

# Foundations of Artificial Intelligence

## C2. Combinatorial Optimization: Advanced Techniques

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April 2, 2025 — C2. Combinatorial Optimization: Advanced Techniques

C2.1 Dealing with Local Optima

C2.2 Outlook: Simulated Annealing

C2.3 Outlook: Genetic Algorithms

C2.4 Summary

# Combinatorial Optimization: Overview

## Chapter overview: combinatorial optimization

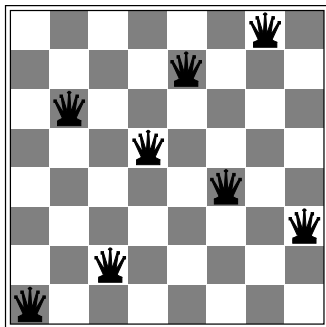
- ▶ C1. Introduction and Hill-Climbing
- ▶ C2. Advanced Techniques

## C2.1 Dealing with Local Optima

# Example: Local Minimum in the 8 Queens Problem

local minimum:

- ▶ candidate has 1 conflict
- ▶ all neighbors have at least 2



# Weaknesses of Local Search Algorithms

difficult situations for hill climbing:

- ▶ **local optima**: all neighbors worse than current candidate
- ▶ **plateaus**: many neighbors equally good as current candidate; none better

German: lokale Optima, Plateaus

consequence:

- ▶ algorithm gets stuck at current candidate

# Combating Local Optima

possible remedies to combat local optima:

- ▶ allow **stagnation** (steps without improvement)
- ▶ include **random aspects** in the **search neighborhood**
- ▶ (sometimes) make **random** steps
- ▶ **breadth-first search** to better candidate
- ▶ **restarts** (with new random initial candidate)

# Allowing Stagnation

allowing stagnation:

- ▶ do not terminate when no neighbor is an improvement
- ▶ limit number of steps to guarantee termination
- ▶ at end, return best visited candidate
  - ▶ pure search problems: terminate as soon as solution found

Example 8 queens problem:

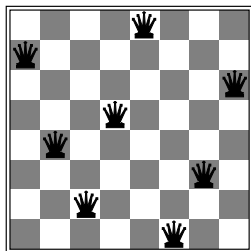
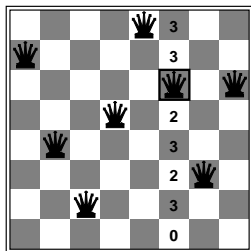
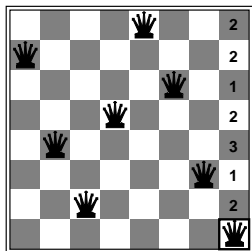
- ▶ with a bound of 100 steps solution found in 96% of the cases
- ▶ on average 22 steps until solution found
- ↪ works very well for this problem;  
for more difficult problems often not good enough



# Random Aspects in the Search Neighborhood

a possible variation of hill climbing for 8 queens:

**Randomly** select a file; move queen in this file to square with minimal number of conflicts (null move possible).



↪ Good local search approaches often combine **randomness (exploration)** with **heuristic guidance (exploitation)**.

**German:** Exploration, Exploitation

## C2.2 Outlook: Simulated Annealing

# Simulated Annealing

**Simulated annealing** is a local search algorithm that systematically injects **noise**, beginning with high noise, then lowering it over time.

- ▶ walk with fixed number of steps  $N$  (variations possible)
- ▶ initially it is “hot”, and the walk is mostly random
- ▶ over time temperature drops (controlled by a **schedule**)
- ▶ as it gets colder, moves to worse neighbors become less likely

very successful in some applications, e.g., VLSI layout

**German:** simulierte Abkühlung, Rauschen

# Simulated Annealing: Pseudo-Code

## Simulated Annealing (for Maximization Problems)

*curr* := a random candidate

*best* := **none**

**for each**  $t \in \{1, \dots, N\}$ :

**if** `is_solution`(*curr*) **and** (**best is none** **or**  $v(\text{curr}) > v(\text{best})$ ):

*best* := *curr*

$T$  := `schedule`( $t$ )

*next* := a random neighbor of *curr*

$\Delta E$  :=  $h(\text{next}) - h(\text{curr})$

**if**  $\Delta E \geq 0$  **or** with probability  $e^{\frac{\Delta E}{T}}$ :

*curr* := *next*

**return** *best*

## C2.3 Outlook: Genetic Algorithms

# Genetic Algorithms

**Evolution** often finds good solutions.

**idea:** simulate evolution by **selection**, **crossover** and **mutation** of individuals

**ingredients:**

- ▶ encode each candidate as a string of symbols (**genome**)
- ▶ **fitness function:** evaluates strength of candidates (= heuristic)
- ▶ **population** of  $k$  (e.g. 10–1000) **individuals** (candidates)

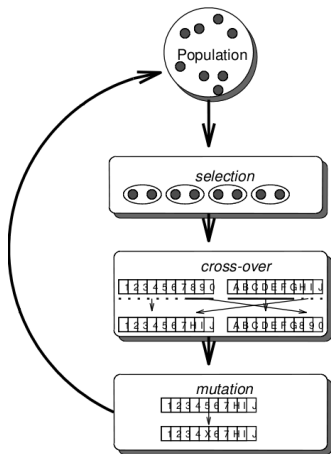
**German:** Evolution, Selektion, Kreuzung, Mutation, Genom, Fitnessfunktion, Population, Individuen

# Genetic Algorithm: Example

example 8 queens problem:

- ▶ **genome**: encode candidate as string of 8 numbers
- ▶ **fitness**: number of non-attacking queen pairs
- ▶ use population of 100 candidates

# Selection, Mutation and Crossover



many variants:

How to select?

How to perform crossover?

How to mutate?

select according to fitness function,  
followed by pairing

determine crossover points,  
then recombine

mutation: randomly modify  
each string position with  
a certain probability



## C2.4 Summary

# Summary

- ▶ weakness of local search: **local optima** and **plateaus**
- ▶ remedy: balance **exploration** against **exploitation** (e.g., with **randomness** and **restarts**)
- ▶ **simulated annealing** and **genetic algorithms** are more complex search algorithms using the typical ideas of local search (randomization, keeping promising candidates)