

Foundations of Artificial Intelligence

B12. State-Space Search: Greedy BFS, A*, Weighted A*

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms
 - B9. Heuristics
 - B10. Analysis of Heuristics
 - B11. Best-first Graph Search
 - B12. Greedy Best-first Search, A*, Weighted A*
 - B13. IDA*
 - B14. Properties of A*, Part I
 - B15. Properties of A*, Part II

Introduction

What Is It About?

In this chapter we study last chapter's algorithms in more detail:

- greedy best-first search
- A*
- weighted A*

Greedy Best-first Search

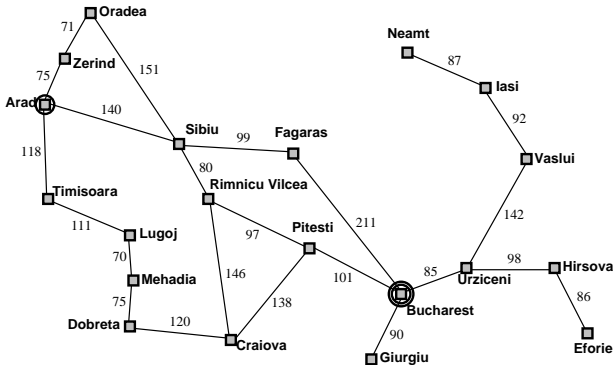
Greedy Best-first Search

Greedy Best-first Search

only consider the heuristic: $f(n) = h(n.state)$

Note: usually *without reopening* (for reasons of efficiency)

Example: Greedy Best-first Search for Route Planning

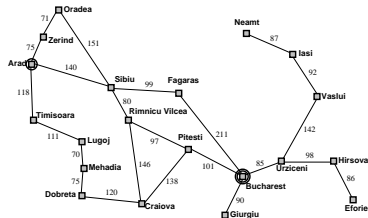


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Drobeta	242
Eforie	161
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Example: Greedy Best-first Search for Route Planning

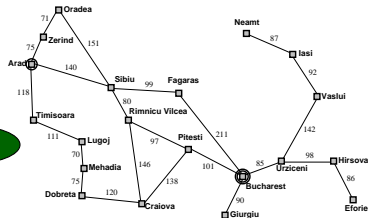
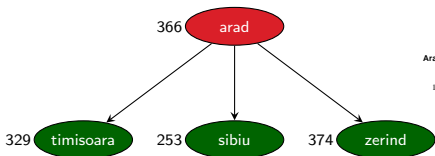
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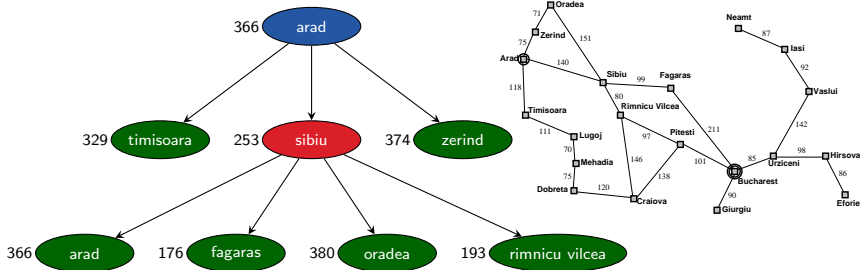
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Example: Greedy Best-first Search for Route Planning



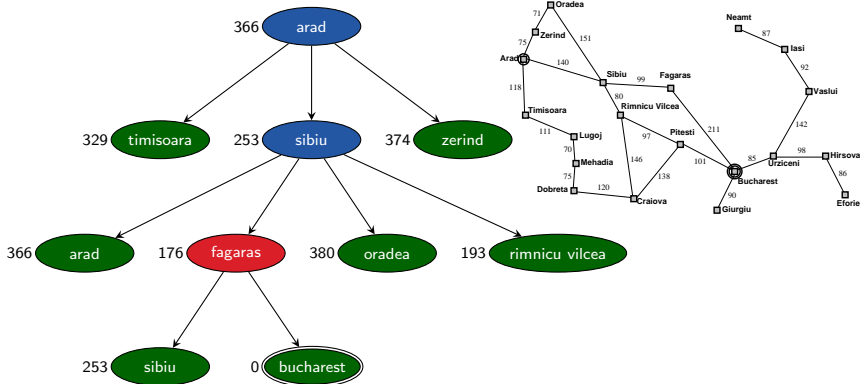
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Example: Greedy Best-first Search for Route Planning



