Foundations of Artificial Intelligence

B7. State-Space Search: Uniform Cost Search

Malte Helmert

University of Basel

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State-Space Search: Overview

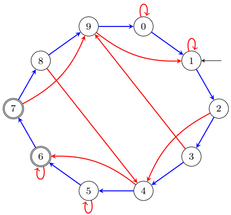
Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
    - B4. Data Structures for Search Algorithms
    - B5. Tree Search and Graph Search
    - B6. Breadth-first Search
    - B7. Uniform Cost Search
    - B8. Depth-first Search and Iterative Deepening
- B9–B15. Heuristic Algorithms

Introduction

Uniform Cost Search

- breadth-first search optimal if all action costs equal
- otherwise no optimality guarantee ⇝ example:



- consider bounded inc-and-square problem with $cost(inc) = 1, cost(sqr) = 3$
- solution of breadth-first search still $\langle inc, sqr, sqr \rangle$ (cost: 7)
- but: $\langle inc, inc, inc, inc, inc \rangle$ (cost: 5) is cheaper!

Uniform Cost Search

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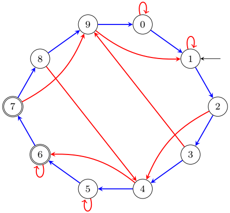

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- but: $\langle inc, inc, inc, inc, inc \rangle$ (cost: 5) is cheaper!

remedy: uniform cost search

- always expand a node with minimal path cost (n.path_cost a.k.a. $g(n)$)
- implementation: priority queue (min-heap) for open list

Algorithm

Reminder: Generic Graph Search Algorithm

reminder from Chapter B5:

Generic Graph Search

```
$open$ := new OpenList
$open$.insert(make_root_node())
$closed$ := new ClosedList
while not $open$.is_empty():
    $n$ := $open$.pop()
    if $closed$.lookup($n$.state) = none:
        $closed$.insert($n$)
        if is_goal($n$.state):
            return extract_path($n$)
        for each $\langle a, s' \rangle \in$ succ($n$.state):
            $n'$ := make_node($n, a, s'$)
            $open$.insert($n'$)
return unsolvable
```

Uniform Cost Search

Uniform Cost Search

```
$open$ := new MinHeap ordered by $g$
$open$.insert(make_root_node())
$closed$ := new HashSet
while not $open$.is_empty():
    $n$ := $open$.pop_min()
    if $n$.state $\notin$ $closed$:
        $closed$.insert($n$.state)
        if is_goal($n$.state):
            return extract_path($n$)
        for each $\langle a, s' \rangle \in$ succ($n$.state):
            $n'$ := make_node($n$, $a$, $s'$)
            $open$.insert($n'$)
return unsolvable
```

Uniform Cost Search: Discussion

Adapting generic graph search to uniform cost search:

- here, early goal tests/early updates of the closed list not a good idea. (Why not?)
- as in BFS-Graph, a set is sufficient for the closed list
- a tree search variant is possible, but rare:
  has the same disadvantages as BFS-Tree
  and in general not even semi-complete (Why not?)

Remarks:

- identical to Dijkstra's algorithm for shortest paths
- for both: variants with/without delayed duplicate elimination

Example

bounded inc-and-square variant: $\text{cost}(sqr) = 3$

next ↓

open: [①:0]

closed: { }

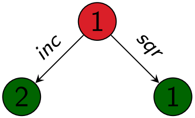
Example

next

open: [②:1 ①:3]

closed: $\{1\}$

bounded inc-and-square variant: cost(sqr) $= 3$



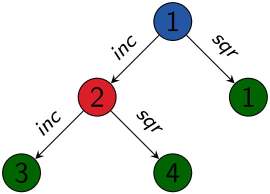
Example

next

open: [③:2 ①:3 ④:4]

closed: $\{1, 2\}$

bounded inc-and-square variant: cost(sqr) $= 3$



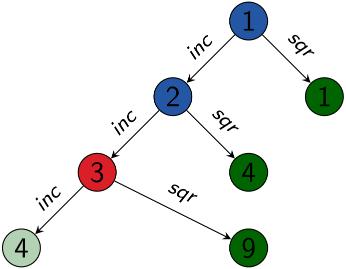
Example

bounded inc-and-square variant: cost(*sqr*) = 3

next

open: [①:3 ④:3 ④:4 ⑨:5]

closed: {1, 2, 3}



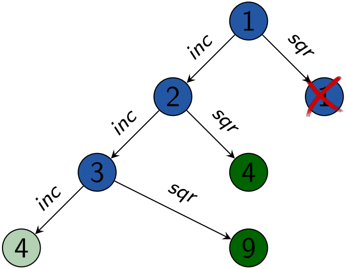
Example

bounded inc-and-square variant: $\text{cost}(sqr) = 3$

next

open: [④:3 ④:4 ⑨:5]

