Foundations of Artificial Intelligence
B10. State-Space Search: Analysis of Heuristics

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B10.1 Properties of Heuristics

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State-Space Search: Overview

Chapter overview: state-space search

- ▶ B1–B3. Foundations
- ▶ B4–B8. Basic Algorithms
- ▶ B9–B15. Heuristic Algorithms
  - ▶ B9. Heuristics
  - ▶ B10. Analysis of Heuristics
  - ▶ B11. Best-first Graph Search
  - ▶ B12. Greedy Best-first Search, A*, Weighted A*
  - ▶ B13. IDA*
  - ▶ B14. Properties of A*, Part I
  - ▶ B15. Properties of A*, Part II

Reminder: Heuristics

> ### Definition (heuristic)
> Let $\mathcal{S}$ be a state space with states S.
> A heuristic function or heuristic for $\mathcal{S}$ is a function
>
> $$h : S \to \mathbb{R}_0^+ \cup \{\infty\},$$
>
> mapping each state to a nonnegative number (or ∞).

B10.1 Properties of Heuristics

Perfect Heuristic

Definition (perfect heuristic)

Let $\mathcal{S}$ be a state space with states S.

The perfect heuristic for $\mathcal{S}$, written h^*, maps each state $s \in S$

▶ to the cost of an optimal solution for s, or

▶ to ∞ if no solution for s exists.

German: perfekte Heuristik

Properties of Heuristics

Definition (safe, goal-aware, admissible, consistent)

Let $\mathcal{S}$ be a state space with states S.

A heuristic h for $\mathcal{S}$ is called

- ▶ safe if $h^*(s) = \infty$ for all $s \in S$ with $h(s) = \infty$
- ▶ goal-aware if $h(s) = 0$ for all goal states s
- ▶ admissible if $h(s) \leq h^*(s)$ for all states $s \in S$
- ▶ consistent if $h(s) \leq cost(a) + h(s')$ for all transitions $s \xrightarrow{a} s'$

German: sicher, zielerkennend, zulässig, konsistent
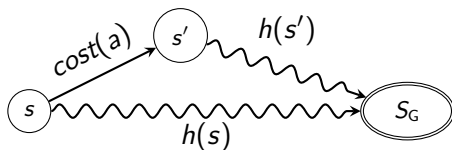
B10.2 Examples

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Route Planning in Romania

straight-line distance:

▶ safe

▶ goal-aware

▶ admissible

▶ consistent

Why?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

> ### Blocks World
>
> misplaced blocks:
> - ► safe?
> - ► goal-aware?
> - ► admissible?
> - ► consistent?

Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Missionaries and Cannibals

people on wrong river bank:

▶ safe?

▶ goal-aware?

▶ admissible?

▶ consistent?

B10.3 Connections

Properties of Heuristics: Connections (1)

Theorem (admissible $\Longrightarrow$ safe + goal-aware)

Let h be an admissible heuristic.

Then h is safe and goal-aware.

Why?

Properties of Heuristics: Connections (2)

Theorem (goal-aware + consistent $\implies$ admissible)

Let h be a goal-aware and consistent heuristic.

Then h is admissible.

Why?

Showing All Four Properties

How can one show most easily that a heuristic
has all four properties?

B10.4 Summary

Summary

- perfect heuristic h^*: true cost to the goal
- important properties: safe, goal-aware, admissible, consistent
- connections between these properties
    - admissible $\implies$ safe and goal-aware
    - goal-aware and consistent $\implies$ admissible