

# Foundations of Artificial Intelligence

## F6. Automated Planning: Abstraction Heuristics

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# Automated Planning: Overview

## Chapter overview: automated planning

- F1. Introduction
- F2. Planning Formalisms
- F3. Delete Relaxation
- F4. Delete Relaxation Heuristics
- F5. Abstraction
- F6. Abstraction Heuristics

# Abstraction Heuristics

# Abstraction Heuristic

Given an abstraction function  $\alpha$  for a state space  $\mathcal{S}$ , use **abstract solution cost** (solution cost of  $\alpha(s)$  in  $\mathcal{S}^\alpha$ ) as heuristic for **concrete solution cost** (solution cost of  $s$  in  $\mathcal{S}$ ).

## Definition (abstraction heuristic)

The **abstraction heuristic** for abstraction  $\alpha$  maps each state  $s$  to its abstract solution cost

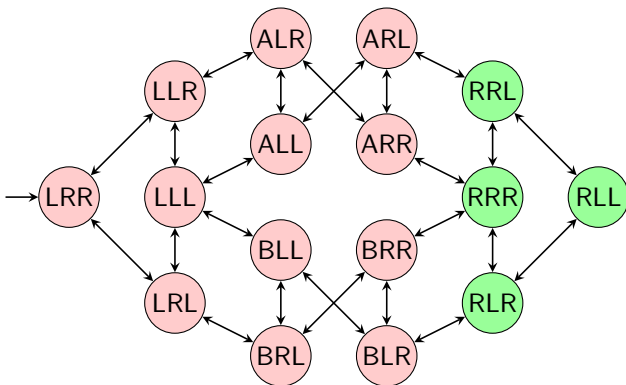
$$h^\alpha(s) = h_{\mathcal{S}^\alpha}^*(\alpha(s)),$$

where  $h_{\mathcal{S}^\alpha}^*$  is the perfect heuristic in  $\mathcal{S}^\alpha$ .

**German:** abstrakte/konkrete Zielabstände, Abstraktionsheuristik

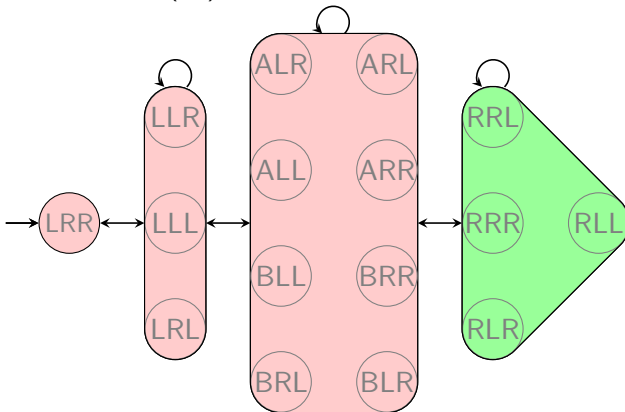
# Abstraction: Example

concrete state space



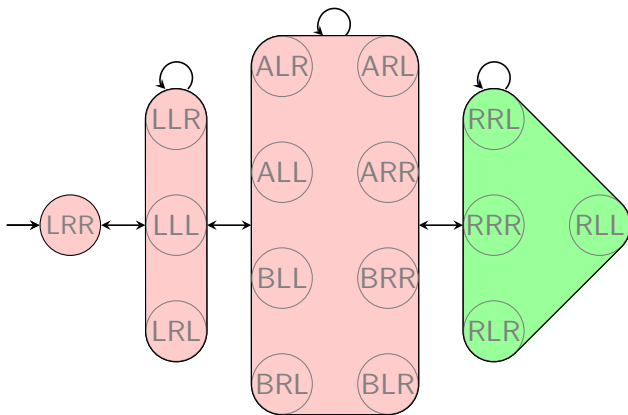
# Abstraction: Example

(an) abstract state space



**Remark:** Most arcs correspond to several (parallel) transitions with different labels.

# Abstraction Heuristic: Example



$$h^\alpha(\{p \mapsto L, t_A \mapsto R, t_B \mapsto R\}) = 3$$

# Abstraction Heuristics: Discussion

- Every abstraction heuristic is **admissible** and **consistent**.  
(proof idea?)
- The choice of the **abstraction function  $\alpha$**  is very important.
  - **Every**  $\alpha$  yields an admissible and consistent heuristic.
  - But most  $\alpha$  lead to poor heuristics.
- An effective  $\alpha$  must yield an **informative heuristic** ...
- ... as well as being **efficiently computable**.
- **How to find a suitable  $\alpha$ ?**