closed: $\{1, 2, 3\}$



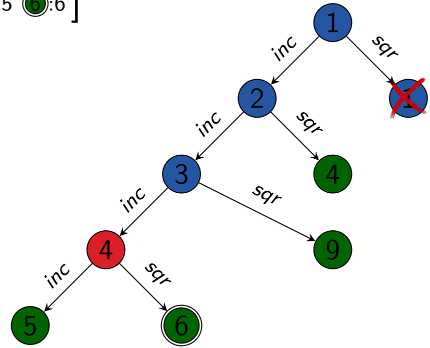
Example

bounded inc-and-square variant: $\text{cost}(\mathit{sqr}) = 3$

next ↓

open: $\big[$ 4:4 5:4 9:5 6:6 $\big]$

closed: $\{1, 2, 3, 4\}$



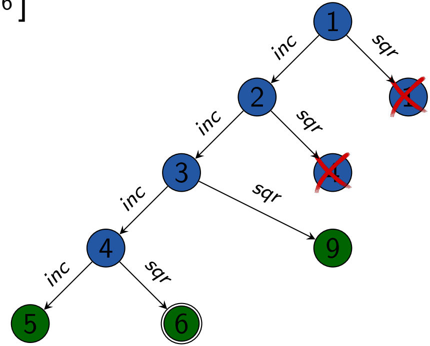
Example

bounded inc-and-square variant: $\mathrm{cost}(sqr) = 3$

next

open: [5:4 9:5 6:6]

closed: $\{1, 2, 3, 4\}$



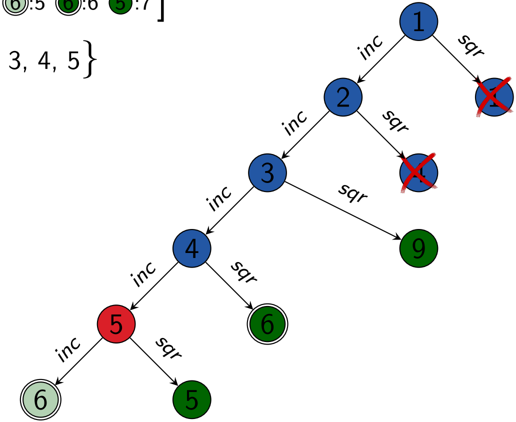
Example

bounded inc-and-square variant: cost(*sqr*) = 3

next

open: [9:5 6:5 6:6 5:7]

closed: $\{1, 2, 3, 4, 5\}$



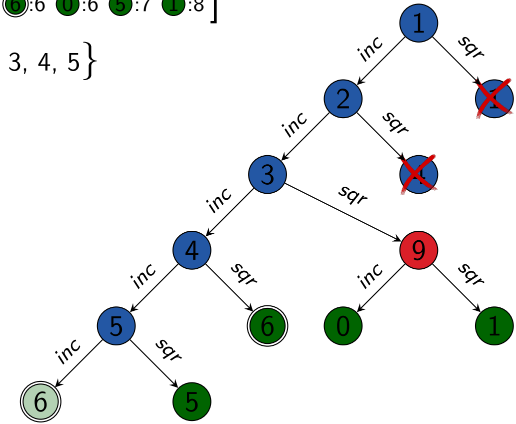
Example

bounded inc-and-square variant: $\text{cost}(sqr) = 3$

next

open: [6:5 6:6 0:6 5:7 1:8]

closed: $\{1, 2, 3, 4, 5\}$



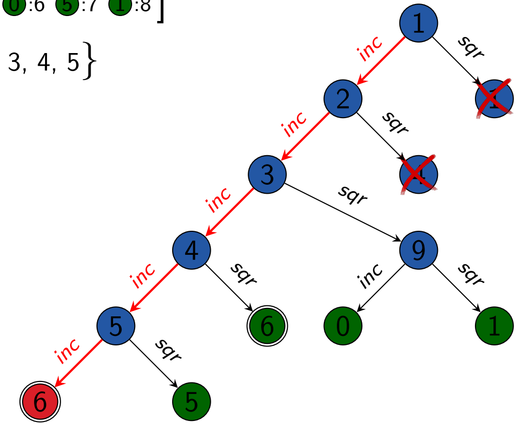
Example

bounded inc-and-square variant: cost(*sqr*) = 3

next

open: [6:6 0:6 5:7 1:8]

closed: {1, 2, 3, 4, 5}



Uniform Cost Search: Improvements

possible improvements:

- if action costs are small integers, bucket heaps often more efficient
- additional early duplicate tests for generated nodes can reduce memory requirements
    - can be beneficial or detrimental for runtime
    - must be careful to keep shorter path to duplicate state

Properties

Completeness and Optimality

properties of uniform cost search:

- uniform cost search is complete (Why?)
- uniform cost search is optimal (Why?)

Time and Space Complexity

properties of uniform cost search:

- Time complexity depends on distribution of action costs (no simple and accurate bounds).
    - Let $\varepsilon := \min_{a \in A} cost(a)$ and consider the case $\varepsilon > 0$.
    - Let c^* be the optimal solution cost.
    - Let b be the branching factor and consider the case $b \geq 2$.
    - Then the time complexity is at most $O(b^{\lfloor c^*/\varepsilon \rfloor + 1})$. (Why?)
    - often a very weak upper bound
- space complexity = time complexity

Summary

Summary

uniform cost search: expand nodes in order of ascending path costs

- usually as a graph search
- then corresponds to Dijkstra's algorithm
- complete and optimal