## Foundations of Artificial Intelligence E2. Propositional Logic: Equivalence and Normal Forms

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### Foundations of Artificial Intelligence April 23, 2025 — E2. Propositional Logic: Equivalence and Normal Forms

E2.1 Equivalence

# E2.2 Normal Forms

E2.3 Summary

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# Propositional Logic: Overview

#### Chapter overview: propositional logic

- ▶ E1. Syntax and Semantics
- E2. Equivalence and Normal Forms
- E3. Reasoning and Resolution
- E4. DPLL Algorithm
- E5. Local Search and Outlook

# E2.1 Equivalence

# Logical Equivalance

### Definition (logically equivalent)

Formulas  $\varphi$  and  $\psi$  are called logically equivalent ( $\varphi \equiv \psi$ ) if for all interpretations *I*:  $I \models \varphi$  iff  $I \models \psi$ .

German: logisch äquivalent

# Equivalences

Logical Equivalences Let  $\varphi$ ,  $\psi$ , and  $\eta$  be formulas.  $(\varphi \land \psi) \equiv (\psi \land \varphi)$  and  $(\varphi \lor \psi) \equiv (\psi \lor \varphi)$ (commutativity) ( $(\varphi \land \psi) \land \eta$ )  $\equiv (\varphi \land (\psi \land \eta))$  and  $((\varphi \lor \psi) \lor \eta) \equiv (\varphi \lor (\psi \lor \eta))$ (associativity) ( $(\varphi \land \psi) \lor \eta$ )  $\equiv ((\varphi \lor \eta) \land (\psi \lor \eta))$  and  $((\varphi \lor \psi) \land \eta) \equiv ((\varphi \land \eta) \lor (\psi \land \eta))$ (distributivity)  $\blacktriangleright \neg(\varphi \land \psi) \equiv (\neg \varphi \lor \neg \psi)$  and  $\neg(\varphi \lor \psi) \equiv (\neg \varphi \land \neg \psi)$ (De Morgan)  $\blacktriangleright$   $\neg \neg \varphi \equiv \varphi$ (double negation)  $\blacktriangleright (\varphi \to \psi) \equiv (\neg \varphi \lor \psi)$  $((\rightarrow)$ -elimination)

Commutativity and associativity are often used implicitly  $\rightsquigarrow$  We write  $(X_1 \land X_2 \land X_3 \land X_4)$  instead of  $(X_1 \land (X_2 \land (X_3 \land X_4)))$ 

# E2.2 Normal Forms

## Normal Forms: Terminology

# Definition (literal) If $P \in \Sigma$ , then the formulas P and $\neg P$ are called literals. P is called positive literal, $\neg P$ is called negative literal. The complementary literal to P is $\neg P$ and vice versa. For a literal $\ell$ , the complementary literal to $\ell$ is denoted with $\overline{\ell}$ .

German: Literal, positives/negatives/komplementäres Literal Question: What is the difference between  $\bar{\ell}$  and  $\neg \ell$ ?

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## Normal Forms: Terminology

Definition (clause) A disjunction of 0 or more literals is called a clause. The empty clause (with 0 literals) is  $\perp$ . Clauses consisting of exactly one literal are called unit clauses.

German: Klausel, leere Klausel, Einheitsklausel

Definition (monomial)

A conjunction of 0 or more literals is called a monomial.

German: Monom

### Normal Forms

#### Definition (normal forms)

A formula  $\varphi$  is in conjunctive normal form (CNF, clause form) if  $\varphi$  is a conjunction of 0 or more clauses:

$$\varphi = \bigwedge_{i=1}^{n} \left( \bigvee_{j=1}^{m_i} \ell_{i,j} \right)$$

A formula  $\varphi$  is in disjunctive normal form (DNF) if  $\varphi$  is a disjunction of 0 or more monomials:

$$\varphi = \bigvee_{i=1}^{n} \left( \bigwedge_{j=1}^{m_i} \ell_{i,j} \right)$$

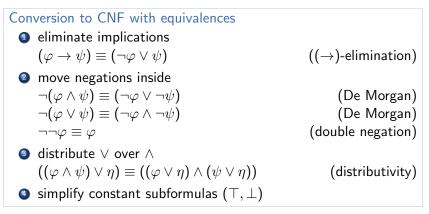
### German: konjunktive Normalform, disjunktive Normalform

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## Normal Forms

For every propositional formula, there exists a logically equivalent propositional formula in CNF and in DNF.



There are formulas  $\varphi$  for which every logically equivalent formula in CNF and DNF is exponentially longer than  $\varphi.$ 

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# E2.3 Summary

## Summary

- two formulas are logically equivalent if they have the same models
- different kinds of formulas:
  - atomic formulas and literals
  - clauses and monomials
  - conjunctive normal form (CNF) and disjunctive normal form (DNF)
- for every formula, there is a logically equivalent formula in CNF and a logically equivalent formula in DNF