Foundations of Artificial Intelligence

B11. State-Space Search: Best-first Graph Search

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms
  - B9. Heuristics
  - B10. Analysis of Heuristics
  - B11. Best-first Graph Search
  - B12. Greedy Best-first Search, A*, Weighted A*
  - B13. IDA*
  - B14. Properties of A*, Part I
  - B15. Properties of A*, Part II

B11.1 Introduction

Heuristic Search Algorithms

Heuristic Search Algorithms

Heuristic search algorithms use heuristic functions to (partially or fully) determine the order of node expansion.

German: heuristische Suchalgorithmen

- this chapter: short introduction
- next chapters: more thorough analysis

B11.2 Best-first Search

Best-first Search

Best-first search is a class of search algorithms that expand the "most promising" node in each iteration.

- decision which node is most promising uses heuristics...
- ...but not necessarily exclusively.

Best-first Search

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Best-first Search

A best-first search is a heuristic search algorithm that evaluates search nodes with an evaluation function f and always expands a node n with minimal $f(n)$ value.

German: Bestensuche, Bewertungsfunktion

- implementation essentially like uniform cost search
- different choices of $f \rightsquigarrow$ different search algorithms

The Most Important Best-first Search Algorithms

the most important best-first search algorithms:

- $f(n) = h(n.\text{state})$: greedy best-first search
  ⇝ only the heuristic counts
- $f(n) = g(n) + h(n.\text{state})$: A*
  ⇝ combination of path cost and heuristic
- $f(n) = g(n) + w \cdot h(n.\text{state})$: weighted A*
  $w \in \mathbb{R}_0^+$ is a parameter
  ⇝ interpolates between greedy best-first search and A*

German: gierige Bestensuche, A*, Weighted A*
⇝ properties: next chapters

What do we obtain with $f(n) := g(n)$?

Best-first Search: Graph Search or Tree Search?

Best-first search can be graph search or tree search.

- now: graph search (i.e., with duplicate elimination), which is the more common case
- Chapter B13: a tree search variant

B11.3 Algorithm Details

Reminder: Uniform Cost Search

reminder from Chapter B7:

Uniform Cost Search

```
open := new MinHeap ordered by g
open.insert(make_root_node())
closed := new HashSet
while not open.is_empty():
    n := open.pop_min()
    if n.state ∉ closed:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s'⟩ ∈ succ(n.state):
            n' := make_node(n, a, s')
            open.insert(n')
return unsolvable
```

Best-first Search without Reopening (1st Attempt)

Best-first Search without Reopening (1st Attempt)

```
open := new MinHeap ordered by f
open.insert(make_root_node())
closed := new HashSet
while not open.is_empty():
    n := open.pop_min()
    if n.state ∉ closed:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            n′ := make_node(n, a, s′)
            open.insert(n′)
return unsolvable
```


Best-first Search w/o Reopening (1st Attempt): Discussion

Discussion:

This is already an acceptable implementation of best-first search.

two useful improvements:

- discard states considered unsolvable by the heuristic
  ⇝ saves memory in *open*
- if multiple search nodes have identical f values, use h to break ties (preferring low h)
  - not always a good idea, but often
  - obviously unnecessary if $f = h$ (greedy best-first search)

Best-first Search without Reopening (Final Version)

Best-first Search without Reopening

```
open := new MinHeap ordered by ⟨f, h⟩
if h(init()) < ∞:
    open.insert(make_root_node())
closed := new HashSet
while not open.is_empty():
    n := open.pop_min()
    if n.state ∉ closed:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            if h(s′) < ∞:
                n′ := make_node(n, a, s′)
                open.insert(n′)
return unsolvable
```


Best-first Search: Properties

properties:

- complete if h is safe (Why?)
- optimality depends on f ⇝ next chapters

B11.4 Reopening

Reopening

- reminder: uniform cost search expands nodes in order of increasing g values

⇝ guarantees that cheapest path to state of a node has been found when the node is expanded

- with arbitrary evaluation functions f in best-first search this does not hold in general

⇝ in order to find solutions of low cost, we may want to expand duplicate nodes when cheaper paths to their states are found (reopening)

German: Reopening

Best-first Search with Reopening

Best-first Search with Reopening

```
open := new MinHeap ordered by ⟨f, h⟩
if h(init()) < ∞:
    open.insert(make_root_node())
distances := new HashMap
while not open.is_empty():
    n := open.pop_min()
    if distances.lookup(n.state) = none or g(n) < distances[n.state]:
        distances[n.state] := g(n)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            if h(s′) < ∞:
                n′ := make_node(n, a, s′)
                open.insert(n′)
return unsolvable
```

⇝ *distances* controls reopening and replaces *closed*

B11.5 Summary

Summary

- best-first search: expand node with minimal value of evaluation function f
  - $f = h$: greedy best-first search
  - $f = g + h$: A^*
  - $f = g + w \cdot h$ with parameter $w \in \mathbb{R}_0^+$: weighted A^*
- here: best-first search as a graph search
- reopening: expand duplicates with lower path costs to find cheaper solutions