Foundations of Artificial Intelligence

B2. State-Space Search: Representation of State Spaces

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February 26, 2025

State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
  - B1. State Spaces
  - B2. Representation of State Spaces
  - B3. Examples of State Spaces
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms

B2.1 Representation of State Spaces

Representation of State Spaces

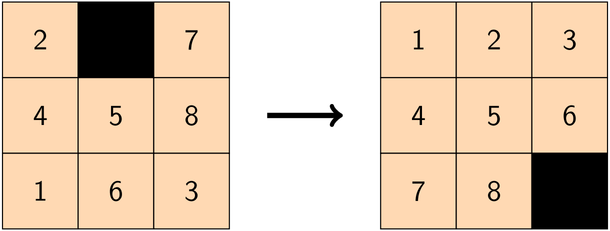
- practically interesting state spaces are often huge (10^{10}, 10^{20}, 10^{100} states)
- How do we represent them, so that we can efficiently deal with them algorithmically?

three main options:

1. as explicit (directed) graphs
2. with declarative representations
3. as a black box

German: explizit, deklarativ, Black Box

Example: 8-Puzzle



B2.2 Explicit Graphs

State Spaces as Explicit Graphs

State Spaces as Explicit Graphs

represent state spaces as explicit directed graphs:

- vertices = states
- directed arcs = transitions

$\leadsto$ represented as adjacency list or adjacency matrix

German: Adjazenzliste, Adjazenzmatrix

Example (explicit graph for bounded inc-and-square)

`ai-b02-bounded-inc-and-square.graph`

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Example (explicit graph for 8-puzzle)

`ai-b02-puzzle8.graph`

State Spaces as Explicit Graphs: Discussion

discussion:

- impossible for large state spaces (too much space required)
- if spaces small enough for explicit representations, solutions easy to compute: Dijkstra's algorithm $O(|S| \log |S| + |T|)$
- interesting for time-critical all-pairs-shortest-path queries (examples: route planning, path planning in video games)

B2.3 Declarative Representations

State Spaces with Declarative Representations

State Spaces with Declarative Representations

represent state spaces declaratively:

- compact description of state space as input to algorithms
  $\rightsquigarrow$ state spaces exponentially larger than the input
- algorithms directly operate on compact description

$\rightsquigarrow$ allows automatic reasoning about problem: reformulation, simplification, abstraction, etc.

Example (declarative representation for 8-puzzle)

`puzzle8-domain.pddl` + `puzzle8-problem.pddl`

B2.4 Black Box

State Spaces as Black Boxes

State Spaces as Black Boxes

Define an abstract interface for state spaces.

For state space $\mathcal{S} = \langle S, A, cost, T, s_I, S_G \rangle$
we need these methods:

- init(): generate initial state
  result: state s_I
- is_goal(s): test if s is a goal state
  result: **true** if $s \in S_G$; **false** otherwise
- succ(s): generate applicable actions and successors of s
  result: sequence of pairs $\langle a, s' \rangle$ with $s \xrightarrow{a} s'$
- cost(a): gives cost of action a
  result: $cost(a)$ ($\in \mathbb{N}_0$)

Remark: we will extend the interface later
in a small but important way

State Spaces as Black Boxes: Example and Discussion

Example (Black Box Representation for 8-Puzzle)

demo: `puzzle8.py`

- in the following: focus on black box model
- explicit graphs only as illustrating examples
- near end of semester: declarative state spaces (classical planning)

B2.5 Summary

Summary

- state spaces often huge ($> 10^{10}$ states)
  $\rightsquigarrow$ how to represent?
- explicit graphs: adjacency lists or matrices; only suitable for small problems
- declaratively: compact description as input to search algorithms
- black box: implement an abstract interface