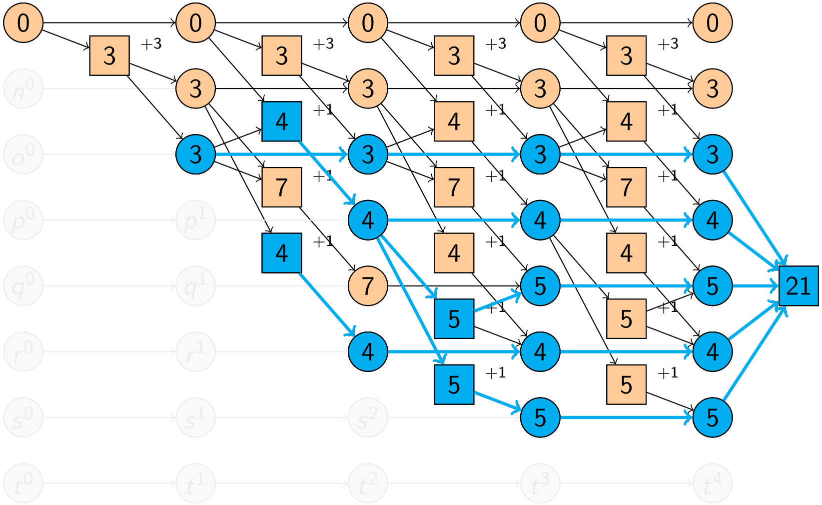

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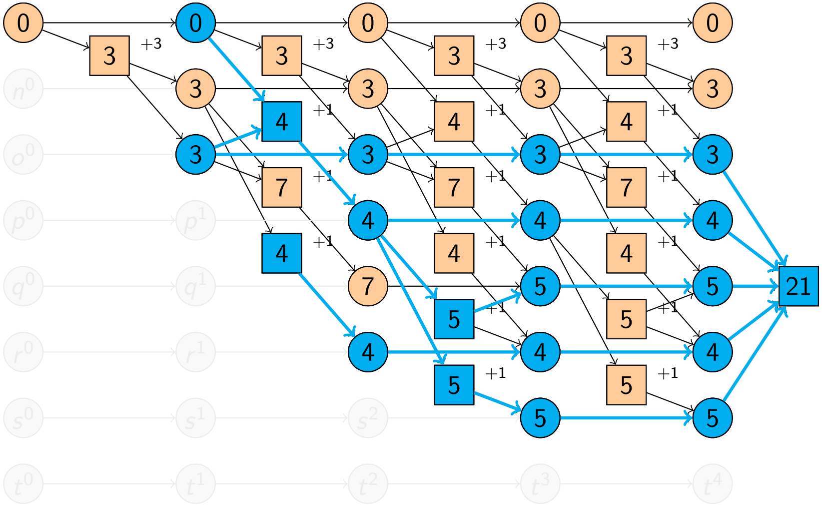

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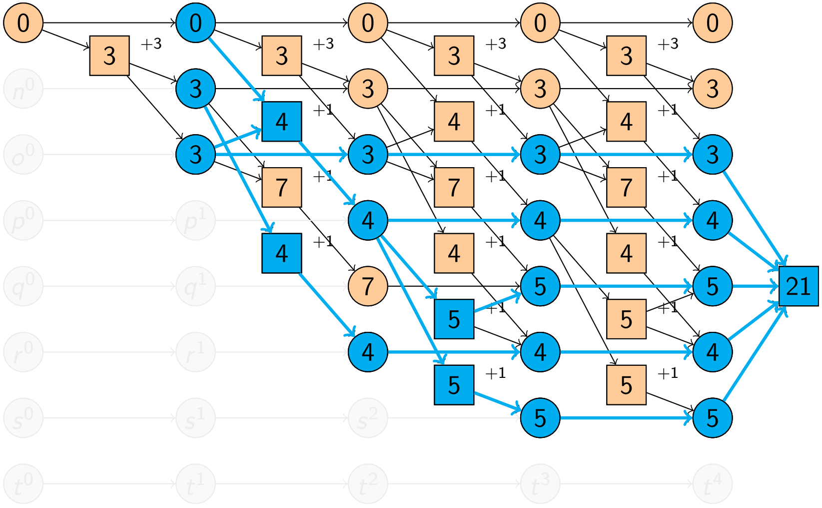

