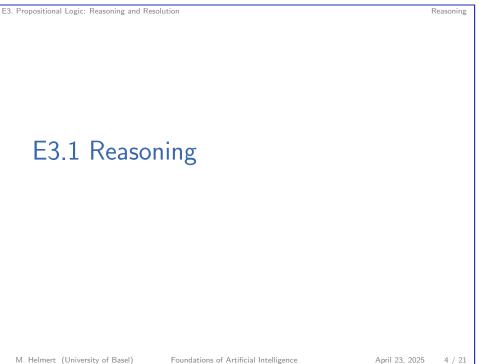
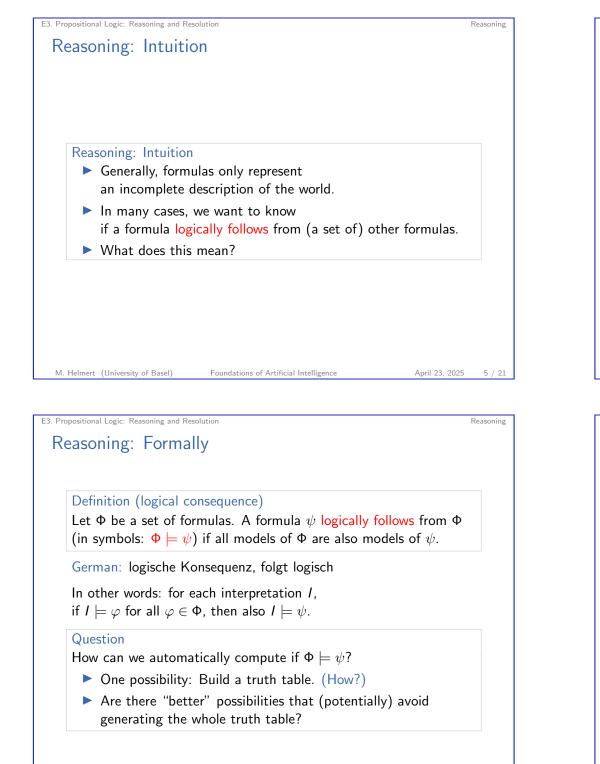


# Propositional Logic: Overview Chapter overview: propositional logic ► E1. Syntax and Semantics ▶ E2. Equivalence and Normal Forms ► E3. Reasoning and Resolution ► E4. DPLL Algorithm ► E5. Local Search and Outlook

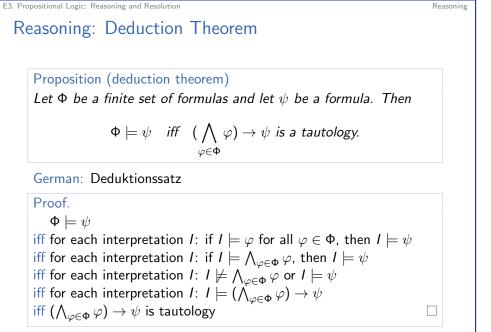
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Propositional Logic: Reasoning and Re			reasoninį
Reasoning: Intuiti	on		
• example: $\varphi =$	$(P \lor Q) \land (R \lor \neg P) \land S$		
S holds in ever	y model of $arphi$ .		
What about P	, Q and R?		
$\rightsquigarrow$ consider all mo	dels of $\varphi$ :		
$ I_2 = \{P \mapsto I_3 \in I_3 \} \} $	$\begin{array}{l} F, Q \mapsto T, R \mapsto F, S \mapsto T \} \\ F, Q \mapsto T, R \mapsto T, S \mapsto T \} \\ T, Q \mapsto F, R \mapsto T, S \mapsto T \} \\ T, Q \mapsto T, R \mapsto T, S \mapsto T \} \end{array}$		
Observation			
In all models o	f $arphi$ , the formula ${\it Q} ee {\it R}$ holds a	s well.	
• We say: " $Q \lor$	R logically follows from $\varphi$ ."		
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Propositional Logic: Reasoning and Re			Dooconing



