

# Foundations of Artificial Intelligence

## B2. State-Space Search: Representation of State Spaces

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# State-Space Search: Overview

## Chapter overview: state-space search

- B1–B3. Foundations
  - B1. State Spaces
  - B2. Representation of State Spaces
  - B3. Examples of State Spaces
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms

# Representation of State Spaces

# Representation of State Spaces

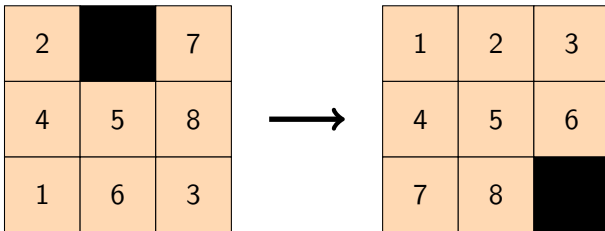
- practically interesting state spaces are often **huge** ( $10^{10}$ ,  $10^{20}$ ,  $10^{100}$  states)
- How do we **represent** them, so that we can efficiently deal with them algorithmically?

three main options:

- 1 as **explicit** (directed) graphs
- 2 with **declarative** representations
- 3 as a **black box**

**German:** explizit, deklarativ, Black Box

# Example: 8-Puzzle



# Explicit Graphs

# State Spaces as Explicit Graphs

## State Spaces as Explicit Graphs

represent state spaces as **explicit directed graphs**:

- vertices = states
- directed arcs = transitions

↔ represented as **adjacency list** or **adjacency matrix**

**German:** Adjazenzliste, Adjazenzmatrix

**Example (explicit graph for bounded inc-and-square)**

`ai-b02-bounded-inc-and-square.graph`

# State Spaces as Explicit Graphs

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**German:** Adjazenzliste, Adjazenzmatrix

**Example (explicit graph for 8-puzzle)**

`ai-b02-puzzle8.graph`



# State Spaces as Explicit Graphs: Discussion

## discussion:

- **impossible** for **large** state spaces (too much space required)
- if spaces small enough for explicit representations, solutions easy to compute: **Dijkstra's algorithm**  
 $O(|S| \log |S| + |T|)$
- interesting for time-critical **all-pairs-shortest-path** queries (examples: route planning, path planning in video games)

# Declarative Representations

# State Spaces with Declarative Representations

## State Spaces with Declarative Representations

represent state spaces **declaratively**:

- **compact** description of state space as input to algorithms  
     $\rightsquigarrow$  state spaces **exponentially larger** than the input
  - algorithms directly operate on compact description
- $\rightsquigarrow$  allows automatic reasoning about problem:  
reformulation, simplification, abstraction, etc.

## Example (declarative representation for 8-puzzle)

```
puzzle8-domain.pddl + puzzle8-problem.pddl
```

# Black Box

# State Spaces as Black Boxes

## State Spaces as Black Boxes

Define an **abstract interface** for state spaces.

For state space  $\mathcal{S} = \langle S, A, cost, T, s_1, S_G \rangle$

we need these methods:

- **init()**: generate initial state  
result: state  $s_1$
- **is\_goal(s)**: test if  $s$  is a goal state  
result: **true** if  $s \in S_G$ ; **false** otherwise
- **succ(s)**: generate applicable actions and successors of  $s$   
result: sequence of pairs  $\langle a, s' \rangle$  with  $s \xrightarrow{a} s'$
- **cost(a)**: gives cost of action  $a$   
result:  $cost(a) (\in \mathbb{N}_0)$

**Remark:** we will extend the interface later  
in a small but important way

# State Spaces as Black Boxes: Example and Discussion

## Example (Black Box Representation for 8-Puzzle)

demo: `puzzle8.py`

- in the following: focus on black box model
- explicit graphs only as illustrating examples
- near end of semester: declarative state spaces  
(classical planning)

# Summary

# Summary

- state spaces often **huge** ( $> 10^{10}$  states)  
     $\rightsquigarrow$  **how to represent?**
- **explicit graphs**: adjacency lists or matrices;  
    only suitable for small problems
- **declaratively**: compact description as input  
    to search algorithms
- **black box**: implement an abstract interface