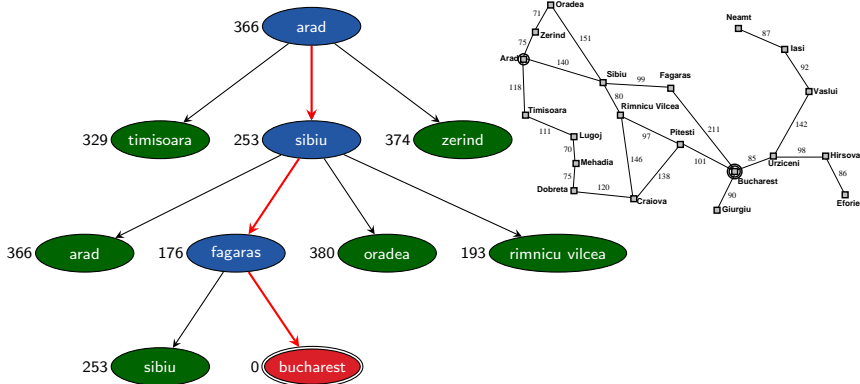
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Example: Greedy Best-first Search for Route Planning



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Example: Greedy Best-first Search for Route Planning



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Greedy Best-first Search: Properties

- **complete** with **safe** heuristics
(like all variants of best-first graph search)
- **suboptimal**: solutions can be **arbitrarily bad**
- often **very fast**: one of the fastest search algorithms in practice
- monotonic transformations of h (e.g. scaling, additive constants) do not affect behaviour ([Why is this interesting?](#))

A*

A*

A*

combine greedy best-first search with uniform cost search:

$$f(n) = g(n) + h(n.state)$$

- **trade-off** between path cost and proximity to goal
- $f(n)$ estimates overall cost of cheapest solution **from initial state via n to the goal**

A*: Citations



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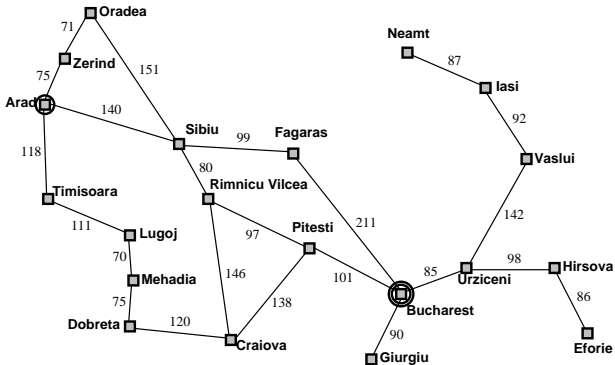
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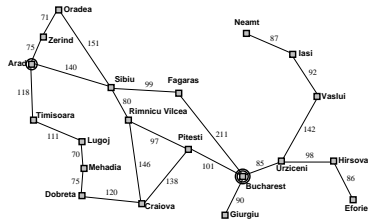
Example: A* for Route Planning



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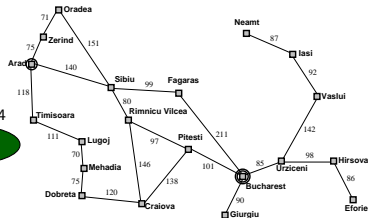
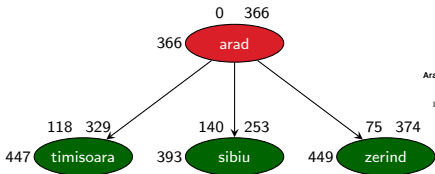
Example A* for Route Planning