# Automatic Computation of Suitable Abstractions

## Main Problem with Abstraction Heuristics

### How to find a good abstraction?

Several successful methods:

- **pattern databases (PDBs)**  $\rightsquigarrow$  [this course](#)  
(Culberson & Schaeffer, 1996)
- **merge-and-shrink** abstractions  
(Dräger, Finkbeiner & Podelski, 2006)
- **Cartesian** abstractions (Seipp & Helmert, 2013)
- **domain** abstractions (Kreft et al., 2023)

**German:** Pattern Databases, Merge-and-Shrink-Abstraktionen, Kartesische Abstraktionen, Domänenabstraktionen

# Pattern Databases

# Pattern Databases: Background

- The most common abstraction heuristics are **pattern database heuristics**.
- originally introduced for the **15-puzzle** (Culberson & Schaeffer, 1996) and for **Rubik's Cube** (Korf, 1997)
- introduced for **automated planning** by Edelkamp (2001)
- for many search problems the **best known** heuristics
- many many research papers studying
  - theoretical properties
  - efficient implementation and application
  - pattern selection
  - ...

# Pattern Databases: Projections

A PDB heuristic for a planning task is an abstraction heuristic where

- some aspects (= state variables) of the task are preserved **with perfect precision** while
- all other aspects are not preserved **at all**.

formalized as **projections** to a **pattern**  $P \subseteq V$ :

$$\pi_P(s) = \{v \mapsto s(v) \mid v \in P\}$$

example:

- $s = \{p \mapsto L, t_A \mapsto R, t_B \mapsto R\}$
- **projection** on  $P = \{p\}$  (= ignore trucks):  
 $\pi_P(s) = \{p \mapsto L\}$
- **projection** on  $P = \{p, t_A\}$  (= ignore truck B):  
 $\pi_P(s) = \{p \mapsto L, t_A \mapsto R\}$

German: Projektionen

# Pattern Databases: Definition

## Definition (pattern database heuristic)

Let  $P$  be a subset of the variables of a planning task.

The abstraction heuristic induced by the **projection**  $\pi_P$  on  $P$  is called **pattern database heuristic** (**PDB heuristic**) with **pattern**  $P$ .

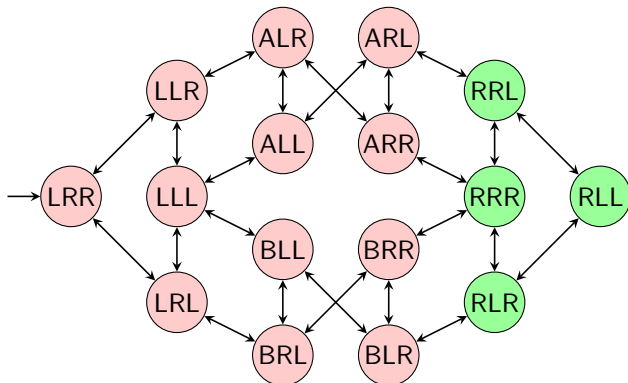
abbreviated notation:  $h^P$  for  $h^{\pi_P}$

German: Pattern-Database-Heuristik

remark:

- “pattern databases” in analogy to **endgame databases** (which have been successfully applied in 2-person-games)

