Algorithms and Data Structures
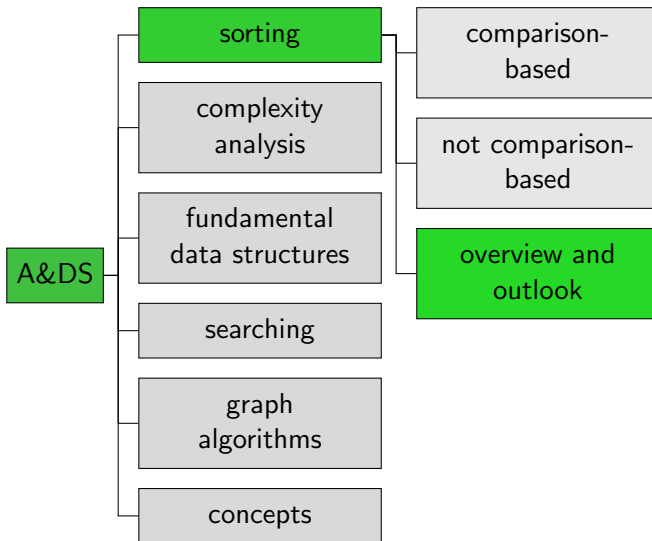
A15. Sorting: Overview & Outlook

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Content of the Course

Overview

Comparison-based Sorting: Overview

Algorithm	Running time $O(\cdot)$	Memory $O(\cdot)$	stable
	best/avg./worst	best/avg./worst	
Selection sort	n^2	1	no
Insertion sort	$n/n^2/n^2$	1	yes
Merge sort	$n \log n$	n	yes
Quicksort	$n \log n/n \log n/n^2$	$\log n/\log n/n$	no
Heapsort	$n \log n$	1	no

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Very nice visualization of the algorithms at
https://www.toptal.com/developers/sorting-algorithms/

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- Equal asymptotic running time does not mean that algorithms take equally long (different hidden constants in $O(\cdot)$).
 Heapsort needs twice as many comparisons as merge sort.

Outlook

Partially Sorted Data

- Often some subsequences of the input are already sorted (so-called runs).
- Insertion sort directly benefits from this.
- For some other approaches, there are variants that exploit runs, e.g. natural merge sort.

Many Equivalent Keys

- Quite common in practical applications.
  e.g. sorting students by place of residence
- There are special variants for some algorithms.
- For example, 3-way partitioning in quicksort

$< P$	$= P$	$> P$
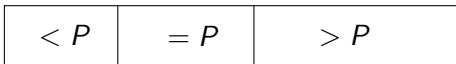
Sorting Complex Objects

- Most of the time, we do not want to sort numbers but complex objects.
- It would be extremely expensive to move them in memory for every swap.
- Instead: Sort elements that only consist of the key and a pointer/reference to the actual object.

Not So Correct Algorithms

full comic at https://xkcd.com/1185/
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Solve other Problems by Sorting

k-smallest element

- For example, identifying the median ($k = \lfloor n/2 \rfloor$).
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Duplicates

- How many different keys are there? Which value is most common? Are there duplicate keys?
- Can be solved directly with quadratic algorithms.
- Or – more clever – sort first and then use a single scan.

Quiz

Quiz

kahoot.it