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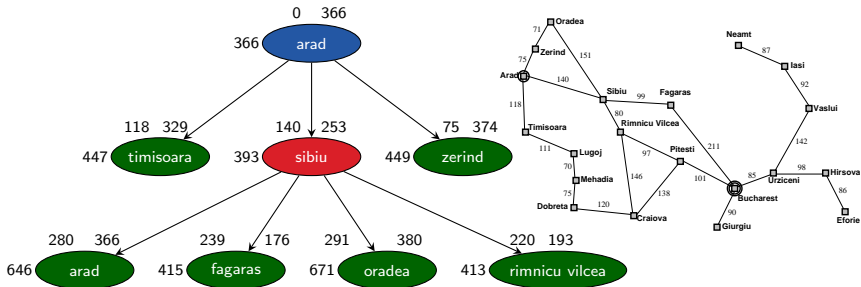
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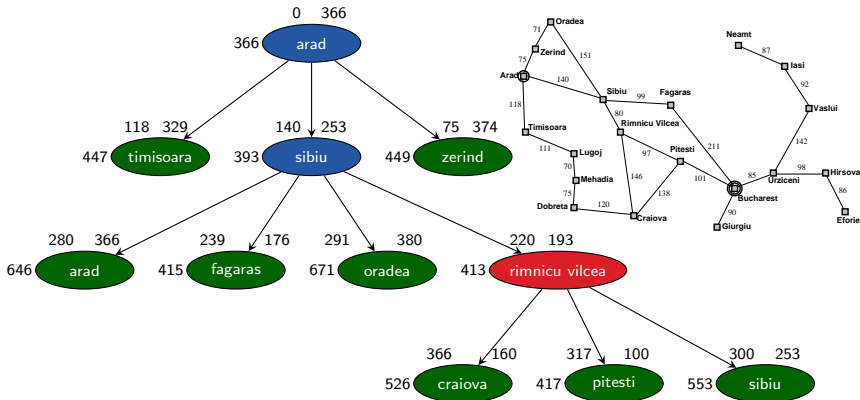
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Example A* for Route Planning



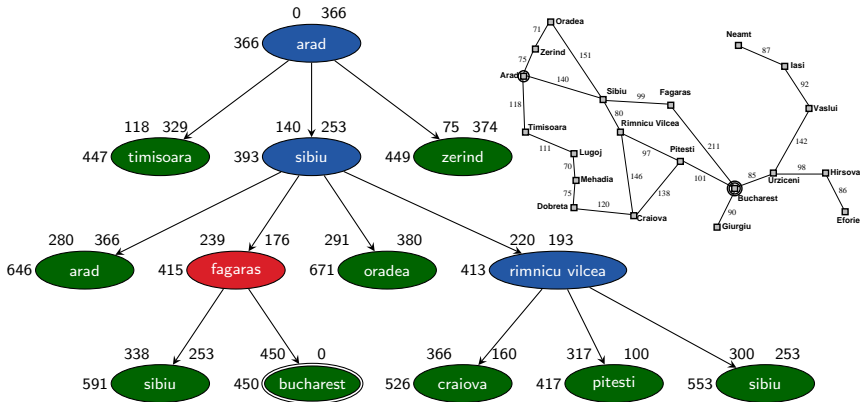
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Example A* for Route Planning



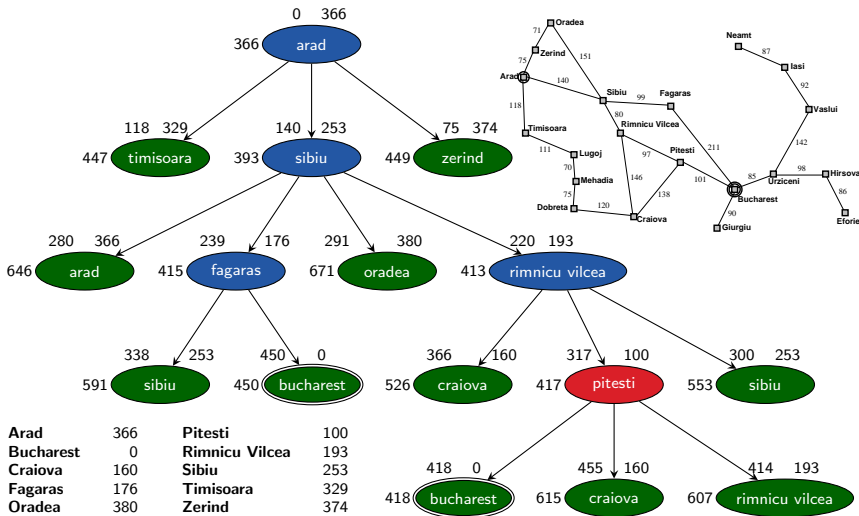
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Example A* for Route Planning