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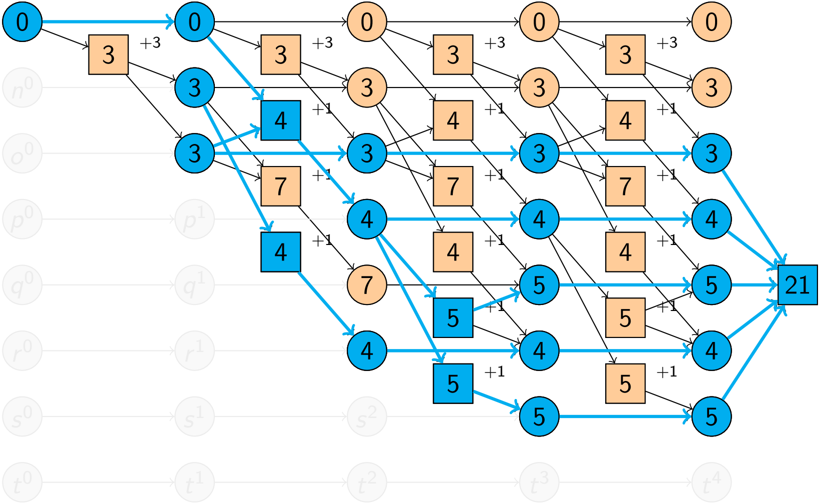

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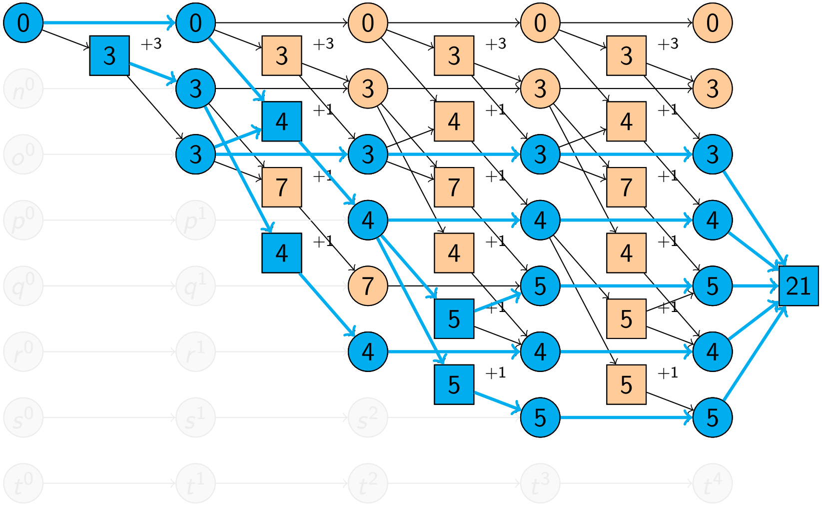

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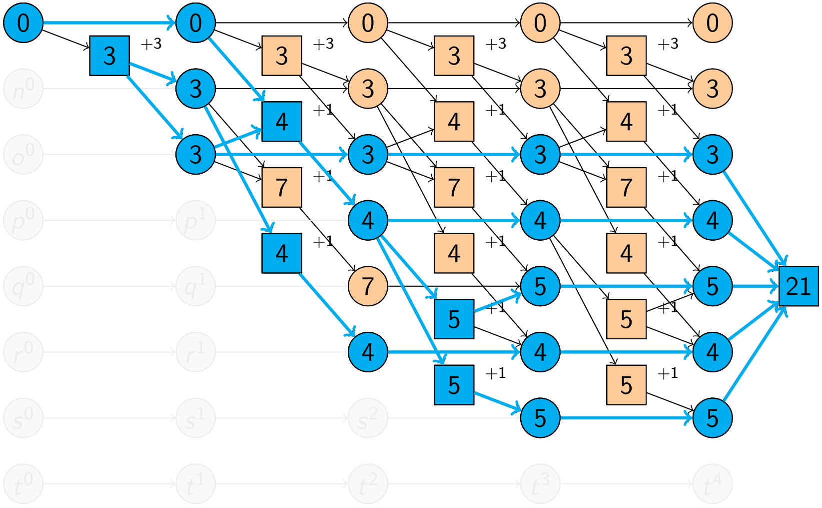

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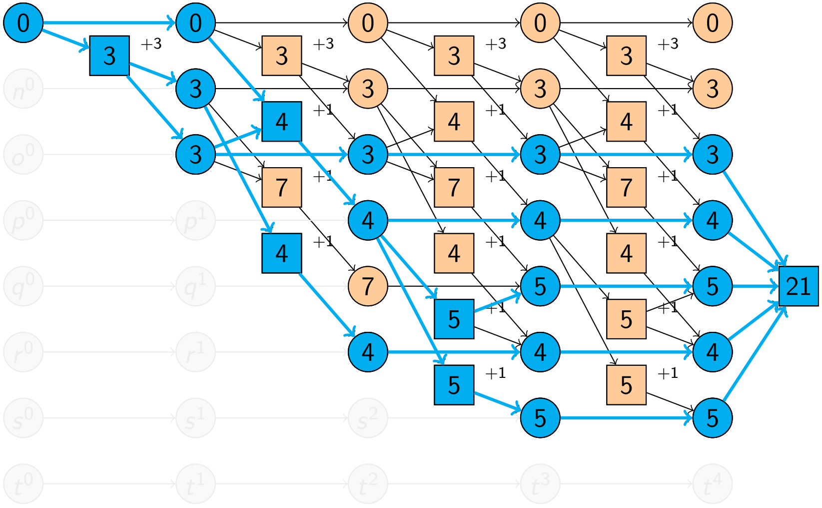

