Foundations of Artificial Intelligence

B7. State-Space Search: Uniform Cost Search

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State-Space Search: Overview

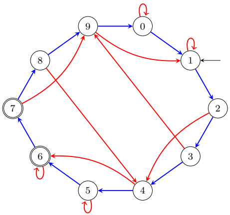
Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
  - B4. Data Structures for Search Algorithms
  - B5. Tree Search and Graph Search
  - B6. Breadth-first Search
  - B7. Uniform Cost Search
  - B8. Depth-first Search and Iterative Deepening
- B9–B15. Heuristic Algorithms

B7.1 Introduction

Uniform Cost Search

- breadth-first search optimal if all action costs equal
- otherwise no optimality guarantee ⇝ example:



- consider bounded inc-and-square problem with $cost(inc) = 1$, $cost(sqr) = 3$
- solution of breadth-first search still $\langle inc, sqr, sqr \rangle$ (cost: 7)
- but: $\langle inc, inc, inc, inc, inc \rangle$ (cost: 5) is cheaper!

remedy: uniform cost search

- always expand a node with minimal path cost (n.path_cost a.k.a. $g(n)$)
- implementation: priority queue (min-heap) for open list

B7.2 Algorithm

Reminder: Generic Graph Search Algorithm

reminder from Chapter B5:

Generic Graph Search

```
open := new OpenList
open.insert(make_root_node())
closed := new ClosedList
while not open.is_empty():
    n := open.pop()
    if closed.lookup(n.state) = none:
        closed.insert(n)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s'⟩ ∈ succ(n.state):
            n' := make_node(n, a, s')
            open.insert(n')
return unsolvable
```

Uniform Cost Search

Uniform Cost Search

```
open := new MinHeap ordered by g
open.insert(make_root_node())
closed := new HashSet
while not open.is_empty():
    n := open.pop_min()
    if n.state ∉ closed:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            n′ := make_node(n, a, s′)
            open.insert(n′)
return unsolvable
```

Uniform Cost Search: Discussion

Adapting generic graph search to uniform cost search:

- here, early goal tests/early updates of the closed list not a good idea. (Why not?)
- as in BFS-Graph, a set is sufficient for the closed list
- a tree search variant is possible, but rare:
  has the same disadvantages as BFS-Tree
  and in general not even semi-complete (Why not?)

Remarks:

- identical to Dijkstra's algorithm for shortest paths
- for both: variants with/without delayed duplicate elimination

Example

bounded inc-and-square variant: $\text{cost}(sqr) = 3$

next ↓

open: [1:0]

closed: { }

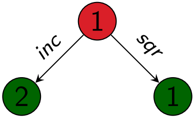
Example

bounded inc-and-square variant: cost(*sqr*) $= 3$

next

open: [2:1 1:3]

closed: $\{1\}$



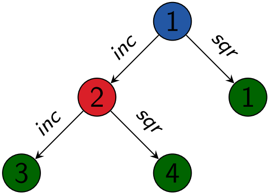
Example

bounded inc-and-square variant: $\text{cost}(sqr) = 3$

next

open: [3:2 1:3 4:4]

closed: $\{1, 2\}$



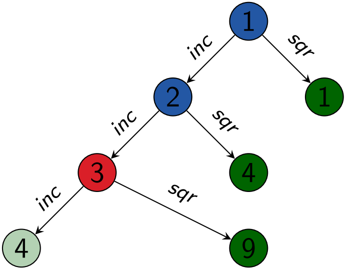
Example

bounded inc-and-square variant: cost(sqr) = 3

next

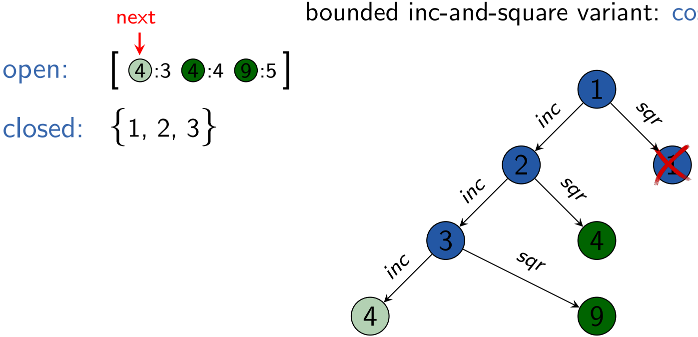
open: [1:3 4:3 4:4 9:5]

closed: $\{1, 2, 3\}$



Example

bounded inc-and-square variant: cost(sqr) $= 3$



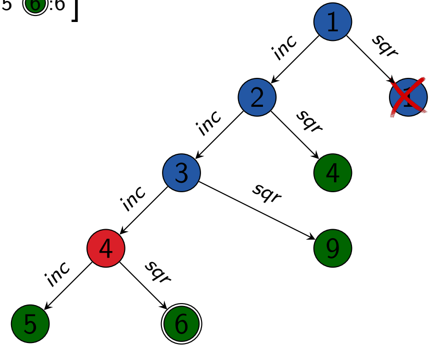
Example

bounded inc-and-square variant: cost(*sqr*) = 3

next

open: [4:4 5:4 9:5 6:6]

