

Foundations of Artificial Intelligence

B10. State-Space Search: Analysis of Heuristics

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms
 - B9. Heuristics
 - B10. Analysis of Heuristics
 - B11. Best-first Graph Search
 - B12. Greedy Best-first Search, A^* , Weighted A^*
 - B13. IDA*
 - B14. Properties of A^* , Part I
 - B15. Properties of A^* , Part II

Reminder: Heuristics

Definition (heuristic)

Let \mathcal{S} be a state space with states S .

A **heuristic function** or **heuristic** for \mathcal{S} is a function

$$h : \mathcal{S} \rightarrow \mathbb{R}_0^+ \cup \{\infty\},$$

mapping each state to a nonnegative number (or ∞).

Properties of Heuristics

Perfect Heuristic

Definition (perfect heuristic)

Let \mathcal{S} be a state space with states S .

The **perfect heuristic** for \mathcal{S} , written h^* , maps each state $s \in S$

- to the cost of an **optimal solution** for s , or
- to ∞ if no solution for s exists.

German: perfekte Heuristik

Properties of Heuristics

Definition (safe, goal-aware, admissible, consistent)

Let \mathcal{S} be a state space with states S .

A heuristic h for \mathcal{S} is called

- **safe** if $h^*(s) = \infty$ for all $s \in S$ with $h(s) = \infty$
- **goal-aware** if $h(s) = 0$ for all goal states s
- **admissible** if $h(s) \leq h^*(s)$ for all states $s \in S$
- **consistent** if $h(s) \leq \text{cost}(a) + h(s')$ for all transitions $s \xrightarrow{a} s'$

German: sicher, zielerkennend, zulässig, konsistent

Properties of Heuristics

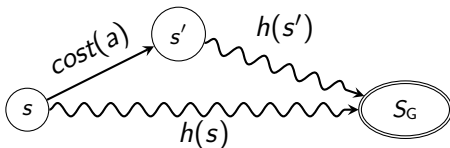
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Examples

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Route Planning in Romania

straight-line distance:

- safe
- goal-aware
- admissible
- consistent

Why?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Blocks World

misplaced blocks:

- safe?
- goal-aware?
- admissible?
- consistent?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Missionaries and Cannibals

people on wrong river bank:

- safe?
- goal-aware?
- admissible?
- consistent?

Connections

Properties of Heuristics: Connections (1)

Theorem (admissible \implies safe + goal-aware)

Let h be an admissible heuristic.

Then h is safe and goal-aware.

Why?

Properties of Heuristics: Connections (2)

Theorem (goal-aware + consistent \implies admissible)

Let h be a goal-aware and consistent heuristic.

Then h is admissible.

Why?

Showing All Four Properties

How can one show most easily that a heuristic has all four properties?

Summary

Summary

- **perfect heuristic h^*** : true cost to the goal
- important properties: **safe, goal-aware, admissible, consistent**
- **connections** between these properties
 - admissible \implies safe and goal-aware
 - goal-aware and consistent \implies admissible