Foundations of Artificial Intelligence

B3. State-Space Search: Examples of State Spaces

Malte Helmert

University of Basel

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State-Space Search: Overview

Chapter overview: state-space search

- B1–B3. Foundations
  - B1. State Spaces
  - B2. Representation of State Spaces
  - B3. Examples of State Spaces
- B4–B8. Basic Algorithms
- B9–B15. Heuristic Algorithms

Three Examples

In this chapter we introduce three state spaces that we will use as illustrating examples:

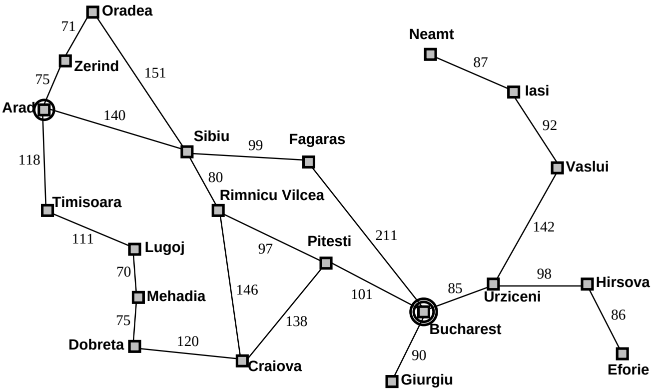
1. route planning in Romania
2. blocks world
3. missionaries and cannibals

Route Planning in Romania

Route Planning in Romania

Setting: Route Planning in Romania

We are on holiday in Romania and are currently located in Arad. Our flight home leaves from Bucharest. How to get there?



Romania Formally

State Space Route Planning in Romania

- states S: $\{\text{arad}, \text{bucharest}, \text{craiova}, \ldots, \text{zerind}\}$
- actions A: $move_{c,c'}$ for any two cities c and c' connected by a single road segment
- action costs $cost$: see figure, e.g., $cost(move_{\text{iasi},\text{vaslui}}) = 92$
- transitions T: $s \xrightarrow{a} s'$ iff $a = move_{s,s'}$
- initial state: $s_\text{I} = \text{arad}$
- goal states: $S_\text{G} = \{\text{bucharest}\}$

Blocks World

Blocks World

Blocks world is a traditional example problem in AI.

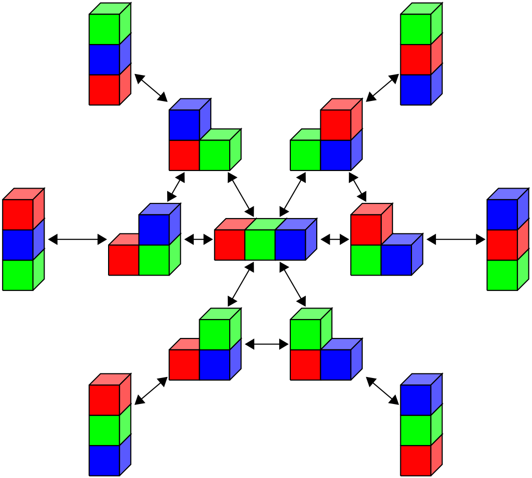
Setting: Blocks World

- Colored blocks lie on a table.
- They can be stacked into towers, moving one block at a time.
- Our task is to create a given goal configuration.

Example: Blocks World with Three Blocks

Action names omitted for readability. All actions cost 1.

Initial state and goal can be arbitrary.



Blocks World: Formal Definition

state space $\langle S, A, cost, T, s_I, S_G \rangle$ for blocks world with n blocks

State Space Blocks World

states S:

partitions of $\{1, 2, \dots, n\}$ into nonempty ordered lists

example $n = 3$:

- $\{\langle 1,2,3 \rangle\}$, $\{\langle 1,3,2 \rangle\}$, $\{\langle 2,1,3 \rangle\}$, $\{\langle 2,3,1 \rangle\}$, $\{\langle 3,1,2 \rangle\}$, $\{\langle 3,2,1 \rangle\}$
- $\{\langle 1,2 \rangle, \langle 3 \rangle\}$, $\{\langle 2,1 \rangle, \langle 3 \rangle\}$, $\{\langle 1,3 \rangle, \langle 2 \rangle\}$, $\{\langle 3,1 \rangle, \langle 2 \rangle\}$, $\{\langle 2,3 \rangle, \langle 1 \rangle\}$, $\{\langle 3,2 \rangle, \langle 1 \rangle\}$
- $\{\langle 1 \rangle, \langle 2 \rangle, \langle 3 \rangle\}$