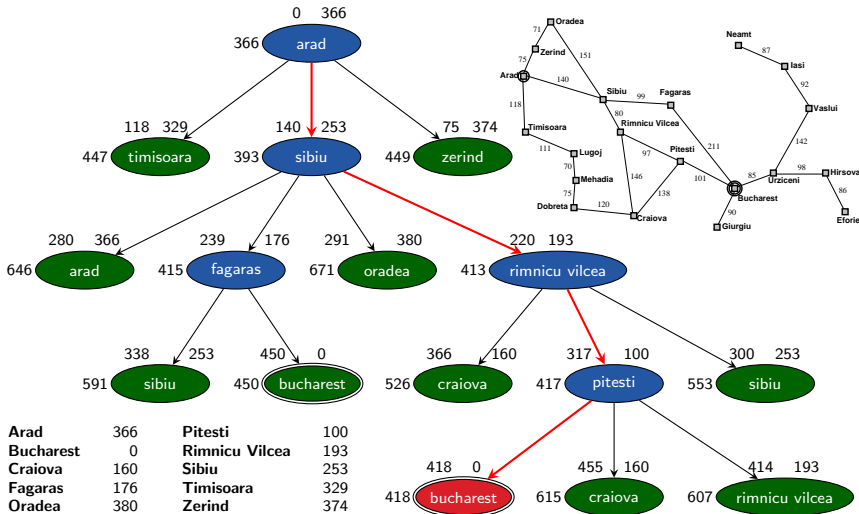
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Example A* for Route Planning



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Example A* for Route Planning



A*: Properties

- **complete** with **safe** heuristics
(like all variants of best-first graph search)
- **with reopening: optimal** with **admissible** heuristics
- **without reopening: optimal** with heuristics
that are **admissible** and **consistent**

↪ proofs: Chapters B14 and B15

A*: Implementation Aspects

some practical remarks on implementing A*:

- **common bug:** reopening not implemented although heuristic is not consistent
- **common bug:** duplicate test “too early” (upon generation of search nodes)
- **common bug:** goal test “too early” (upon generation of search nodes)
- all these bugs lead to loss of optimality and can remain undetected for a long time

Weighted A*

Weighted A*

Weighted A*

A* with more heavily weighted heuristic:

$$f(n) = g(n) + w \cdot h(n.state),$$

where **weight** $w \in \mathbb{R}_0^+$ with $w \geq 1$ is a freely choosable parameter

Note: $w < 1$ is conceivable, but usually not a good idea
(Why not?)

Weighted A*: Properties

weight parameter controls “greediness” of search:

- $w = 0$: like uniform cost search
- $w = 1$: like A*
- $w \rightarrow \infty$: like greedy best-first search

with $w \geq 1$ properties analogous to A*:

- **h admissible:**
found solution guaranteed to be at most w times
as expensive as optimum when reopening is used
- **h admissible and consistent:**
found solution guaranteed to be at most w times
as expensive as optimum; no reopening needed

(without proof)

Summary

Summary

best-first graph search with evaluation function f :

- $f = h$: **greedy best-first search**
suboptimal, often very fast
- $f = g + h$: **A***
optimal if h admissible and consistent
or if h admissible and **reopening** is used
- $f = g + w \cdot h$: **weighted A***
for $w \geq 1$ suboptimality factor at most w
under same conditions as for optimality of A*