

# Foundations of Artificial Intelligence

## B10. State-Space Search: Analysis of Heuristics

Malte Helmert

University of Basel

March 19, 2025

# Foundations of Artificial Intelligence

March 19, 2025 — B10. State-Space Search: Analysis of Heuristics

## B10.1 Properties of Heuristics

## B10.2 Examples

## B10.3 Connections

## B10.4 Summary

## State-Space Search: Overview

### Chapter overview: state-space search

- ▶ B1–B3. Foundations
- ▶ B4–B8. Basic Algorithms
- ▶ B9–B15. Heuristic Algorithms
  - ▶ B9. Heuristics
  - ▶ B10. Analysis of Heuristics
  - ▶ B11. Best-first Graph Search
  - ▶ B12. Greedy Best-first Search, A\*, Weighted A\*
  - ▶ B13. IDA\*
  - ▶ B14. Properties of A\*, Part I
  - ▶ B15. Properties of A\*, Part II

## Reminder: Heuristics

### Definition (heuristic)

Let  $S$  be a state space with states  $S$ .

A **heuristic function** or **heuristic** for  $S$  is a function

$$h : S \rightarrow \mathbb{R}_0^+ \cup \{\infty\},$$

mapping each state to a nonnegative number (or  $\infty$ ).

## B10.1 Properties of Heuristics

## Perfect Heuristic

### Definition (perfect heuristic)

Let  $\mathcal{S}$  be a state space with states  $S$ .

The **perfect heuristic** for  $\mathcal{S}$ , written  $h^*$ , maps each state  $s \in S$

- ▶ to the cost of an **optimal solution** for  $s$ , or
- ▶ to  $\infty$  if no solution for  $s$  exists.

German: perfekte Heuristik

## Properties of Heuristics

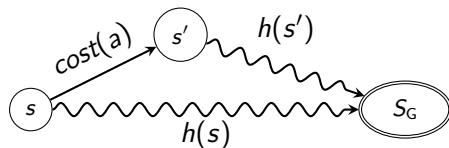
### Definition (safe, goal-aware, admissible, consistent)

Let  $\mathcal{S}$  be a state space with states  $S$ .

A heuristic  $h$  for  $\mathcal{S}$  is called

- ▶ **safe** if  $h^*(s) = \infty$  for all  $s \in S$  with  $h(s) = \infty$
- ▶ **goal-aware** if  $h(s) = 0$  for all goal states  $s$
- ▶ **admissible** if  $h(s) \leq h^*(s)$  for all states  $s \in S$
- ▶ **consistent** if  $h(s) \leq \text{cost}(a) + h(s')$  for all transitions  $s \xrightarrow{a} s'$

German: sicher, zielerkennend, zulässig, konsistent



## B10.2 Examples

## Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Route Planning in Romania

straight-line distance:

- ▶ safe
- ▶ goal-aware
- ▶ admissible
- ▶ consistent

Why?

## Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Blocks World

misplaced blocks:

- ▶ safe?
- ▶ goal-aware?
- ▶ admissible?
- ▶ consistent?

## Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Missionaries and Cannibals

people on wrong river bank:

- ▶ safe?
- ▶ goal-aware?
- ▶ admissible?
- ▶ consistent?

## B10.3 Connections

## Properties of Heuristics: Connections (1)

Theorem (admissible  $\implies$  safe + goal-aware)

*Let  $h$  be an admissible heuristic.*

*Then  $h$  is safe and goal-aware.*

Why?

## Properties of Heuristics: Connections (2)

Theorem (goal-aware + consistent  $\implies$  admissible)

*Let  $h$  be a goal-aware and consistent heuristic.*

*Then  $h$  is admissible.*

Why?

## Showing All Four Properties

How can one show most easily that a heuristic  
has all four properties?

## B10.4 Summary

## Summary

- ▶ **perfect heuristic  $h^*$** : true cost to the goal
- ▶ important properties: **safe**, **goal-aware**, **admissible**, **consistent**
- ▶ **connections** between these properties
  - ▶ admissible  $\implies$  safe and goal-aware
  - ▶ goal-aware and consistent  $\implies$  admissible