Blocks World: Formal Definition

state space $\langle S, A, cost, T, s_\mathrm{I}, S_\mathrm{G} \rangle$ for blocks world with n blocks

State Space Blocks World

actions A:

- $\{move_{u,v} \mid u, v \in \{1, \dots, n\}$ with $u \neq v\}$
  - move block u onto block v.
  - both must be uppermost blocks in their towers
- $\{\textit{to-table}_u \mid u \in \{1, \dots, n\}\}$
  - move block u onto the table ($\rightsquigarrow$ forming a new tower)
  - must be uppermost block in its tower

action costs $cost$:

$cost(a) = 1$ for all actions $a \in A$

Blocks World: Formal Definition

state space $\langle S, A, cost, T, s_I, S_G \rangle$ for blocks world with n blocks

State Space Blocks World

transitions:

- transition $s \xrightarrow{a} s'$ with $a = move_{u,v}$ exists iff
  - $s = \{\langle b_1, \dots, b_k, u \rangle, \langle c_1, \dots, c_m, v \rangle\} \cup X$ and
  - if $k > 0$: $s' = \{\langle b_1, \dots, b_k \rangle, \langle c_1, \dots, c_m, v, u \rangle\} \cup X$
  - if $k = 0$: $s' = \{\langle c_1, \dots, c_m, v, u \rangle\} \cup X$
- transition $s \xrightarrow{a} s'$ with $a = \textit{to-table}_u$ exists iff
  - $s = \{\langle b_1, \dots, b_k, u \rangle\} \cup X$ and
  - if $k > 0$: $s' = \{\langle b_1, \dots, b_k \rangle, \langle u \rangle\} \cup X$
  - if $k = 0$: $s' = s$

Blocks World: Formal Definition

state space $\langle S, A, cost, T, s_I, S_G \rangle$ for blocks world with n blocks

State Space Blocks World

initial state s_I and goal states S_G:

one possible scenario for $n = 3$:

- $s_I = \{\langle 1, 3\rangle, \langle 2\rangle\}$
- $S_G = \{\{\langle 3, 2, 1\rangle\}\}$

(in general can have arbitrary scenarios)

Blocks World: Properties

blocks	states	blocks	states
1	1	10	58941091
2	3	11	824073141
3	13	12	12470162233
4	73	13	202976401213
5	501	14	3535017524403
6	4051	15	65573803186921
7	37633	16	1290434218669921
8	394353	17	26846616451246353
9	4596553	18	588633468315403843

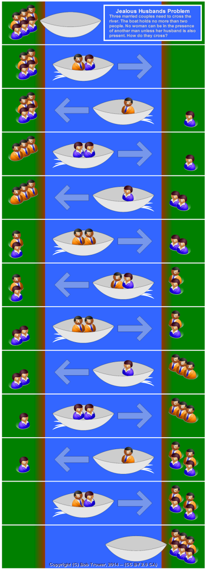
- For every given initial and goal state with n blocks, simple algorithms find a solution in time $O(n)$. (How?)
- Finding optimal solutions is NP-complete (with a compact problem description).

Missionaries and Cannibals

Missionaries and Cannibals

Setting: Missionaries and Cannibals

- Six people must cross a river.
- Their rowing boat can carry one or two people across the river at a time. (It is too small for three.)
- Three people are missionaries, three are cannibals.
- Missionaries may never stay with a majority of cannibals.



Missionaries and Cannibals Formally

State Space Missionaries and Cannibals

states S:

triples of numbers $\langle m, c, b\rangle \in \{0, 1, 2, 3\} \times \{0, 1, 2, 3\} \times \{0, 1\}$:

- number of missionaries m,
- cannibals c and
- boats b

on the left river bank

initial state: $s_{\mathrm{I}} = \langle 3, 3, 1\rangle$

goal: $S_{\mathrm{G}} = \{\langle 0, 0, 0\rangle, \langle 0, 0, 1\rangle\}$

actions, action costs, transitions: ?

Summary

Summary

illustrating examples for state spaces:

- route planning in Romania:
  - small example of explicitly representable state space
- blocks world:
  - family of tasks where n blocks on a table must be rearranged
  - traditional example problem in AI
  - number of states explodes quickly as n grows
- missionaries and cannibals:
  - traditional brain teaser with small state space
    (32 states, of which many unreachable)