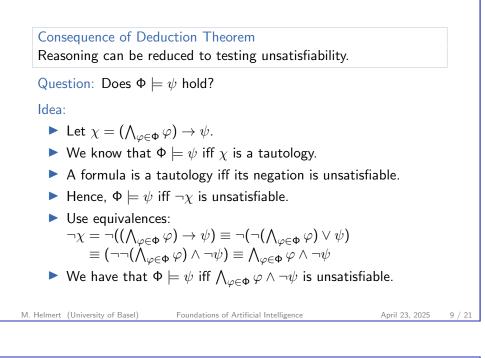
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E3. Propositional Logic: Reasoning and Resolution

Reasoning

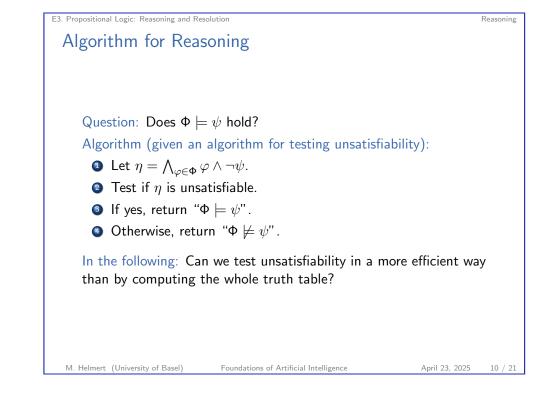


### Reasoning by Unsatisfiability Testing



Reasoning

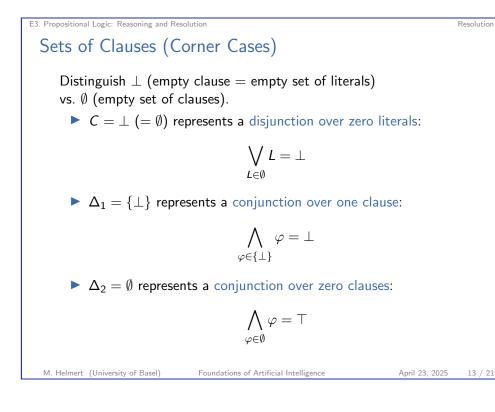




# E3. Propositional Logic: Reasoning and Resolution Resolution Sets of Clauses for the rest of this chapter: prerequisite: formulas in conjunctive normal form clause represented as a set C of literals $\blacktriangleright$ formula represented as a set $\Delta$ of clauses Example Let $\varphi = (P \lor Q) \land \neg P$ . $\blacktriangleright \varphi$ in conjunctive normal form • $\varphi$ consists of clauses ( $P \lor Q$ ) and $\neg P$ • representation of $\varphi$ as set of sets of literals: $\{\{P, Q\}, \{\neg P\}\}$

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E3. Propositional Logic: Reasoning and Resolution

The Resolution Rule

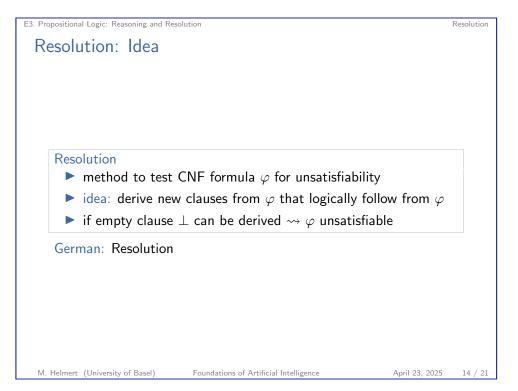
 $\frac{C_1 \cup \{\ell\}, C_2 \cup \{\bar{\ell}\}}{C_1 \cup C_2}$ 

- "From  $C_1 \cup \{\ell\}$  and  $C_2 \cup \{\overline{\ell}\}$ , we can conclude  $C_1 \cup C_2$ ."
- $C_1 \cup C_2$  is resolvent of parent clauses  $C_1 \cup \{\ell\}$  and  $C_2 \cup \{\bar{\ell}\}$ .
- $\blacktriangleright$  The literals  $\ell$  and  $\overline{\ell}$  are called resolution literals. the corresponding proposition is called resolution variable.
- resolvent follows logically from parent clauses (Why?)