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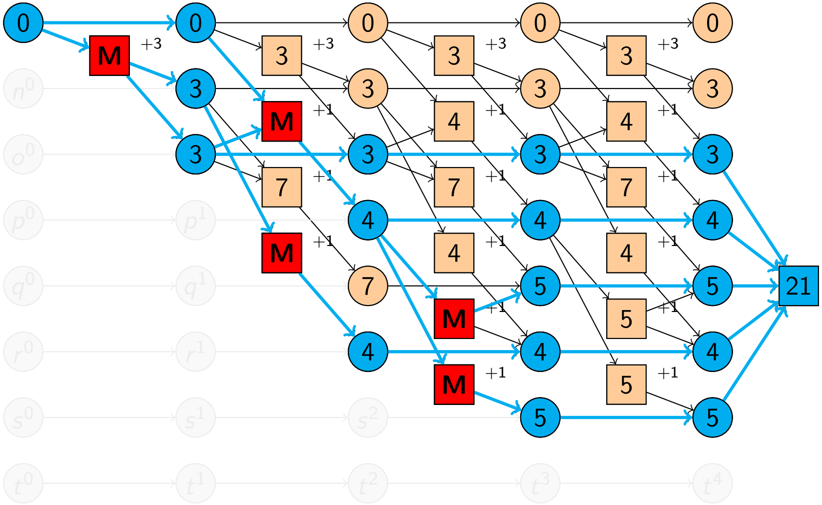

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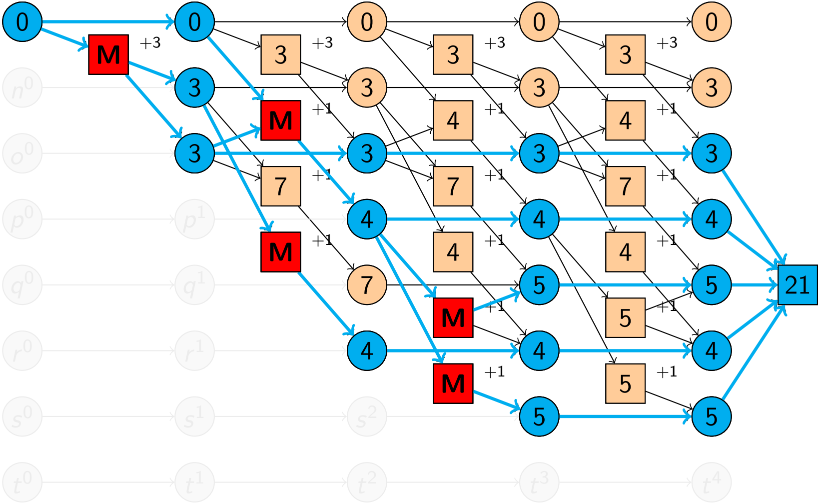

$$h^{\mathsf{FF}}(\{m\}) = 3 + 1 + 1 + 1 + 1 = 7$$

FF Heuristic: Remarks

- Like h^{add}, h^{FF} is safe and goal-aware, but neither admissible nor consistent.
- approximation of h^{+} which is always at least as good as h^{add}
- usually significantly better
- can be computed in almost linear time ($O(n \log n)$) in the size of the description of the planning task

FF Heuristic: Remarks

- Like h^{add}, h^{FF} is safe and goal-aware, but neither admissible nor consistent.
- approximation of h^{+} which is always at least as good as h^{add}
- usually significantly better
- can be computed in almost linear time ($O(n \log n)$) in the size of the description of the planning task
- computation of heuristic value depends on tie-breaking of marking rules (h^{FF} not well-defined)
- one of the most successful planning heuristics

Comparison of Relaxation Heuristics

Relationships of Relaxation Heuristics

Let s be a state in the STRIPS planning task $\langle V, I, G, A \rangle$.

Then

- $h^{\max}(s) \leq h^{+}(s) \leq h^{*}(s)$
- $h^{\max}(s) \leq h^{+}(s) \leq h^{\mathrm{FF}}(s) \leq h^{\mathrm{add}}(s)$
- h^{*} and h^{FF} are incomparable
- h^{*} and h^{add} are incomparable

further remarks:

- For non-admissible heuristics, it is generally neither good nor bad to compute higher values than another heuristic.
- For relaxation heuristics, the objective is to approximate h^{+} as closely as possible.

Summary

Summary

- Many delete relaxation heuristics can be viewed as computations on relaxed planning graphs (RPGs).
- examples: h^{max}, h^{add}, h^{FF}
- h^{max} and h^{add} propagate numeric values in the RPGs
  - difference: h^{max} computes the maximum of predecessor costs for action and goal vertices; h^{add} computes the sum
- h^{FF} marks vertices and sums the costs of marked action vertices.
- generally: $h^{\text{max}}(s) \leq h^{+}(s) \leq h^{\text{FF}}(s) \leq h^{\text{add}}(s)$