closed: {1, 2, 3, 4}



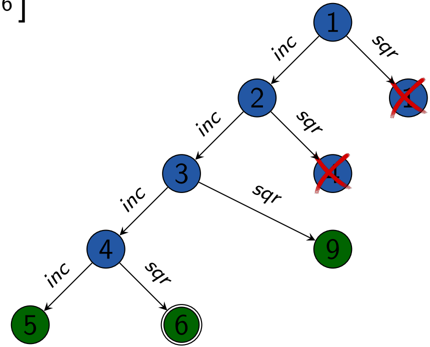
Example

bounded inc-and-square variant: cost(sqr) = 3

next

open: [5:4 9:5 6:6]

closed: {1, 2, 3, 4}



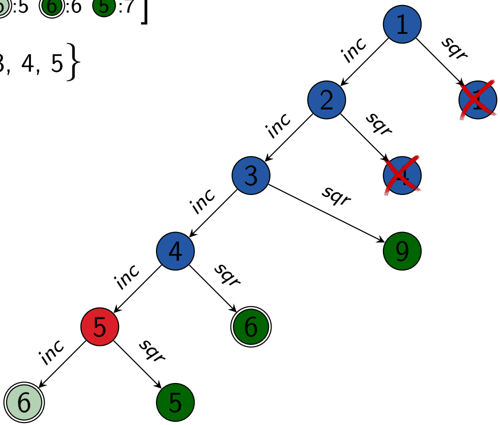
Example

bounded inc-and-square variant: cost(*sqr*) = 3

next

open: [9:5 6:5 6:6 5:7]

closed: {1, 2, 3, 4, 5}



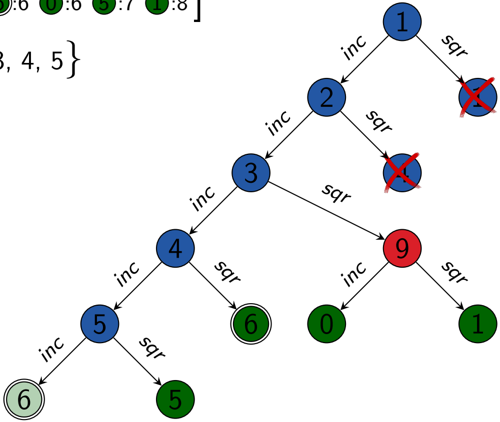
Example

bounded inc-and-square variant: $\text{cost}(sqr) = 3$

next

open: [6:5 6:6 0:6 5:7 1:8]

closed: $\{1, 2, 3, 4, 5\}$



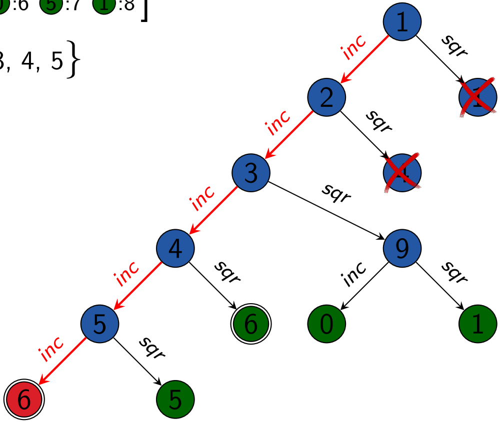
Example

bounded inc-and-square variant: $\text{cost}(\mathit{sqr}) = 3$

next

open: [6:6 0:6 5:7 1:8]

closed: $\{1, 2, 3, 4, 5\}$



Uniform Cost Search: Improvements

possible improvements:

- if action costs are small integers, bucket heaps often more efficient
- additional early duplicate tests for generated nodes can reduce memory requirements
  - can be beneficial or detrimental for runtime
  - must be careful to keep shorter path to duplicate state

B7.3 Properties

Completeness and Optimality

properties of uniform cost search:

- uniform cost search is complete (Why?)
- uniform cost search is optimal (Why?)

Time and Space Complexity

properties of uniform cost search:

- Time complexity depends on distribution of action costs (no simple and accurate bounds).
  - Let $\varepsilon := \min_{a \in A} cost(a)$ and consider the case $\varepsilon > 0$.
  - Let c^* be the optimal solution cost.
  - Let b be the branching factor and consider the case $b \geq 2$.
  - Then the time complexity is at most $O(b^{\lfloor c^*/\varepsilon \rfloor + 1})$. (Why?)
  - often a very weak upper bound
- space complexity = time complexity

B7.4 Summary

Summary

uniform cost search: expand nodes in order of ascending path costs

- usually as a graph search
- then corresponds to Dijkstra's algorithm
- complete and optimal