Foundations of Artificial Intelligence

B11. State-Space Search: Best-first Graph Search

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms
  - B9. Heuristics
  - B10. Analysis of Heuristics
  - B11. Best-first Graph Search
  - B12. Greedy Best-first Search, A^*, Weighted A^*
  - B13. IDA^*
  - B14. Properties of A^*, Part I
  - B15. Properties of A^*, Part II

Introduction

Heuristic Search Algorithms

Heuristic Search Algorithms

Heuristic search algorithms use heuristic functions
to (partially or fully) determine the order of node expansion.

German: heuristische Suchalgorithmen

- this chapter: short introduction
- next chapters: more thorough analysis

Best-first Search

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Best-first Search

A best-first search is a heuristic search algorithm that evaluates search nodes with an evaluation function f and always expands a node n with minimal $f(n)$ value.

German: Bestensuche, Bewertungsfunktion

- implementation essentially like uniform cost search
- different choices of f $\rightsquigarrow$ different search algorithms

The Most Important Best-first Search Algorithms

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- $f(n) = g(n) + w \cdot h(n.\text{state})$: weighted A^*
  $w \in \mathbb{R}_0^+$ is a parameter
  $\rightsquigarrow$ interpolates between greedy best-first search and A^*

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What do we obtain with $f(n) := g(n)$?

Best-first Search: Graph Search or Tree Search?

Best-first search can be graph search or tree search.

- now: graph search (i.e., with duplicate elimination), which is the more common case
- Chapter B13: a tree search variant

Algorithm Details

Reminder: Uniform Cost Search

reminder from Chapter B7:

Uniform Cost Search

open := **new** MinHeap ordered by g
open.insert(make_root_node())
closed := **new** HashSet
while not *open*.is_empty():
        n := *open*.pop_min()
        if n.state $\notin$ *closed*:
                closed.insert(n.state)
                if is_goal(n.state):
                        return extract_path(n)
                for each $\langle a, s' \rangle \in$ succ(n.state):
                        n' := make_node(n, a, s')
                        open.insert(n')
return unsolvable

Best-first Search without Reopening (1st Attempt)

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This is already an acceptable implementation of best-first search.

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two useful improvements:

- discard states considered unsolvable by the heuristic
  $\rightsquigarrow$ saves memory in *open*
- if multiple search nodes have identical f values,
  use h to break ties (preferring low h)
  - not always a good idea, but often
  - obviously unnecessary if $f = h$ (greedy best-first search)

Best-first Search without Reopening (Final Version)

Best-first Search without Reopening

```
open := new MinHeap ordered by ⟨f, h⟩
if h(init()) < ∞:
    open.insert(make_root_node())
closed := new HashSet
while not open.is_empty():
    n := open.pop_min()
    if n.state ∉ closed:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            if h(s′) < ∞:
                n′ := make_node(n, a, s′)
                open.insert(n′)
return unsolvable
```

Best-first Search: Properties

properties:

- complete if h is safe (Why?)
- optimality depends on f $\rightsquigarrow$ next chapters

Reopening

Reopening

- reminder: uniform cost search expands nodes in order of increasing g values

⇝ guarantees that cheapest path to state of a node has been found when the node is expanded

- with arbitrary evaluation functions f in best-first search this does not hold in general

⇝ in order to find solutions of low cost, we may want to expand duplicate nodes when cheaper paths to their states are found (reopening)

German: Reopening

Best-first Search with Reopening

Best-first Search with Reopening

```
open := new MinHeap ordered by ⟨f, h⟩
if h(init()) < ∞:
    open.insert(make_root_node())
distances := new HashMap
while not open.is_empty():
    n := open.pop_min()
    if distances.lookup(n.state) = none or g(n) < distances[n.state]:
        distances[n.state] := g(n)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s'⟩ ∈ succ(n.state):
            if h(s') < ∞:
                n' := make_node(n, a, s')
                open.insert(n')
return unsolvable
```

$\rightsquigarrow$ *distances* controls reopening and replaces *closed*

Summary

Summary

- best-first search: expand node with minimal value of evaluation function f
  - $f = h$: greedy best-first search
  - $f = g + h$: A^*
  - $f = g + w \cdot h$ with parameter $w \in \mathbb{R}_0^+$: weighted A^*
- here: best-first search as a graph search
- reopening: expand duplicates with lower path costs to find cheaper solutions