Foundations of Artificial Intelligence

B13. State-Space Search: IDA*

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms
  - B9. Heuristics
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  - B11. Best-first Graph Search
  - B12. Greedy Best-first Search, A*, Weighted A*
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  - B14. Properties of A*, Part I
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IDA*: Idea

IDA*

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Idea: use the concepts of iterative-deepening DFS

IDA*

The main drawback of the presented best-first graph search algorithms is their space complexity.

Idea: use the concepts of iterative-deepening DFS

- depth-limited search with increasing limits
- instead of depth we limit f
  (in this chapter $f(n) := g(n) + h(n.\text{state})$ as in A*)

$\rightsquigarrow$ IDA* (iterative-deepening A*)

- tree search, unlike the previous best-first search algorithms

IDA*: Algorithm

Reminder: Iterative Deepening Depth-first Search

reminder from Chapter B8: iterative deepening depth-first search

Iterative Deepening DFS

for $depth\_limit \in \{0, 1, 2, \dots\}$:
    $solution :=$ depth_limited_search(init(), $depth\_limit$)
    if $solution \neq$ **none**:
        return $solution$

function depth_limited_search(s, $depth\_limit$):

if is_goal(s):
    return $\langle\rangle$
if $depth\_limit > 0$:
    for each $\langle a, s' \rangle \in$ succ(s):
        $solution :=$ depth_limited_search(s', $depth\_limit - 1$)
        if $solution \neq$ **none**:
            $solution$.push_front(a)
            return $solution$
return none

First Attempt: IDA* Main Function

first attempt: iterative deepening A* (IDA*)

IDA* (First Attempt)

for *f_limit* $\in \{0, 1, 2, \dots\}$:
    solution := f_limited_search(init(), 0, *f_limit*)
    if *solution* $\neq$ **none**:
        return *solution*

First Attempt: f-Limited Search

function f_limited_search(s, g, *f_limit*):

if $g + h(s) > f\_limit$:
    return none
if is_goal(s):
    return $\langle\rangle$
for each $\langle a, s'\rangle \in$ succ(s):
    solution := f_limited_search(s', $g + \text{cost}(a)$, *f_limit*)
    if *solution* $\neq$ **none**:
        solution.push_front(a)
        return *solution*
return none

IDA* First Attempt: Discussion

- The pseudo-code can be rewritten to be even more similar to our IDDFS pseudo-code. However, this would make our next modification more complicated.
- The algorithm follows the same principles as IDDFS, but takes path costs and heuristic information into account.
- For unit-cost state spaces and the trivial heuristic $h : s \mapsto 0$ for all states s, it behaves identically to IDDFS.
- For general state spaces, there is a problem with this first attempt, however.

Growing the f Limit

- In IDDFS, we grow the limit from the smallest limit that gives a non-empty search tree (0) by 1 at a time.
- This usually leads to exponential growth of the tree between rounds, so that re-exploration work can be amortized.
- In our first attempt at IDA*, there is no guarantee that increasing the f limit by 1 will lead to a larger search tree than in the previous round.
- This problem becomes worse if we also allow non-integer (fractional) costs, where increasing the limit by 1 would be very arbitrary.

Setting the Next f Limit

idea: let the f-limited search compute the next sensible f limit

- Start with $h(\text{init}())$, the smallest f limit that results in a non-empty search tree.
- In every round, increase the f limit to the smallest value that ensures that in the next round at least one additional path will be considered by the search.

⇝ f_limited_search now returns two values:

- the next f limit that would include at least one new node in the search tree (∞ if no such limit exists; **none** if a solution was found), and
- the solution that was found (or **none**).

Final Algorithm: IDA* Main Function

final algorithm: iterative deepening A* (IDA*)

IDA*

$f\_limit = h(\text{init}())$
while $f\_limit \neq \infty$:
    $\langle f\_limit, solution \rangle := \text{f\_limited\_search}(\text{init}(), 0, f\_limit)$
    if $solution \neq$ **none**:
        return $solution$
return unsolvable

Final Algorithm: f-Limited Search

function f_limited_search(s, g, f_limit):

```
if g + h(s) > f_limit:
    return ⟨g + h(s), none⟩
if is_goal(s):
    return ⟨none, ⟨⟩⟩
new_limit := ∞
for each ⟨a, s'⟩ ∈ succ(s):
    ⟨child_limit, solution⟩ := f_limited_search(s', g + cost(a), f_limit)
    if solution ≠ none:
        solution.push_front(a)
        return ⟨none, solution⟩
    new_limit := min(new_limit, child_limit)
return ⟨new_limit, none⟩
```

Final Algorithm: f-Limited Search

function f_limited_search(s, g, f_limit):

if $g + h(s) > f\_limit$:
    return $\langle g + h(s), \textbf{none}\rangle$
if is_goal(s):
    return $\langle \textbf{none}, \langle\rangle\rangle$
$new\_limit := \infty$
for each $\langle a, s'\rangle \in$ succ(s):
    $\langle child\_limit, solution\rangle :=$ f_limited_search(s', $g + \text{cost}(a)$, f_limit)
    if $solution \neq$ **none**:
        $solution$.push_front(a)
        return $\langle \textbf{none}, solution\rangle$
    $new\_limit := \min(new\_limit, child\_limit)$
return $\langle new\_limit, \textbf{none}\rangle$

IDA*: Properties

IDA*: Properties

Inherits important properties of A* and depth-first search:

- semi-complete if h safe and $cost(a) > 0$ for all actions a
- optimal if h admissible
- space complexity $O(\ell b)$, where
  - ℓ: length of longest generated path
    (for unit cost problems: bounded by optimal solution cost)
  - b: branching factor

We state these without proof.

IDA*: Discussion

- compared to A^* potentially considerable overhead because no duplicates are detected
  - ⇝ exponentially slower in many state spaces
  - ⇝ often combined with partial duplicate elimination (cycle detection, transposition tables)
- overhead due to iterative increases of f limit often negligible, but not always
  - especially problematic if action costs vary a lot: then it can easily happen that each new f limit only considers a small number of new paths

Summary

Summary

- IDA* is a tree search variant of A*
  based on iterative deepening depth-first search
- main advantage: low space complexity
- disadvantage: repeated work can be significant
- most useful when there are few duplicates