

Foundations of Artificial Intelligence

C2. Combinatorial Optimization: Advanced Techniques

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Combinatorial Optimization: Overview

Chapter overview: combinatorial optimization

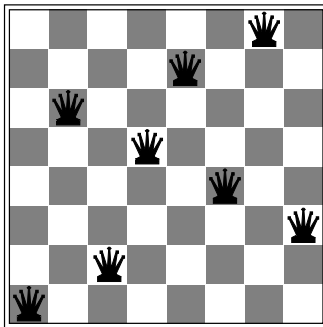
- C1. Introduction and Hill-Climbing
- C2. Advanced Techniques

Dealing with Local Optima

Example: Local Minimum in the 8 Queens Problem

local minimum:

- candidate has 1 conflict
- all neighbors have at least 2



Weaknesses of Local Search Algorithms

difficult situations for hill climbing:

- **local optima:** all neighbors worse than current candidate
- **plateaus:** many neighbors equally good as current candidate; none better

German: lokale Optima, Plateaus

consequence:

- algorithm gets stuck at current candidate

Combating Local Optima

possible remedies to combat local optima:

- allow **stagnation** (steps without improvement)
- include **random aspects** in the **search neighborhood**
- (sometimes) make **random** steps
- **breadth-first search** to better candidate
- **restarts** (with new random initial candidate)

Allowing Stagnation

allowing stagnation:

- do not terminate when no neighbor is an improvement
- limit number of steps to guarantee termination
- at end, return best visited candidate
 - pure search problems: terminate as soon as solution found

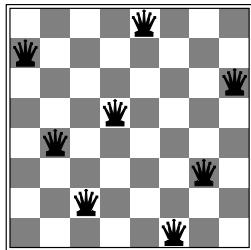
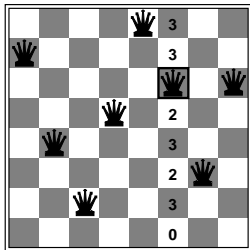
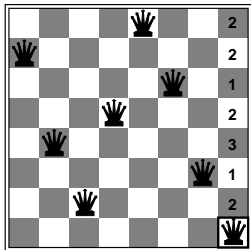
Example 8 queens problem:

- with a bound of 100 steps solution found in **96%** of the cases
 - on average 22 steps until solution found
- ↪ works very well for this problem;
for more difficult problems often not good enough

Random Aspects in the Search Neighborhood

a possible variation of hill climbing for 8 queens:

Randomly select a file; move queen in this file to square with minimal number of conflicts (null move possible).



↪ Good local search approaches often combine **randomness (exploration)** with **heuristic guidance (exploitation)**.

German: Exploration, Exploitation

Outlook: Simulated Annealing

Simulated Annealing

Simulated annealing is a local search algorithm that systematically injects **noise**, beginning with high noise, then lowering it over time.

- walk with fixed number of steps N (variations possible)
- initially it is “hot”, and the walk is mostly random
- over time temperature drops (controlled by a **schedule**)
- as it gets colder, moves to worse neighbors become less likely

very successful in some applications, e.g., VLSI layout

German: simulierte Abkühlung, Rauschen

Simulated Annealing: Pseudo-Code

Simulated Annealing (for Maximization Problems)

curr := a random candidate

best := **none**

for each $t \in \{1, \dots, N\}$:

if *is_solution*(*curr*) **and** (**best is none or** $v(curr) > v(best)$):

best := *curr*

$T := \text{schedule}(t)$

next := a random neighbor of *curr*

$\Delta E := h(next) - h(curr)$

if $\Delta E \geq 0$ **or** with probability $e^{\frac{\Delta E}{T}}$:

curr := *next*

return *best*

Outlook: Genetic Algorithms

Genetic Algorithms

Evolution often finds good solutions.

idea: simulate evolution by **selection**, **crossover** and **mutation** of individuals

ingredients:

- encode each candidate as a string of symbols (**genome**)
- **fitness function:** evaluates strength of candidates (= heuristic)
- **population** of k (e.g. 10–1000) **individuals** (candidates)

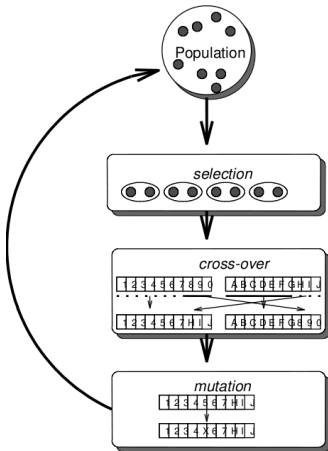
German: Evolution, Selektion, Kreuzung, Mutation, Genom, Fitnessfunktion, Population, Individuen

Genetic Algorithm: Example

example 8 queens problem:

- **genome:** encode candidate as string of 8 numbers
- **fitness:** number of non-attacking queen pairs
- use population of 100 candidates

Selection, Mutation and Crossover



many variants:

How to select?

How to perform crossover?

How to mutate?

select according to fitness function,
followed by pairing

determine crossover points,
then recombine

mutation: randomly modify
each string position with
a certain probability

Summary

Summary

- weakness of local search: **local optima** and **plateaus**
- remedy: balance **exploration** against **exploitation** (e.g., with **randomness** and **restarts**)
- **simulated annealing** and **genetic algorithms** are more complex search algorithms using the typical ideas of local search (randomization, keeping promising candidates)