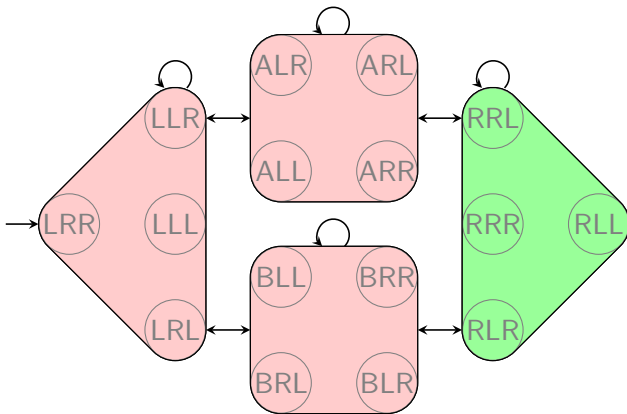
# Example: Concrete State Space



- state variable *package*: {L, R, A, B}
- state variable *truck A*: {L, R}
- state variable *truck B*: {L, R}

# Example: Projection (1)

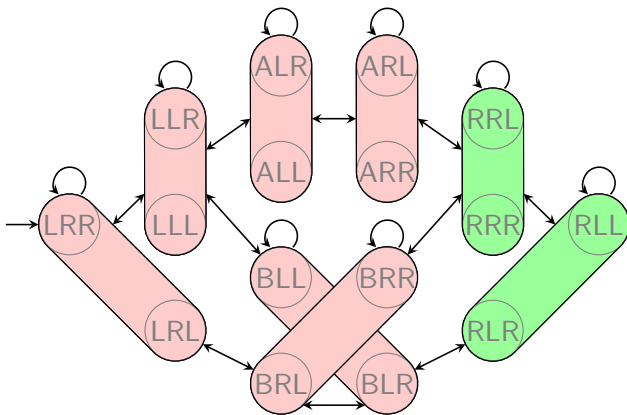
abstraction induced by  $\pi_{\{package\}}$ :



$$h^{\{package\}}(LRR) = 2$$

# Example: Projection (2)

abstraction induced by  $\pi_{\{package, truck A\}}$ :

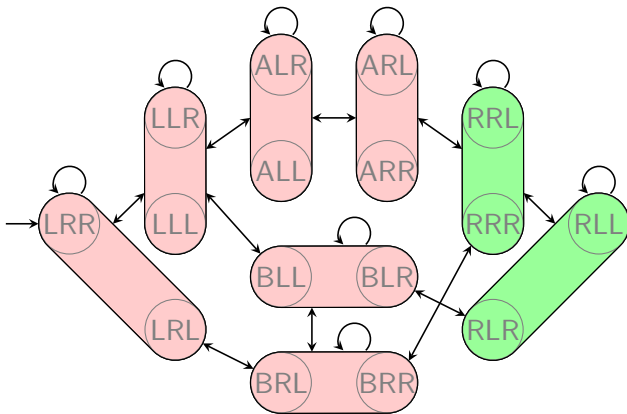


$$h_{\{package, truck A\}}(LRR) = 2$$



# Example: Projection (2)

abstraction induced by  $\pi_{\{package, truck A\}}$ :



$$h^{\{package, truck A\}}(LRR) = 2$$

# Pattern Databases in Practice

practical aspects which we do not discuss in detail:

- How to automatically find **good patterns**?
- How to combine **multiple** PDB heuristics?
- How to **implement** PDB heuristics efficiently?
  - good implementations efficiently handle **abstract** state spaces with  $10^7$ ,  $10^8$  or more abstract states
  - effort independent of the size of the **concrete** state space
  - usually all heuristic values are precomputed
    - ~> space complexity = number of abstract states

# Summary

# Summary

- basic idea of **abstraction heuristics**: estimate solution cost by considering a **smaller** planning task.
- formally: **abstraction function**  $\alpha$  maps states to **abstract states** and thus defines which states can be distinguished by the resulting heuristic.
- induces **abstract state space** whose solution costs are used as heuristic
- **Pattern database heuristics** are abstraction heuristics based on **projections** onto state variable subsets (**patterns**): states are distinguishable iff they differ on the pattern.