German: Resolutionsregel, Resolvent, Elternklauseln, Resolutionsliterale. Resolutionsvariable

#### Example

- ▶ resolvent of  $\{A, B, \neg C\}$  and  $\{A, D, C\}$ ?
- resolvents of  $\{\neg A, B, \neg C\}$  and  $\{A, D, C\}$ ?



# Resolution: Derivations Definition (derivation) Notation: $R(\Delta) = \Delta \cup \{C \mid C \text{ is resolvent of two clauses in } \Delta\}$ A clause D can be derived from $\Delta$ (in symbols $\Delta \vdash D$ ) if there is a sequence of clauses $C_1, \ldots, C_n = D$ such that for all $i \in \{1, \ldots, n\}$ we have $C_i \in R(\Delta \cup \{C_1, ..., C_{i-1}\})$ . German: Ableitung, abgeleitet Lemma (soundness of resolution) If $\Delta \vdash D$ , then $\Delta \models D$ . Does the converse direction hold as well (completeness)?

German: Korrektheit, Vollständigkeit

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E3. Propositional Logic: Reasoning and Resolution

Resolution

### **Resolution:** Completeness?

The converse of the lemma does not hold in general. example:

- $\{\{A, B\}, \{\neg B, C\}\} \models \{A, B, C\}$ , but
- ▶  $\{\{A, B\}, \{\neg B, C\}\} \not\vdash \{A, B, C\}$

but: converse holds for special case of empty clause  $\perp$  (no proof)



German: Widerlegungsvollständigkeit

#### consequences:

- Resolution is a complete proof method for testing unsatisfiability of CNF formulas.
- Resolution can be used for general reasoning by reducing to a test for unsatisfiability of CNF formulas.

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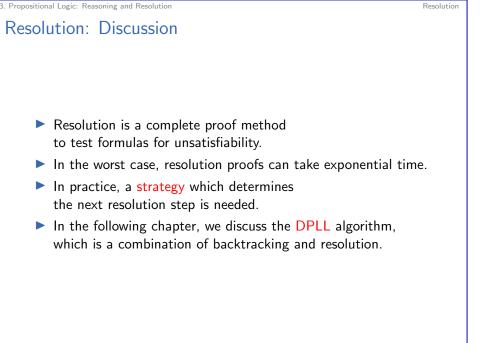
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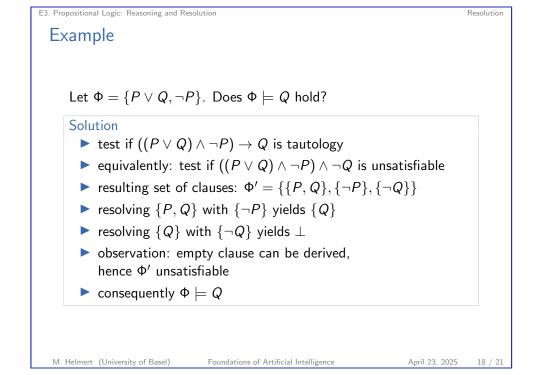
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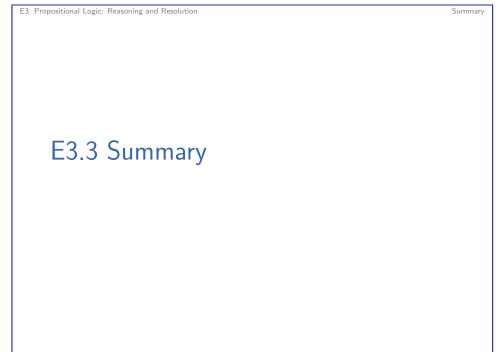
Resolution

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E3. Propositional Logic: Reasoning and Resolution







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E3.	Propositional	Logic:	Reasoning	and	Resolution

Summary

### Summary

- Reasoning: the formula ψ follows from the set of formulas Φ if all models of Φ are also models of ψ.
- Reasoning can be reduced to testing validity (with the deduction theorem).
- Testing validity can be reduced to testing unsatisfiability.
- Resolution is a refutation-complete proof method applicable to formulas in conjunctive normal form.
- $\rightsquigarrow\,$  can be used to test if a set of clauses is unsatisfiable

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