Theory of Computer Science

B3. Finite Automata

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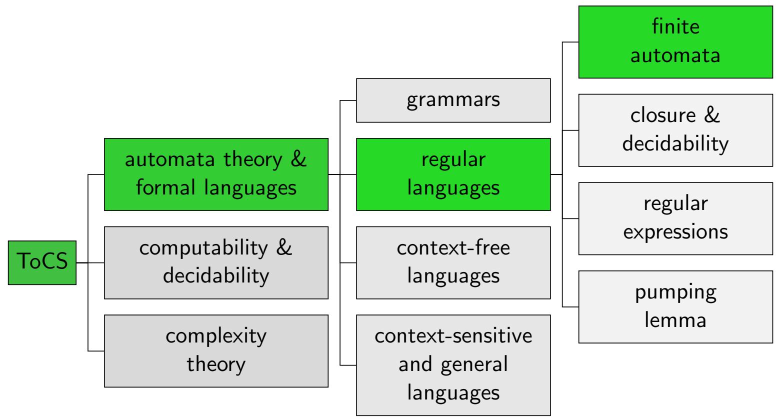
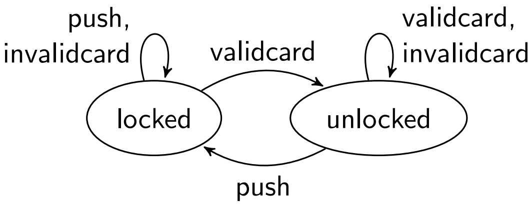
B3.1 Introduction

Content of the Course



A Controller for a Turnstile

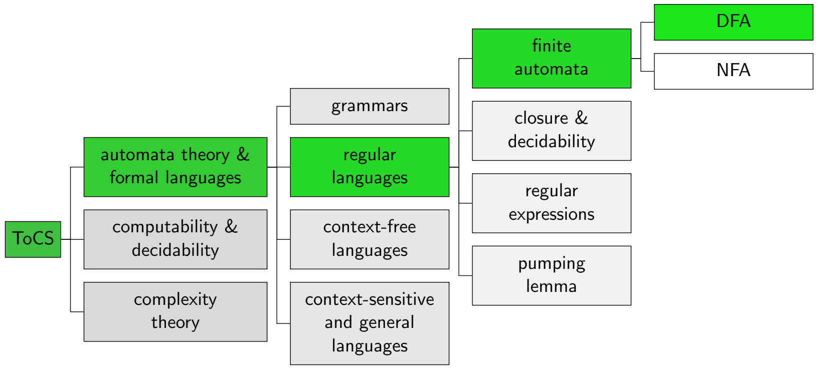
- simple access control
- card reader and push sensor
- card can either be valid or invalid



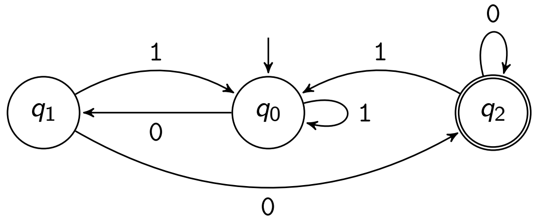
- Finite automata are a good model for computers with very limited memory.
  Where can the turnstile controller store information about what it has seen in the past?
- We will not consider automata that run forever but that process a finite input sequence and then classify it as accepted or not.

B3.2 DFAs

Content of the Course

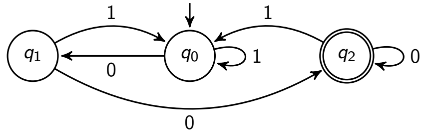


Finite Automaton: Example



When reading the input 01100 the automaton visits the states q_0, q_1, q_0, q_0, q_1, q_2.

Finite Automata: Terminology and Notation



- states $Q = \{q_0, q_1, q_2\}$
- input alphabet $\Sigma = \{0, 1\}$
- transition function δ
- start state q_0
- accept states $\{q_2\}$

$\delta(q_0, 0) = q_1$
$\delta(q_0, 1) = q_0$
$\delta(q_1, 0) = q_2$
$\delta(q_1, 1) = q_0$
$\delta(q_2, 0) = q_2$
$\delta(q_2, 1) = q_0$

δ	0	1
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_0

table form of δ

Deterministic Finite Automaton: Definition

Definition (Deterministic Finite Automata)

A deterministic finite automaton (DFA) is a 5-tuple $M = \langle Q, \Sigma, \delta, q_0, F\rangle$ where

- Q is the finite set of states
- Σ is the input alphabet
- $\delta : Q \times \Sigma \to Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states (or final states)

DFA: Accepted Words

Intuitively, a DFA accepts a word if its computation terminates in an accept state.

