Foundations of Artificial Intelligence

B5. State-Space Search: Tree Search and Graph Search

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
  - B4. Data Structures for Search Algorithms
  - B5. Tree Search and Graph Search
  - B6. Breadth-first Search
  - B7. Uniform Cost Search
  - B8. Depth-first Search and Iterative Deepening
- B9–B15. Heuristic Algorithms

Introduction

Search Algorithms

General Search Algorithm

iteratively create a search tree:

- starting with the initial state,
- repeatedly expand a state by generating its successors (which state depends on the used search algorithm)
- stop when a goal state is expanded (sometimes: generated)
- or all reachable states have been considered

Search Algorithms

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In this chapter, we study two essential classes of search algorithms:

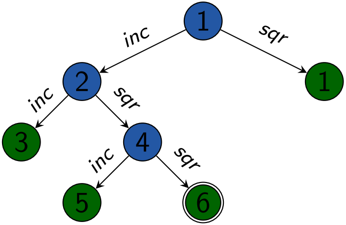
- tree search
- graph search

Each class consists of a large number of concrete algorithms.

German: expandieren, erzeugen, Baumsuche, Graphensuche

Tree Search

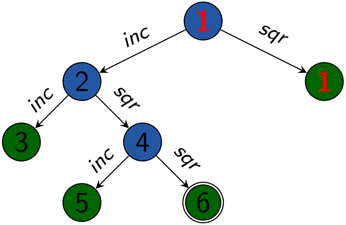
Tree Search: General Idea



- possible paths to be explored organized in a tree (search tree)
- search nodes correspond 1:1 to paths from initial state
- duplicates a.k.a. transpositions (i.e., multiple nodes with identical state) possible
- search tree can have unbounded depth

German: Suchbaum, Duplikate, Transpositionen

Tree Search: General Idea



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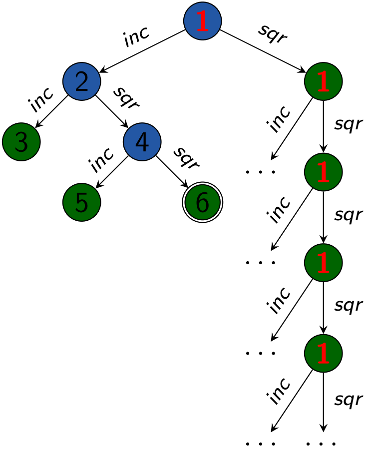

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Generic Tree Search Algorithm

Generic Tree Search Algorithm

```
$open$ := new OpenList
$open$.insert(make_root_node())
while not $open$.is_empty():
    $n$ := $open$.pop()
    if is_goal($n$.state):
        return extract_path($n$)
    for each $\langle a, s' \rangle \in$ succ($n$.state):
        $n'$ := make_node($n, a, s'$)
        $open$.insert($n'$)
return unsolvable
```

Generic Tree Search Algorithm: Discussion

discussion:

- generic template for tree search algorithms

$\rightsquigarrow$ for concrete algorithm, we must (at least) decide how to implement the open list

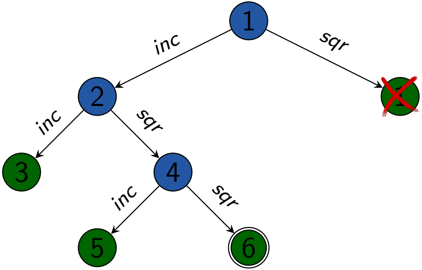
- concrete algorithms often conceptually follow template, (= generate the same search tree), but deviate from details for efficiency reasons

Graph Search

Graph Search

differences to tree search:

- recognize duplicates: when a state is reached on multiple paths, only keep one search node
- search nodes correspond 1:1 to reachable states
- depth of search tree bounded



remarks:

- some graph search algorithms do not immediately eliminate all duplicates (⇝ later)
- one possible reason: find optimal solutions when a path to state s found later is cheaper than one found earlier

Generic Graph Search Algorithm

Generic Graph Search Algorithm

```
$open$ := new OpenList
$open$.insert(make_root_node())
$closed$ := new ClosedList
while not $open$.is_empty():
    $n$ := $open$.pop()
    if $closed$.lookup($n$.state) = none:
        $closed$.insert($n$)
        if is_goal($n$.state):
            return extract_path($n$)
        for each $\langle a, s' \rangle \in$ succ($n$.state):
            $n'$ := make_node($n, a, s'$)
            $open$.insert($n'$)
return unsolvable
```

Generic Graph Search Algorithm: Discussion

discussion:

- same comments as for generic tree search apply
- in "pure" algorithm, closed list does not actually need to store the search nodes
    - sufficient to implement *closed* as set of states
    - advanced algorithms often need access to the nodes, hence we show this more general version here
- some variants perform goal and duplicate tests elsewhere (earlier) ⇝ following chapters

Evaluating Search Algorithms

Criteria: Completeness

four criteria for evaluating search algorithms:

Completeness

Is the algorithm guaranteed to find a solution if one exists?

Does it terminate if no solution exists?

first property: semi-complete
both properties: complete

German: Vollständigkeit, semi-vollständig, vollständig

Criteria: Optimality

four criteria for evaluating search algorithms:

Optimality

Are the solutions returned by the algorithm always optimal?

German: Optimalität

Criteria: Time Complexity

four criteria for evaluating search algorithms:

Time Complexity

How much time does the algorithm need until termination?

- usually worst case analysis
- usually measured in generated nodes

often a function of the following quantities:

- b: (branching factor) of state space
  (max. number of successors of a state)
- d: search depth
  (length of longest path in generated search tree)

German: Zeitaufwand, Verzweigungsgrad, Suchtiefe

Criteria: Space Complexity

four criteria for evaluating search algorithms:

Space Complexity

How much memory does the algorithm use?

- usually worst case analysis
- usually measured in (concurrently) stored nodes

often a function of the following quantities:

- b: (branching factor) of state space
  (max. number of successors of a state)
- d: search depth
  (length of longest path in generated search tree)

German: Speicheraufwand

Analyzing the Generic Search Algorithms

Generic Tree Search Algorithm

- Is it complete? Is it semi-complete?
- Is it optimal?
- What is its worst-case time complexity?
- What is its worst-case space complexity?

Generic Graph Search Algorithm

- Is it complete? Is it semi-complete?
- Is it optimal?
- What is its worst-case time complexity?
- What is its worst-case space complexity?

Summary

Summary (1)

tree search:

- search nodes correspond 1:1 to paths from initial state

graph search:

- search nodes correspond 1:1 to reachable states

⇝ duplicate elimination

generic methods with many possible variants

Summary (2)

evaluating search algorithms:

- completeness and semi-completeness
- optimality
- time complexity and space complexity