Theory of Computer Science B12. Type-1 and Type-0 Languages: Closure & Decidability

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Turing Machines vs. Grammars

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B12.1 Turing Machines vs. Grammars

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B12.1 Turing Machines vs. Grammars

B12.2 Closure Properties and Decidability

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Turing Machines vs. Grammars

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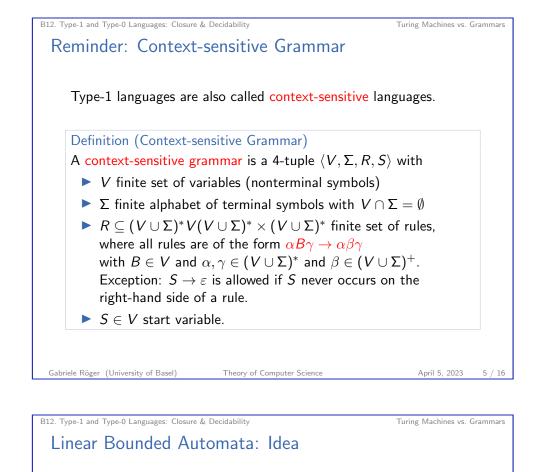
Turing Machines

We have seen several variants of Turing machines:

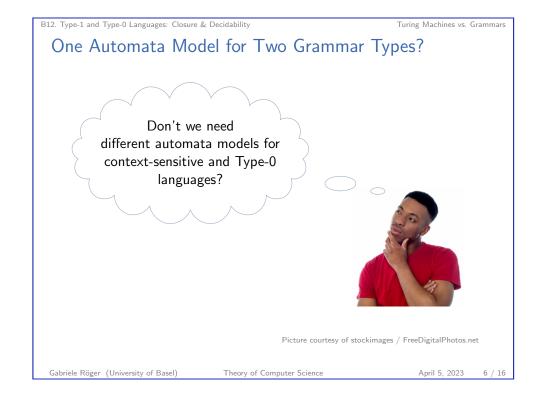
- Deterministic TM with head movements left or right
- Deterministic TM with head movements left, right or neutral
- Multitape Turing machines
- Nondeterministic Turing machines

All variants recognize the same languages.

We mentioned earlier that we can relate Turing machines to the Type-1 and Type-0 languages.



- Linear bounded automata are NTMs that may only use the part of the tape occupied by the input word.
- one way of formalizing this: NTMs where blank symbol may never be replaced by a different symbol





Definition (Linear Bounded Automata) An NTM $M = \langle Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}} \rangle$ is called a linear bounded automaton (LBA) if for all $q \in Q \setminus \{q_{\text{accept}}, q_{\text{reject}}\}$ and all transition rules $\langle q', c, y \rangle \in \delta(q, \Box)$ we have $c = \Box$.

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LBAs Recognize Type-1 Languages

Theorem

The languages that can be recognized by linear bounded automata are exactly the context-sensitive (type-1) languages.

Without proof.

proof sketch for grammar \Rightarrow NTM direction:

computation of the NTM follows the production of the word in the grammar in opposite order

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- accept when only the start symbol (and blanks) are left on the tape
- because the language is context-sensitive, we never need additional space on the tape (empty word needs special treatment)

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What about the Deterministic Variants?

We know that DTMs and NTMs recognize the same languages. Hence:

Corollary

The Turing-recognizable languages are exactly the Type-0 languages.

Note: It is an open problem whether deterministic LBAs can recognize exactly the type-1 languages.

NTMs Recognize Type-0 Languages

Theorem

The languages that can be recognized by nondeterministic Turing machines are exactly the type-0 languages.

Without proof.

proof sketch for grammar \Rightarrow NTM direction:

- analogous to previous proof
- For grammar rules w₁ → w₂ with |w₁| > |w₂|, we must "insert" symbols into the existing tape content; this is a bit tedious, but not very difficult

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Closure Properties and Decidability

B12.2 Closure Properties and Decidability

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Closure Properties and Decidability

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Summary

Closure Properties

	Intersection	Union	Complement	Concatenation	Star
Type 3	Yes	Yes	Yes	Yes	Yes
Type 2	No	Yes	No	Yes	Yes
Type 1	Yes ⁽²⁾	$Yes^{(1)}$	Yes ⁽²⁾	Yes ⁽¹⁾	Yes ⁽¹⁾
Type 0	Yes ⁽²⁾	$Yes^{(1)}$	No ⁽³⁾	Yes ⁽¹⁾	$Yes^{(1)}$

Proofs?

(1) proof via grammars, similar to context-free cases

(2) without proof

(3) proof in later chapters (part C)

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Summary

- Turing machines recognize exactly the type-0 languages.
- Linear bounded automata recognize exactly the context-sensitive languages.
- The context-sensitive and type-0 languages are closed under almost all usual operations.
 - exception: type-0 not closed under complement
- For context-sensitive and type-0 languages almost no problem is decidable.
 - exception: word problem for context-sensitive lang. decidable

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Closure Properties and Decidability

Decidability

	Word problem	Emptiness problem	Equivalence problem	Intersection problem					
Туре 3	Yes	Yes	Yes	Yes					
Type 2	Yes	Yes	No	No					
Type 1	Yes ⁽¹⁾	No ⁽³⁾	No ⁽²⁾	No ⁽²⁾					
Type 0	No ⁽⁴⁾	No ⁽⁴⁾	No ⁽²⁾	No ⁽²⁾					
 Proofs? (1) same argument we used for context-free languages (2) because already undecidable for context-free languages (3) without proof (4) proofs in later chapters (part C) 									
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What's Next?

contents of this course:

- A. background √
 ▷ mathematical foundations and proof techniques
 B. automata theory and formal languages √
 - \triangleright What is a computation?
- C. Turing computability
 - \triangleright What can be computed at all?
- D. complexity theory▷ What can be computed efficiently?
- E. more computability theory
 - ▷ Other models of computability

Summarv