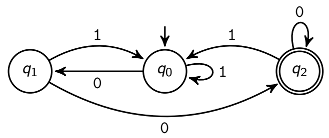
Definition (Words Accepted by a DFA)

DFA $M = \langle Q, \Sigma, \delta, q_0, F\rangle$ accepts the word $w = a_1 \dots a_n$ if there is a sequence of states $q'_0, \dots, q'_n \in Q$ with

1. $q'_0 = q_0$,
2. $\delta(q'_{i-1}, a_i) = q'_i$ for all $i \in \{1, \dots, n\}$ and
3. $q'_n \in F$.

Example

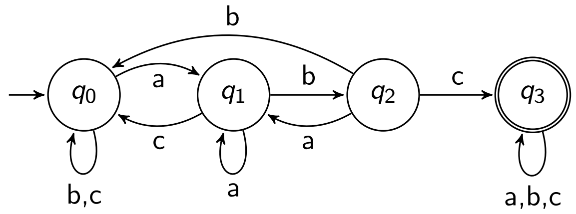
Example

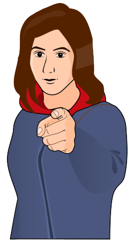


accepts:	does not accept:
00	ε
10010100	1001010
01000	010001

Exercise (slido)

Consider the following DFA:





Which of the following words does it accept?

- abc
- ababcb
- babbc

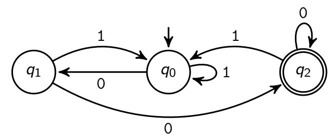
DFA: Recognized Language

Definition (Language Recognized by a DFA)

Let M be a deterministic finite automaton.
The language recognized by M is defined as
$\mathcal{L}(M) = \{w \in \Sigma^* \mid w \text{ is accepted by } M\}$.

Example

Example



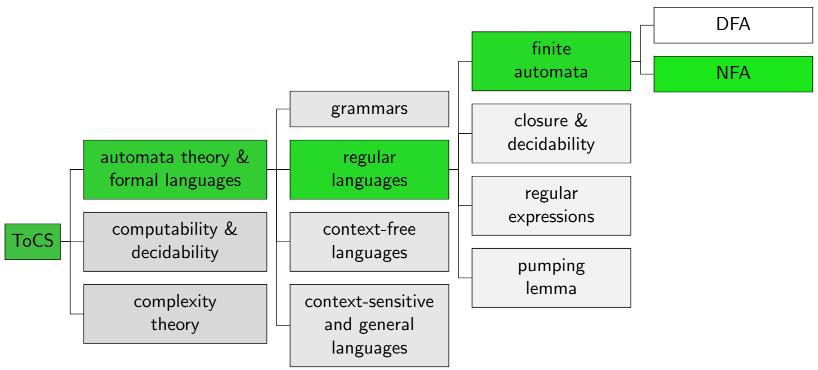
The DFA recognizes the language
$\{w \in \{0,1\}^* \mid w \text{ ends with } 00\}$.

A Note on Terminology

- In the literature, "accept" and "recognize" are sometimes used synonymously or the other way around.
  DFA recognizes a word or accepts a language.
- We try to stay consistent using the previous definitions (following the text book by Sipser).

B3.3 NFAs

Content of the Course



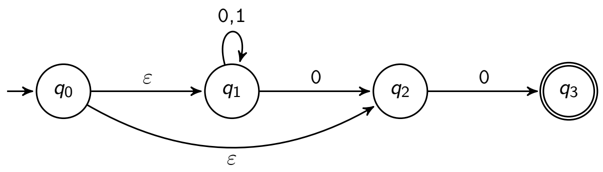
Nondeterministic Finite Automata

Picture courtesy of stockimages / FreeDigitalPhotos.net

In what Sense is a DFA Deterministic?

- A DFA has a single fixed state from which the computation starts.
- When a DFA is in a specific state and reads an input symbol, we know what the next state will be.
- For a given input, the entire computation is determined.
- This is a deterministic computation.

Nondeterministic Finite Automata: Example



differences to DFAs:

- transition function δ can lead to zero or more successor states for the same $a \in \Sigma$
- ε-transitions can be taken without "consuming" a symbol from the input
- the automaton accepts a word if there is at least one accepting sequence of states

Nondeterministic Finite Automaton: Definition

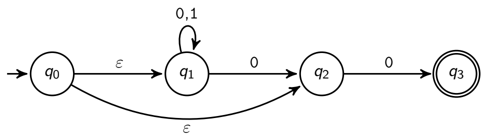
Definition (Nondeterministic Finite Automata)

A nondeterministic finite automaton (NFA) is a 5-tuple $M = \langle Q, \Sigma, \delta, q_0, F \rangle$ where

- Q is the finite set of states
- Σ is the input alphabet
- $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$ is the transition function (mapping to the power set of Q)
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

DFAs are (essentially) a special case of NFAs.

Accepting Computation: Example



$w = 0100$

$\rightsquigarrow$ computation tree on blackboard

ε-closure of a State

For a state $q \in Q$, we write $E(q)$ to denote the set of states that are reachable from q via ε-transitions in δ.

Definition (ε-closure)

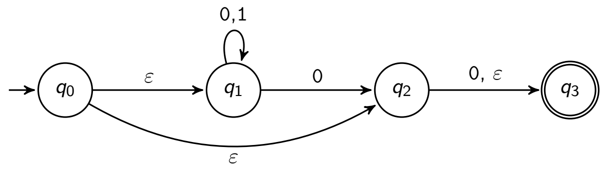
For NFA $M = \langle Q, \Sigma, \delta, q_0, F\rangle$ and state $q \in Q$, state p is in the ε-closure $E(q)$ of q iff there is a sequence of states $q'_0, \dots, q'_n$ with

1. $q'_0 = q$,
2. $q'_i \in \delta(q'_{i-1}, \varepsilon)$ for all $i \in \{1, \dots, n\}$ and
3. $q'_n = p$.

$q \in E(q)$ for every state q

Exercise (slido)

Consider the following NFA:



Which states are in the ε-closure $E(q_0)$?

- q_0
- q_1
- q_2
- q_3

NFA: Accepted Words

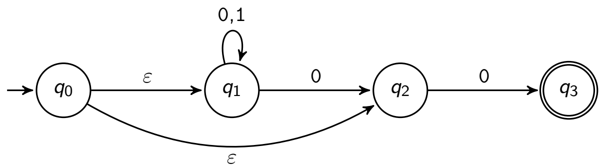
Definition (Words Accepted by an NFA)

NFA $M = \langle Q, \Sigma, \delta, q_0, F\rangle$ accepts the word $w = a_1 \dots a_n$ if there is a sequence of states $q'_0, \dots, q'_n \in Q$ with

1. $q'_0 \in E(q_0)$,
2. $q'_i \in \bigcup_{q \in \delta(q'_{i-1}, a_i)} E(q)$ for all $i \in \{1, \dots, n\}$ and
3. $q'_n \in F$.

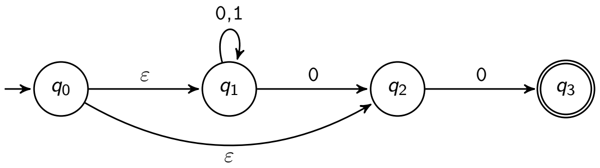
Example: Accepted Words

Example



accepts:	does not accept:
0	ε
10010100	1001010
01000	010001

Exercise (slido)



Does this NFA accept input 01010?

NFA: Recognized Language

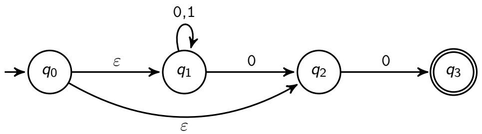
Definition (Language Recognized by an NFA)

Let M be an NFA with input alphabet Σ.

The language recognized by M is defined as
$\mathcal{L}(M) = \{w \in \Sigma^* \mid w \text{ is accepted by } M\}$.

Example: Recognized Language

Example



The NFA recognizes the language
$\{w \in \{0,1\}^* \mid w = 0 \text{ or } w \text{ ends with } 00\}$.

Summary

- DFAs are automata where every state transition is uniquely determined.
- NFAs can have zero, one or more transitions for a given state and input symbol.
- NFAs can have ϵ-transitions that can be taken without reading a symbol from the input.
- NFAs accept a word if there is at least one accepting sequence of states.